PMU-based Model-free Approach for Real-time Rotor Angle Monitoring

Sambarta Dasgupta, *Student Member, IEEE*, Magesh Paramasivam, *Student Member, IEEE*, Umesh Vaidya, *Member, IEEE* and Venkataramana Ajjarapu, *Fellow, IEEE*

***Abstract*—In this paper, we present an algorithm for rotor angle stability monitoring of power system in real-time. The proposed algorithm is model-free and can make use of high resolution Phasor Measurement Units (PMUs) data to provide reliable, timely information about the system's stability. The theoretical basis behind the proposed algorithm is adopted from dynamical systems theory. In particular, the algorithm approximately computes the system's Lyapunov exponent (LE), thereby measuring the exponential convergence or divergence rate of the rotor angle trajectories. The LE serves as a certificate of stability where the positive (negative) value of the LE implies exponential divergence (convergence) of nearby system trajectories; hence, unstable (stable) rotor angle dynamics. We also show the proposed model-free algorithm can be used for the identification of the coherent sets of generators. The simulation results are presented to verify the developed results in the paper on the modified IEEE 162 bus system.**

***Index Terms*—Online Stability Monitoring, Lyapunov Exponents, Transient Stability.**

I. Introduction

Advancement in sensing technologies in the form of PMUs has made it possible to obtain high resolution, real-time dynamic state information of the power grid. This advancement has presented us with a unique opportunity to develop methods for real-time monitoring and control of the power grid. There are increased research efforts in the community to address stability monitoring and control problems [1]. However, some serious challenges remain to enable the PMU-based sensing technology for real-time monitoring and control. The short-term stability or transient stability problem for the power grid occurs over a very short $4-10$ sec time period following a fault. This relatively short time period combined with the large size of the power network makes it difficult to develop a reliable method and provide timely information about the system's stability. Existing methods employing the power system model are not particularly appropriate for real-time stability monitoring application because of the computational complexity associated with the system's size. Furthermore, presence of various sources of parametric and modeling uncertainties in power system dynamics could also be a cause for unreliable stability prediction.

To circumvent the problem associated with using this model, we present a novel model-free algorithm for real-time rotor angle stability monitoring. The LE from the ergodic theory of dynamical systems theory is used as the stability certificate. The positive (negative) value of largest LE implies exponential divergence (convergence) of nearby system trajectories; hence, unstable (stable) rotor angle dynamics [2]. LE has been utilized in [3] to analyze the rotor angle stability. Furthermore, an LE-based algorithm has been proposed to identify coherent generators in [4]. In both [3] and [4], the model-based algorithm is proposed for LE computation. In [2], a model-free LE computation from the time series has been proposed for short-term voltage stability monitoring. In this work, we extend the algorithm, proposed in [2], for real-time rotor angle stability monitoring.

Authors are with the Department of Electrical and Computer Engineering Iowa State University, Ames, Iowa 50011. U. Vaidya and S. Dasgupta would like to acknowledge financial support from the National Science Foundation CAREER grant ECCS 1150405 and PSERC grant S-50. Email: dasgupta@iastate.edu, mageshp@iastate.edu, ugvaidya@iastate.edu, and vajjarap@iastate.edu.

II. Algorithm for LE Computation

The transient stability is defined as the convergence of the angle difference between the pair of generators. Let there be n number of generators in the network. The generator angle time series can be denoted as $[\theta_1(t), \theta_2(t), \ldots, \theta_n(t)]^T \in \mathbb{R}^n$, where $t = 0, \Delta t, 2\Delta t, \ldots, M\Delta t$, where Δt is the sampling period. Next, the angle stability is defined following [5].

Definition 1. *The system shows asymptotic angle stability if for all i, j, there exists a $\kappa_{ij} < \infty$ such that,* $\lim_{t\to\infty}|\theta_i(t) - \theta_j(t)| = \kappa_{ij}$.

Following Definition 1, we observe for rotor angle stability the required relative angle differences tend to be a constant value for all possible pairs. We take one of the angles, say $\theta_R(t)$, as reference and define the relative angle difference with respect to $\theta_R(t)$ as follows, $\hat{\theta}_i^R(t) = \theta_i(t) - \theta_R(t), i = 1, \ldots, n, \quad i \neq R$. The system achieves stability, if each of $\hat{\theta}_i^R(t)$ converges to a constant value. In [2], it has been demonstrated for stable (unstable) time series the LE goes negative (positive). Here, we have extended the algorithm described in [2] to rotor angles in the following manner.

1) For fixed small numbers, $0 < \varepsilon_1 < \varepsilon_2$, choose N initial conditions, such that $\varepsilon_1 < |\hat{\theta}_i^R(m\Delta t) - \hat{\theta}_i^R((m-1)\Delta t)| < \varepsilon_2$ for $m = 1, 2, \ldots, N$.

2) Define the maximum Lyapunov exponent at time, $k\Delta t$, using the following formula $\lambda_i^R(k\Delta t) := \frac{1}{Nk\Delta t}\sum_{m=1}^{N} \log \frac{|\hat{\theta}_i^R((k+m)\Delta t) - \hat{\theta}_i^R((k+m-1)\Delta t)|}{|\hat{\theta}_i^R(m\Delta t) - \hat{\theta}_i^R((m-1)\Delta t)|}$,

where, $k > N$. The system is transient stable if $\lambda_i^R(k\Delta t)$ is negative for all $i = 1, \ldots n$, for sufficiently large k. The number of initial conditions, N, is a function of parameters ε_1 and ε_2, which in turn, is a function of the sampling frequency.

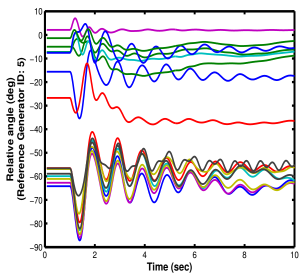

Fig. 1. Relative angles ($t_f = 0.08$s)

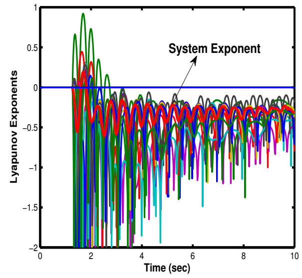

Fig. 2. Evolution of LE - Stable

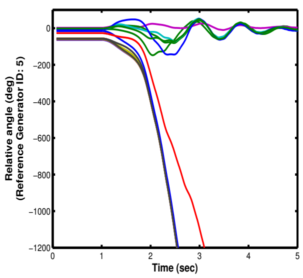

Fig. 3. Relative angles ($t_f = 0.23$s)

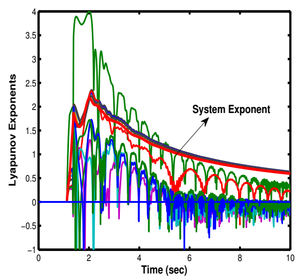

Fig. 4. Evolution of LE - Unstable

The model-based methods proposed in [3] and [4] would be computationally expensive, due to the large system size requiring system-wide rotor angle measurements or their estimated values. On the other hand, the proposed model-free algorithm only requires arithmetic and logarithmic operations. Furthermore, for our proposed algorithm to work, we do not require system-wide rotor angle measurement. With few rotor angle measurements, indeed, stability predictions are restricted to the generators for which the angle measurements are available. More specifically, with few angle measurements our proposed algorithm can identify whether the generators under consideration are moving in synchronism or not. Furthermore, there may be various sources of error in the power system model coming from un-modeled dynamics and parametric uncertainty. These limitations of the model-based approach are overcome using our algorithm.

III. Simulation Results

Simulations have been performed in the IEEE 162 bus system. The test system has 17 generators, 111 loads, 34 shunts, and 238 branches. The power flow and dynamics data for the 162 bus system are available in [6]. For a more accurate load representation, 22 load buses were stepped down through distribution transformers to the 12.47 kV level. The new low voltage buses were assigned numbers 163 through 184. To capture the dynamic behavior of motor loads, the composite load model represented by CMDL [7] was utilized at the new representative load buses in the dynamic simulation studies. The sampling frequency used for the computation of the LE was 2 samples per cycle (120 Hz) and the values of ε_1 and ε_2 are chosen as 0.001 and 0.01 respectively.

A 3-phase fault was created at bus 75 at time, t=1 second. The fault was cleared after 0.08 sec by opening the line between buses 75 and 9. Figure 1 shows the relative difference of all generator angles with respect to the angle corresponding to the generator index 5. Figure 2 shows the corresponding LE evolution, where all LEs become negative, indicating stable behavior. For the same fault scenario if the fault duration (t_f) is 0.23 sec, then the system shows unstable behavior, observed from Fig. 3. The corresponding LEs are shown in Fig. 4. It can be observed that some of the LEs are positive. The conclusion drawn from LEs is the system is unstable. It can be noticed from Fig. 2, and 4, the proposed algorithm can accurately identify stability or instability, using the sign of LE within a time window of 2.5 sec. This demonstrates our algorithm is capable of early detection of instability.

For the unstable scenario, our proposed method can also be used for online identification of generator pairs going out of synchronism. Figure 5 shows the exponent values for generator pairs computed after 4 sec. The rows and columns correspond to the indices of generators in the system. It can be observed the instability is accurately predicted as some of the generator pairs have positive exponents. Furthermore, the generators have been partitioned into two groups, where two generators belonging to the same (different) group have negative (positive) LE values. From Fig. 5, it can be observed the generator pairs with negative (positive) exponents are achieving (losing) synchronism. Comparing Fig. 5 with Fig. 6, we observe the stability prediction and coherent group identification at 4 sec matches with those for the ones computed at 10 sec. Therefore, our algorithm opens the opportunity of identifying the coherent set of generators in transient.

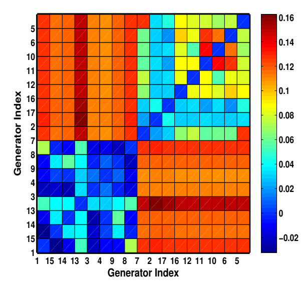

Fig. 5. LEs for generator pairs, after clustering, at $t = 4$ sec.

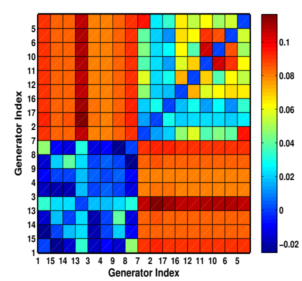

Fig. 6. LEs for generator pairs, after clustering, at $t = 10$ sec.

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