- Laplace and Z Transforms
- Laplace Properties
- Z Xform Properties

Table of Laplace and Z Transforms

(Please email me if you find an error)
Using this table for Z Transforms with Discrete Indices
Shortened 2-page pdf of Laplace Transforms and Properties

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		b^k $\left(b=e^{-aT}\right)$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{\left(z-e^{-aT}\right)^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$	$\frac{z\left(1-e^{-aT}\right)}{a(z-1)\left(z-e^{-aT}\right)}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z\left(e^{-aT}-e^{-bT}\right)}{\left(z-e^{-aT}\right)\left(z-e^{-bT}\right)}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}\left(1-e^{-at}-ate^{-at}\right)$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	

12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
16	$\frac{Bs+C}{(s+a)^2+\omega_n^2}$	$e^{-at}\left[B\cos(\omega_n t)+\frac{C-aB}{\omega_n}\sin(\omega_n t)\right]$ **Bad notation. Better to use *ω_d*.**	
17	**???**	$\sqrt{\frac{a^2d^2+b^2-2abc}{d^2-c^2}}\,d^n\cos(\beta n+\gamma)$ $\beta=\cos^{-1}\left(-\frac{c}{d}\right),\quad \gamma=\tan^{-1}\left(\frac{ac-b}{a\sqrt{d^2-c^2}}\right)$	$\frac{az^2+bz}{z^2+2cz+d^2},\quad d>0$
		Prototype Second Order System ($\zeta<1$, underdamped)	
18	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\cos^{-1}(\zeta)\right)$	
19	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t\right)$	
20	$\frac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t-\cos^{-1}(\zeta)\right)$	

Using this table for Z Transforms with discrete indices

Commonly the "time domain" function is given in terms of a discrete index, k, rather than time. This is easily accommodated by the table. For example if you are given a function:

$$f[k] = k$$

Since t=kT, simply replace k in the function definition by k=t/T. So, in this case,

$$f[k] = \frac{t}{T}$$

and we can use the table entry for the ramp

$$t \overset{\mathcal{Z}}{\longleftrightarrow} \frac{Tz}{(z-1)^2}$$

The answer is then easily obtained

$$f[k] = k = \frac{t}{T} \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z}{(z-1)^2} = F(z)$$

First Derivative	$\frac{df(t)}{dt} \overset{\mathcal{L}}{\leftrightarrow} sF(s) - f(0^-)$
Second Derivative	$\frac{d^2f(t)}{dt^2} \overset{\mathcal{L}}{\leftrightarrow} s^2F(s) - sf(0^-) - \dot{f}(0^-)$
n^{th} Derivative	$\frac{d^nf(t)}{dt^n} \overset{\mathcal{L}}{\leftrightarrow} s^nF(s) - \sum_{i=1}^{n} s^{n-i}f^{(i-1)}(0^-)$
Integration	$\int_0^t f(\lambda)d\lambda \overset{\mathcal{L}}{\leftrightarrow} \frac{1}{s}F(s)$
Multiplication by time	$tf(t) \overset{\mathcal{L}}{\leftrightarrow} -\frac{dF(s)}{ds}$
Time Shift	$f(t-a)\gamma(t-a) \overset{\mathcal{L}}{\leftrightarrow} e^{-as}F(s)$ $(\gamma(t) = \text{unit step function})$
Complex Shift	$f(t)e^{-at} \overset{\mathcal{L}}{\leftrightarrow} F(s+a)$
Time Scaling	$f\left(\frac{t}{a}\right) \overset{\mathcal{L}}{\leftrightarrow} aF(as)$
Convolution ('*' denotes convolution of functions)	$f_1(t) * f_2(t) \overset{\mathcal{L}}{\leftrightarrow} F_1(s)F_2(s)$
Initial Value Theorem	$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$
Final Value Theorem	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$