

CHAPTER 2

Static Stability and Control

"Isn't it astonishing that all these secrets have been preserved for so many years just so that we could discover them!"

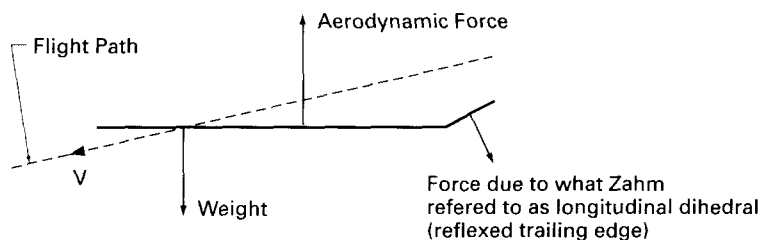
Orville Wright, June 7, 1903

2.1

HISTORICAL PERSPECTIVE

By the start of the 20th century, the aeronautical community had solved many of the technical problems necessary for achieving powered flight of a heavier-than-air aircraft. One problem still beyond the grasp of these early investigators was a lack of understanding of the relationship between stability and control as well as the influence of the pilot on the pilot-machine system. Most of the ideas regarding stability and control came from experiments with uncontrolled hand-launched gliders. Through such experiments, it was quickly discovered that for a successful flight the glider had to be inherently stable. Earlier aviation pioneers such as Albert Zahm in the United States, Alphonse Penaud in France, and Frederick Lanchester in England contributed to the notion of stability. Zahm, however, was the first to correctly outline the requirements for static stability in a paper he presented in 1893. In his paper, he analyzed the conditions necessary for obtaining a stable equilibrium for an airplane descending at a constant speed. Figure 2.1 shows a sketch of a glider from Zahm's paper. Zahm concluded that the center of gravity had to be in front of the aerodynamic force and the vehicle would require what he referred to as "longitudinal dihedral" to have a stable equilibrium point. In the terminology of today, he showed that, if the center of gravity was ahead of the wing aerodynamic center, then one would need a reflexed airfoil to be stable at a positive angle of attack.

In the 20 years prior to the Wright brothers' successful flight, many individuals in the United States and Europe were working with gliders and unpiloted powered models. These investigators were constantly trying to improve their vehicles, with the ultimate goal of achieving powered flight of an airplane under human control. Three men who would leave lasting impressions on the Wright brothers were Otto Lilienthal of Germany and Octave Chanute and Samuel Pierpont Langley of the United States.

**FIGURE 2.1**

Zahm's description of longitudinal stability.

Lilienthal made a significant contribution to aeronautics by his work with model and human-carrying gliders. His experiments included the determination of the properties of curved or cambered wings. He carefully recorded the details of over 2000 glider flights. The information in his journal includes data on materials, construction techniques, handling characteristics of his gliders, and aerodynamics. His successful flights and recorded data inspired and aided many other aviation pioneers. Lilienthal's glider designs were statically stable but had very little control capability. For control, Lilienthal would shift his weight to maintain equilibrium flight, much as hang-glider pilots do today. The lack of suitable control proved to be a fatal flaw for Lilienthal. In 1896, he lost control of his glider; the glider stalled and plunged to earth from an altitude of 50 ft. Lilienthal died a day later from the injuries incurred in the accident.

In the United States, Octave Chanute became interested in gliding flight in the mid 1890s. Initially, he built gliders patterned after Lilienthal's designs. After experimenting with modified versions of Lilienthal's gliders, he developed his own designs. His gliders incorporated biplane and multiplane wings, controls to adjust the wings to maintain equilibrium, and a vertical tail for steering. These design changes represented substantial improvements over Lilienthal's monoplane gliders. Many of Chanute's innovations would be incorporated in the Wright brothers' designs. In addition to corresponding with the Wright brothers, Chanute visited their camp at Kitty Hawk to lend his experience and advice to their efforts.

Another individual who helped the Wright brothers was Samuel Pierpont Langley, secretary of the Smithsonian Institution. The Wright brothers knew of Langley's work and wrote to the Smithsonian asking for the available aeronautical literature. The Smithsonian informed the Wright brothers of the activities of many of the leading aviation pioneers and this information, no doubt, was very helpful to them.

Around 1890 Langley became interested in problems of flight. Initially his work consisted of collecting and examining all the available aerodynamic data. From the study of these data and his own experiments he concluded that heavier-than-air powered flight was possible. Langley then turned his attention to designing and perfecting unpowered models. On May 6, 1896, his powered model flew for $1\frac{1}{2}$ minutes and covered a distance of three-quarters of a mile. Langley's success with powered models pioneered the practicality of mechanical flight.

After his successful model flights, Langley was engaged by the War Department to develop a human-carrying airplane. Congress appropriated \$50,000 for the project. Langley and his engineering assistant, Charles Manley, started work on their own design in 1899. For the next four years, they were busy designing, fabricating, and testing the full-size airplane that was to be launched by a catapult fixed to the top of a houseboat. The first trial was conducted on September 7, 1903, in the middle of the Potomac River near Tidewater, Virginia. The first attempt ended in failure as the airplane pitched down into the river at the end of the launch rails. A second attempt was made on December 8, 1903; this time, the airplane pitched up and fell back into the river. In both trials, the launching system prevented the possibility of a successful flight. For Langley, it was a bitter disappointment and the criticism he received from the press deeply troubled him. He was one of the pioneering geniuses of early aviation, however, and it is a shame that he went to his grave still smarting from the ridicule. Some 20 years later his airplane was modified, a new engine was installed, and the airplane flew successfully.

The time had come for someone to design a powered airplane capable of carrying someone aloft. As we all know, the Wright brothers made their historic first flight on a powered airplane at Kitty Hawk, North Carolina, on December 17, 1903. Orville Wright made the initial flight, which lasted only 12 seconds and covered approximately 125 feet. Taking turns operating the aircraft, Orville and Wilbur made three more flights that day. The final flight lasted 59 seconds and covered a distance of 852 feet while flying into a 20 mph headwind. The airplane tended to fly in a porpoising fashion, with each flight ending abruptly as the vehicle's landing skids struck the ground. The Wright brothers found their powered airplane to be much more responsive than their earlier gliders and, as a result, had difficulty controlling their airplane.

Figure 2.2 shows two photographs of the Kitty Hawk Flyer. The first photograph shows Orville Wright making the historical initial flight and the second shows the airplane after the fourth and last flight of the day. Notice the damaged horizontal rudder (the term used by the Wright brothers). Today we use the term canard to describe a forward control surface. The word canard comes to us from the French word that means "duck." The French used the term canard to describe an early French airplane that had its horizontal tail located far forward of the wing. They thought this airplane looked like a duck with its neck stretched out in flight.

From this very primitive beginning, we have witnessed a remarkable revolution in aircraft development. In less than a century, airplanes have evolved into an essential part of our national defense and commercial transportation system. The success of the Wright brothers can be attributed to their step-by-step experimental approach. After reviewing the experimental data of their contemporaries, the Wright brothers were convinced that additional information was necessary before a successful airplane could be designed. They embarked on an experimental program that included wind-tunnel and flight-test experiments. The Wright brothers designed and constructed a small wind tunnel and made thousands of model tests to determine the aerodynamic characteristics of curved airfoils. They also conducted thousands of glider experiments in developing their airplane. Through their study of the works of others and their own experimental investigations, the Wright

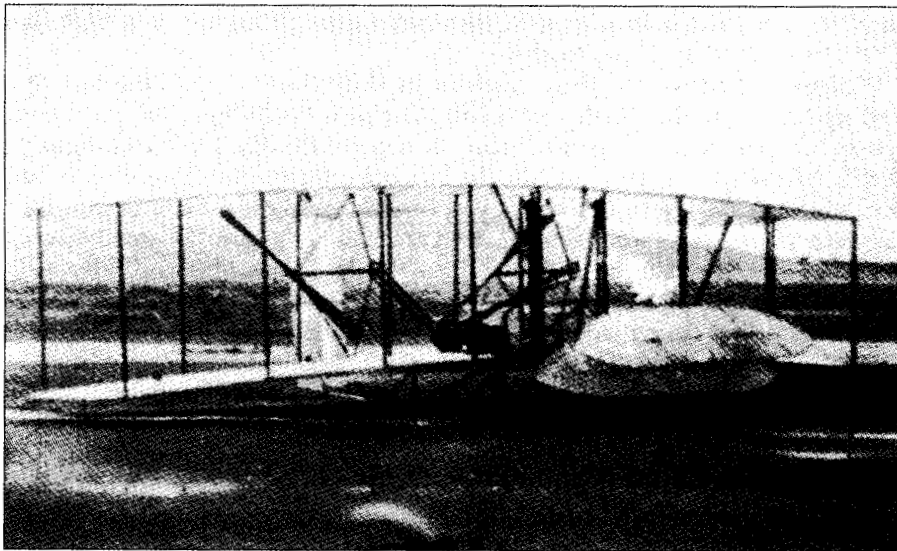
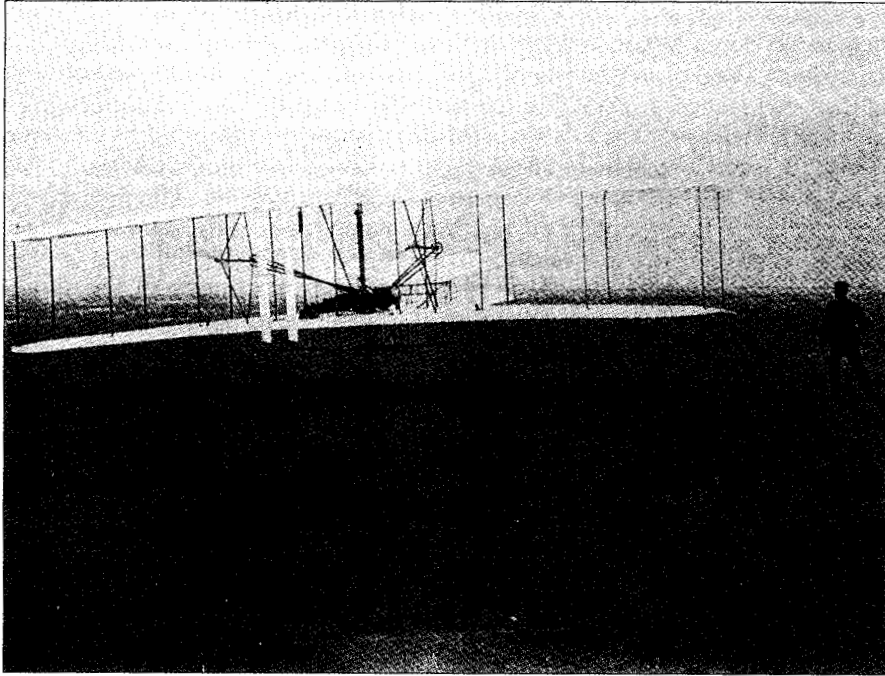


FIGURE 2.2

Photographs of the Wright brothers' airplane, December 17, 1903, Kitty Hawk, North Carolina.

brothers were convinced that the major obstacle to achieving powered flight was the lack of sufficient control. Therefore, much of their work was directed toward improving the control capabilities of their gliders. They felt strongly that powerful controls were essential for the pilot to maintain equilibrium and prevent accidents such as the ones that caused the deaths of Lilienthal and other glider enthusiasts.

This approach represented a radical break with the design philosophy of the day. The gliders and airplanes designed by Lilienthal, Chanute, Langley, and other aviation pioneers were designed to be inherently stable. In these designs, the pilot's only function was to steer the vehicle. Although such vehicles were statically stable, they lacked maneuverability and were susceptible to upset by atmospheric disturbances. The Wright brothers' airplane was statically unstable but quite maneuverable. The lack of stability made their work as pilots very difficult. However, through their glider experiments they were able to teach themselves to fly their unstable airplane.

The Wright brothers succeeded where others failed because of their dedicated scientific and engineering efforts. Their accomplishments were the foundation on which others could build. Some of the major accomplishments follow:

1. They designed and built a wind-tunnel and balance system to conduct aerodynamic tests. With their tunnel they developed a systematic airfoil aerodynamic database.
2. They developed a complete flight control system with adequate control capability.
3. They designed a lightweight engine and an efficient propeller.
4. Finally, they designed an airplane with a sufficient strength-to-weight ratio, capable of sustaining powered flight.

These early pioneers provided much of the understanding we have today regarding static stability, maneuverability, and control. However, it is not clear whether any of these men truly comprehended the relationship among these topics.

2.2 INTRODUCTION

How well an airplane flies and how easily it can be controlled are subjects studied in aircraft stability and control. By stability we mean the tendency of the airplane to return to its equilibrium position after it has been disturbed. The disturbance may be generated by the pilot's actions or atmospheric phenomena. The atmospheric disturbances can be wind gusts, wind gradients, or turbulent air. An airplane must have sufficient stability that the pilot does not become fatigued by constantly having to control the airplane owing to external disturbances. Although airplanes with little or no inherent aerodynamic stability can be flown, they are unsafe to fly unless they are provided artificial stability by an electromechanical device called a stability augmentation system.

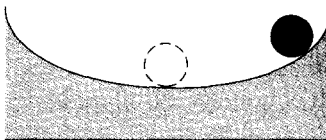
Two conditions are necessary for an airplane to fly its mission successfully. The airplane must be able to achieve equilibrium flight and it must have the capability

to maneuver for a wide range of flight velocities and altitudes. To achieve equilibrium or perform maneuvers, the airplane must be equipped with aerodynamic and propulsive controls. The design and performance of control systems is an integral part of airplane stability and control.

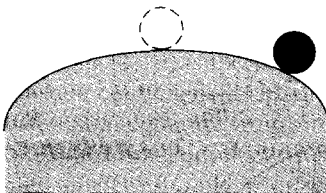
The stability and control characteristics of an airplane are referred to as the vehicle's handling or flying qualities. It is important to the pilot that the airplane possesses satisfactory handling qualities. Airplanes with poor handling qualities will be difficult to fly and could be dangerous. Pilots form their opinions of an airplane on the basis of its handling characteristics. An airplane will be considered of poor design if it is difficult to handle regardless of how outstanding the airplane's performance might be. In the study of airplane stability and control, we are interested in what makes an airplane stable, how to design the control systems, and what conditions are necessary for good handling. In the following sections we will discuss each of these topics from the point of view of how they influence the design of the airplane.

2.2.1 Static Stability

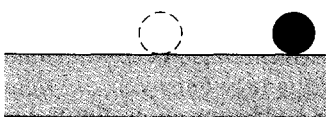
Stability is a property of an equilibrium state. To discuss stability we must first define what is meant by equilibrium. If an airplane is to remain in steady uniform flight, the resultant force as well as the resultant moment about the center of gravity must both be equal to 0. An airplane satisfying this requirement is said to be in a state of equilibrium or flying at a trim condition. On the other hand, if the forces



(a) Statically stable



(b) Statically unstable



(c) Neutral stability

FIGURE 2.3

Sketches illustrating various conditions of static stability.

and moments do not sum to 0, the airplane will be subjected to translational and rotational accelerations.

The subject of airplane stability is generally divided into static and dynamic stability. Static stability is the initial tendency of the vehicle to return to its equilibrium state after a disturbance. An example of the various types of static stability is illustrated in Figure 2.3. If the ball were to be displaced from the bottom of the curved surface (Figure 2.3(a)), by virtue of the gravitational attraction, the ball would roll back to the bottom (i.e., the force and moment would tend to restore the ball to its equilibrium point). Such a situation would be referred to as a stable equilibrium point. On the other hand, if we were able to balance a ball on the curved surface shown in Figure 2.3(b), then any displacement from the equilibrium point would cause the ball to roll off the surface. In this case, the equilibrium point would be classified as unstable. In the last example, shown in Figure 2.3(c), the ball is placed on a flat surface. Now, if the ball were to be displaced from its initial equilibrium point to another position, the ball would remain at the new position. This would be classified as a neutrally stable equilibrium point and represents the limiting (or boundary) between static stability and static instability. The important point in this simple example is that, if we are to have a stable equilibrium point, the vehicle must develop a restoring force or moment to bring it back to the equilibrium condition.

2.2.2 Dynamic Stability

In the study of dynamic stability we are concerned with the time history of the motion of the vehicle after it is disturbed from its equilibrium point. Figure 2.4 shows several airplane motions that could occur if the airplane were disturbed from

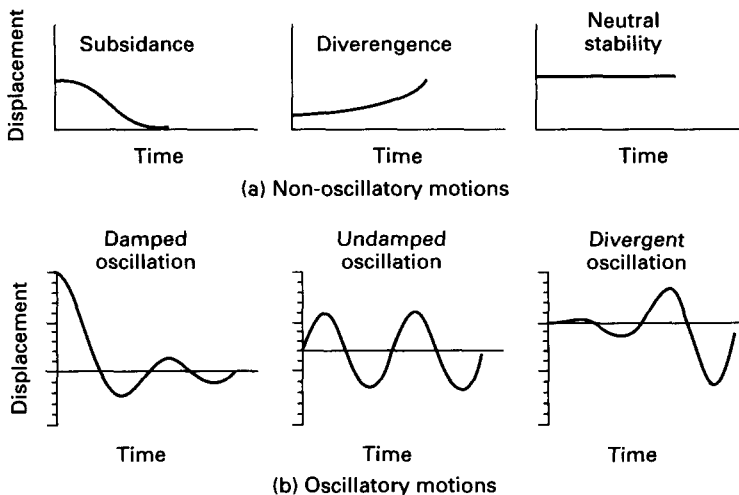


FIGURE 2.4

Examples of stable and unstable dynamic motions.

its equilibrium conditions. Note that the vehicle can be statically stable but dynamically unstable. Static stability, therefore, does not guarantee dynamic stability. However, for the vehicle to be dynamically stable it must be statically stable.

The reduction of the disturbance with time indicates that there is resistance to the motion and, therefore, energy is being dissipated. The dissipation of energy is called positive damping. If energy is being added to the system, then we have a negative damping. Positive damping for an airplane is provided by forces and moments that arise owing to the airplane's motion. In positive damping, these forces and moments will oppose the motion of the airplane and cause the disturbance to damp out with time. An airplane that has negative aerodynamic damping will be dynamically unstable. To fly such an airplane, artificial damping must be designed into the vehicle. The artificial damping is provided by a stability augmentation system (SAS). Basically, a stability augmentation system is an electromechanical device that senses the undesirable motion and moves the appropriate controls to damp out the motion. This usually is accomplished with small control movements and, therefore, the pilot's control actions are not influenced by the system.

Of particular interest to the pilot and designer is the degree of dynamic stability. Dynamic stability usually is specified by the time it takes a disturbance to be damped to half of its initial amplitude or, in the case of an unstable motion, the time it takes for the initial amplitude of the disturbance to double. In the case of an oscillatory motion, the frequency and period of the motion are extremely important.

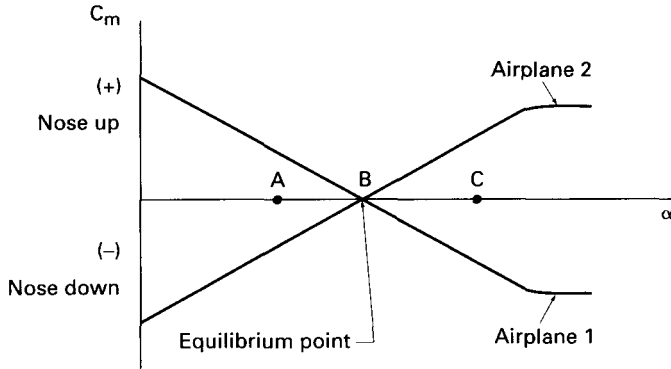
So far, we have been discussing the response of an airplane to external disturbances while the controls are held fixed. When we add the pilot to the system, additional complications can arise. For example, an airplane that is dynamically stable to external disturbances with the controls fixed can become unstable by the pilot's control actions. If the pilot attempts to correct for a disturbance and that control input is out of phase with the oscillatory motion of the airplane, the control actions would increase the motion rather than correct it. This type of pilot-vehicle response is called pilot-induced oscillation (PIO). Many factors contribute to the PIO tendency of an airplane. A few of the major contributions are insufficient aerodynamic damping, insufficient control system damping, and pilot reaction time.

2.3

STATIC STABILITY AND CONTROL

2.3.1 Definition of Longitudinal Static Stability

In the first example we showed that to have static stability we need to develop a restoring moment on the ball when it is displaced from its equilibrium point. The same requirement exists for an airplane. Let us consider the two airplanes and their respective pitching moment curves shown in Figure 2.5. The pitching moment curves have been assumed to be linear until the wing is close to stalling.

**FIGURE 2.5**

Pitching moment coefficient versus angle of attack.

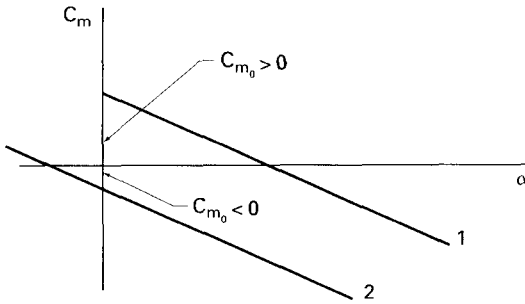
In Figure 2.5, both airplanes are flying at the trim point denoted by B ; that is, $C_{m_{cx}} = 0$. Suppose the airplanes suddenly encounter an upward gust such that the angle of attack is increased to point C . At the angle of attack denoted by C , airplane 1 would develop a negative (nose-down) pitching moment that would tend to rotate the airplane back toward its equilibrium point. However, for the same disturbance, airplane 2 would develop a positive (nose-up) pitching moment that would tend to rotate the aircraft away from the equilibrium point. If we were to encounter a disturbance that reduced the angle of attack, say, to point A , we would find that airplane 1 would develop a nose-up moment that would rotate the aircraft back toward the equilibrium point. On the other hand, airplane 2 would develop a nose-down moment that would rotate the aircraft away from the equilibrium point. On the basis of this simple analysis, we can conclude that to have static longitudinal stability the aircraft pitching moment curve must have a negative slope. That is,

$$\frac{dC_m}{d\alpha} < 0 \quad (2.1)$$

through the equilibrium point.

Another point that we must make is illustrated in Figure 2.6. Here we see two pitching moment curves, both of which satisfy the condition for static stability. However, only curve 1 can be trimmed at a positive angle of attack. Therefore, in addition to having static stability, we also must have a positive intercept, that is, $C_{m_0} > 0$ to trim at positive angles of attack. Although we developed the criterion for static stability from the C_m versus α curve, we just as easily could have accomplished the result by working with a C_m versus C_L curve. In this case, the requirement for static stability would be as follows:

$$\frac{dC_m}{dC_L} < 0 \quad (2.2)$$

**FIGURE 2.6**

Flow field around an airplane created by the wing.

The two conditions are related by the following expression:

$$C_{m_\alpha} = \frac{dC_m}{d\alpha} = \frac{dC_m}{dC_L} \frac{dC_L}{d\alpha} \quad (2.3)$$

which shows that the derivatives differ only by the slope of the lift curve.

2.3.2 Contribution of Aircraft Components

In discussing the requirements for static stability, we so far have considered only the total airplane pitching moment curve. However, it is of interest (particularly to airplane designers) to know the contribution of the wing, fuselage, tail, propulsion system, and the like, to the pitching moment and static stability characteristics of the airplane. In the following sections, each of the components will be considered separately. We will start by breaking down the airplane into its basic components, such as the wing, fuselage, horizontal tail, and propulsion unit. Detailed methods for estimating the aerodynamic stability coefficients can be found in the *United States Air Force Stability and Control Datcom* [2.7]. The *Datcom*, short for data compendium, is a collection of methods for estimating the basic stability and control coefficients for flight regimes of subsonic, transonic, supersonic, and hypersonic speeds. Methods are presented in a systematic body build-up fashion, for example, wing alone, body alone, wing/body and wing/body/tail techniques. The methods range from techniques based on simple expressions developed from theory to correlations obtained from experimental data. In the following sections, as well as in later chapters, we shall develop simple methods for computing the aerodynamic stability and control coefficients. Our emphasis will be for the most part on methods that can be derived from simple theoretical considerations. These methods in general are accurate for preliminary design purposes and show the relationship between the stability coefficients and the geometric and aerodynamic characteristics of the airplane. Furthermore, the methods generally are valid only for the subsonic flight regime. A complete discussion of how to extend these methods to higher-speed flight regimes is beyond the scope of this book and the reader is referred to [2.7] for the high-speed methods.

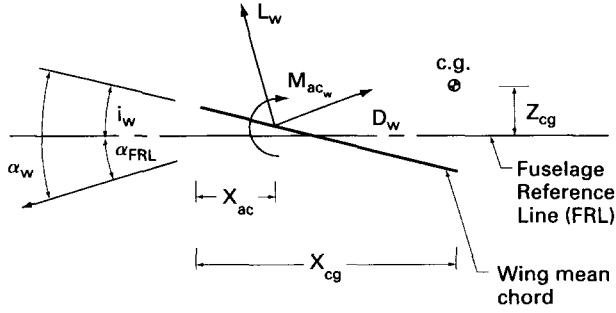


FIGURE 2.7
Wing contribution to the pitching moment.

2.3.3 Wing Contribution

The contribution of the wing to an airplane's static stability can be examined with the aid of Figure 2.7. In this sketch we have replaced the wing by its mean aerodynamic chord $\bar{c}$. The distances from the wing leading edge to the aerodynamic center and the center of gravity are denoted x_{ac} and x_{cg} , respectively. The vertical displacement of the center of gravity is denoted by z_{cg} . The angle the wing chord line makes with the fuselage reference line is denoted as i_w . This is the angle at which the wing is mounted onto the fuselage.

If we sum the moments about the center of gravity, the following equation is obtained:

$$\begin{aligned} \sum \text{Moments} &= M_{cg_w} \\ M_{cg_w} &= L_w \cos(\alpha_w - i_w)[x_{cg} - x_{ac}] + D_w \sin(\alpha_w - i_w)[x_{cg} - x_{ac}] \\ &\quad + L_w \sin(\alpha_w - i_w)[z_{cg}] - D_w \cos(\alpha_w - i_w)[z_{cg}] + M_{ac_w} \end{aligned} \quad (2.4)$$

Dividing by $\frac{1}{2}\rho V^2 S \bar{c}$ yields

$$\begin{aligned} C_{m_{cg_w}} &= C_{L_w} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \cos(\alpha_w - i_w) + C_{D_w} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \sin(\alpha_w - i_w) \\ &\quad + C_{L_w} \frac{z_{cg}}{\bar{c}} \sin(\alpha_w - i_w) - C_{D_w} \frac{z_{cg}}{\bar{c}} \cos(\alpha_w - i_w) + C_{m_{ac_w}} \end{aligned} \quad (2.5)$$

Equation (2.5) can be simplified by assuming that the angle of attack is small. With this assumption the following approximations can be made:

$$\cos(\alpha_w - i_w) \approx 1, \quad \sin(\alpha_w - i_w) \approx \alpha_w - i_w, \quad C_L \gg C_D$$

If we further assume that the vertical contribution is negligible, then Equation (2.5) reduces to

$$C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \quad (2.6)$$

or

$$C_{m_{cg_w}} = C_{m_{ac_w}} + (C_{L_{0_w}} + C_{L_{\alpha_w}} \alpha_w) \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \quad (2.7)$$

where $C_{L_w} = C_{L_{0_w}} + C_{L_{\alpha_w}} \alpha_w$. Applying the condition for static stability yields

$$C_{m_{0_w}} = C_{m_{ac_w}} + C_{L_{0_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \quad (2.8)$$

$$C_{m_{\alpha_w}} = C_{L_{\alpha_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \quad (2.9)$$

For a wing-alone design to be statically stable, Equation (2.9) tells us that the aerodynamic center must lie aft of the center of gravity to make $C_{m_{\alpha}} < 0$. Since we also want to be able to trim the aircraft at a positive angle of attack, the pitching moment coefficient at zero angle of attack, C_{m_0} , must be greater than 0. A positive pitching moment about the aerodynamic center can be achieved by using a negative-cambered airfoil section or an airfoil section that has a reflexed trailing edge. For many airplanes, the center of gravity position is located slightly aft of the aerodynamic center (see data in Appendix B). Also, the wing is normally constructed of airfoil profiles having a positive camber. Therefore, the wing contribution to static longitudinal stability is destabilizing for most conventional airplanes.

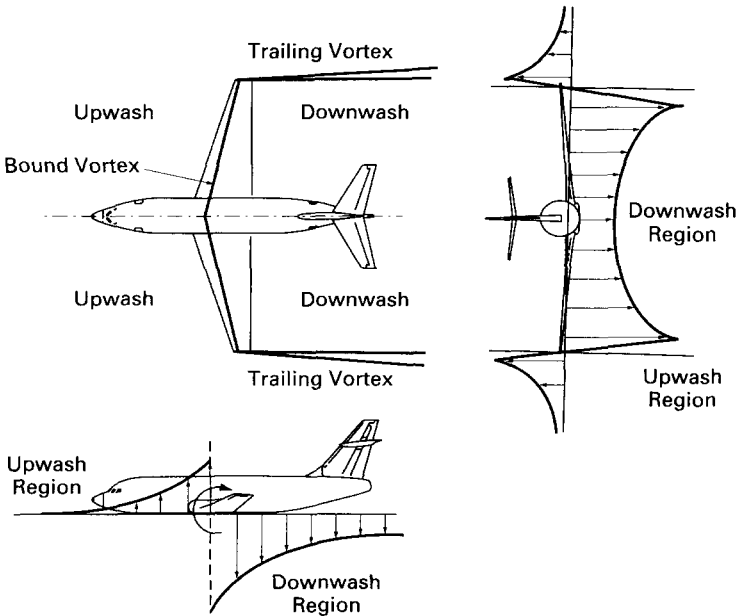


FIGURE 2.8

Flow field around an airplane created by the wing.

2.3.4 Tail Contribution—Aft Tail

The horizontal tail surface can be located either forward or aft of the wing. When the surface is located forward of the wing, the surface is called a canard. Both surfaces are influenced by the flow field created by the wing. The canard surface is affected by the upwash flow from the wing, whereas the aft tail is subjected to the downwash flow. Figure 2.8 is a sketch of the flow field surrounding a lifting wing. The wing flow field is due primarily to the bound and trailing vortices. The magnitude of the upwash or downwash depends on the location of the tail surface with respect to the wing.

The contribution that a tail surface located aft of the wing makes to the airplane's lift and pitching moment can be developed with the aid of Figure 2.9. In this sketch, the tail surface has been replaced by its mean aerodynamic chord. The angle of attack at the tail can be expressed as

$$\alpha_t = \alpha_w - i_w - \varepsilon + i_t \quad (2.10)$$

where ε and i_t are the downwash and tail incidence angles, respectively. If we assume small angles and neglect the drag contribution of the tail, the total lift of the wing and tail can be expressed as

$$L = L_w + L_t \quad (2.11)$$

or

$$C_L = C_{L_w} + \eta \frac{S_t}{S} C_{L_t} \quad (2.12)$$

where

$$\eta = \frac{\frac{1}{2} \rho V_t^2}{\frac{1}{2} \rho V_w^2} = \frac{Q_t}{Q_w} \quad (2.13)$$

The ratio of the dynamic pressures, called the tail efficiency, can have values in the range 0.8–1.2. The magnitude of η depends on the location of the tail surface. If

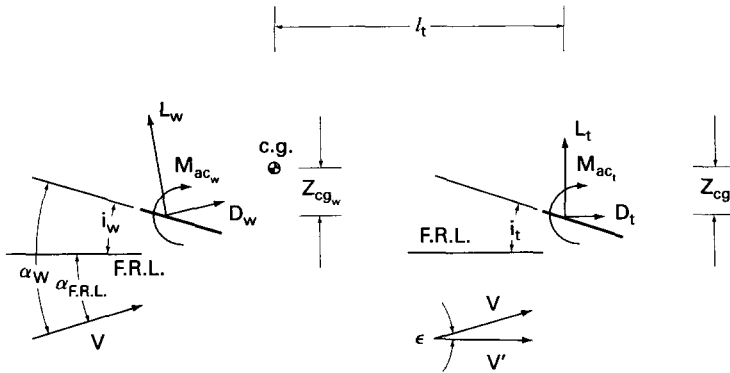


FIGURE 2.9

Aft tail contribution to the pitching moment.

the tail is located in the wake region of the wing or fuselage, η will be less than unity because $Q_t < Q_w$ due to the momentum loss in the wake. On the other hand, if the tail is located in a region where $Q_t > Q_w$, then η will be greater than unity. Such a situation could exist if the tail were located in either the slip stream of the propeller or in the exhaust wake of a jet engine.

The pitching moment due to the tail can be obtained by summing the moments about the center of gravity:

$$M_t = -l_t[L_t \cos(\alpha_{FRL} - \varepsilon) + D_t \sin(\alpha_{FRL} - \varepsilon)] - z_{cg_t}[D_t \cos(\alpha_{FRL} - \varepsilon) - L_t \sin(\alpha_{FRL} - \varepsilon)] + M_{ac_t} \quad (2.14)$$

Usually only the first term of this equation is retained; the other terms generally are small in comparison to the first term. If we again use the small-angle assumption and that $C_{L_t} \gg C_{D_t}$, then Equation (2.14) reduces to

$$M_t = -l_t L_t = -l_t C_{L_t} \frac{1}{2} \rho V_t^2 S_t \quad (2.15)$$

$$C_{m_t} = \frac{M_t}{\frac{1}{2} \rho V^2 S \bar{c}} = -\frac{l_t S_t}{S \bar{c}} \eta C_{L_t} \quad (2.16)$$

or

$$C_{m_t} = -V_H \eta C_{L_t} \quad (2.17)$$

where $V_H = l_t S_t / (S \bar{c})$ is called the horizontal tail volume ratio.

From Figure 2.9, the angle of attack of the tail is seen to be

$$\alpha_t = \alpha_w - i_w - \varepsilon + i_t \quad (2.18)$$

The coefficient C_{L_t} can be written as

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t = C_{L_{\alpha_t}} (\alpha_w - i_w - \varepsilon + i_t) \quad (2.19)$$

where $C_{L_{\alpha_t}}$ is the slope of the tail lift curve. The downwash angle ε can be expressed as

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha} \alpha_w \quad (2.20)$$

where ε_0 is the downwash at zero angle of attack.

The downwash behind a wing with an elliptic lift distribution can be derived from finite-wing theory and shown to be related to the wing lift coefficient and aspect ratio:

$$\varepsilon = \frac{2C_{L_w}}{\pi AR_w} \quad (2.21)$$

where the downwash angle is in radians. The rate of change of downwash angle with angle of attack is determined by taking the derivative of Equation (2.21):

$$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L_{\alpha_w}}}{\pi AR_w} \quad (2.22)$$

where $C_{L_{\alpha_w}}$ is per radian. The preceding expressions do not take into account the

position of the tailplane relative to the wing; that is, its vertical and longitudinal spacing. More accurate methods for estimating the downwash at the tailplane can be found in [2.7]. An experimental technique for determining the downwash using wind-tunnel force and moment measurements will be presented by way of a problem assignment at the end of this chapter.

Rewriting the tail contribution to the pitching moment yields

$$C_{m_{\text{cgt}}} = -V_H \eta C_{L_t} \quad (2.23)$$

$$C_{m_{\text{cgt}}} = \eta V_H C_{L_{\alpha_t}} (\varepsilon_0 + i_w - i_t) - \eta V_H C_{L_{\alpha_t}} \alpha \left(1 - \frac{d\varepsilon}{d\alpha} \right) \quad (2.24)$$

Comparing Equation (2.24) with the linear expression for the pitching moment given as

$$C_{m_{\text{cgt}}} = C_{m_0} + C_{m_\alpha} \alpha \quad (2.25)$$

yields expressions for the intercept and slope:

$$C_{m_0} = \eta V_H C_{L_{\alpha_t}} (\varepsilon_0 + i_w - i_t) \quad (2.26)$$

$$C_{m_\alpha} = -\eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\varepsilon}{d\alpha} \right) \quad (2.27)$$

Recall that earlier we showed that the wing contribution to C_{m_0} was negative for an airfoil having positive camber. The tail contribution to C_{m_0} can be used to ensure that C_{m_0} for the complete airplane is positive. This can be accomplished by adjusting the tail incidence angle i_t . Note that we would want to mount the tail plane at a negative angle of incidence to the fuselage reference line to increase C_{m_0} due to the tail.

The tail contribution to the static stability of the airplane ($C_{m_\alpha} < 0$) can be controlled by proper selection of V_H and $C_{L_{\alpha_t}}$. The contribution of C_{m_α} will become more negative by increasing the tail moment arm l_t or tail surface area S_t and by increasing $C_{L_{\alpha_t}}$. The tail lift curve slope $C_{L_{\alpha_t}}$ can be increased most easily by increasing the aspect ratio of the tail planform. The designer can adjust any one of these parameters to achieve the desired slope. As noted here, a tail surface located aft of the wing can be used to ensure that the airplane has a positive C_{m_0} and a negative C_{m_α} .

EXAMPLE PROBLEM 2.1. The wing-fuselage pitching moment characteristics of a high-wing, single-engine, general aviation airplane follow, along with pertinent geometric data:

$$C_{m_{\text{cgwf}}} = -0.05 - 0.0035\alpha$$

where α is the fuselage reference line angle of attack in degrees and wf means wing-fuselage

$$S_w = 178 \text{ ft}^2 \quad x_{\text{cg}}/c = 0.1$$

$$b_w = 35.9 \text{ ft} \quad AR_w = 7.3$$

$$\bar{c}_w = 5.0 \text{ ft} \quad C_{L_{\alpha \text{ wf}}} = 0.07/\text{deg} \quad i_w = 2.0^\circ \quad C_{L_{\alpha=0}} = 0.26$$

Estimate the horizontal tail area and tail incidence angle, i_t , so that the complete airplane has the following pitching moment characteristics (illustrated in Figure 2.10):

$$C_{m_{cg_{wft}}} = 0.15 - 0.025\alpha$$

where α is in degrees and wft is the wing-fuselage-horizontal tail contribution.

Assume the following with regard to the horizontal tail:

$$l_t = 14.75 \text{ ft} \quad \eta = 1$$

$$AR_t = 4.85 \quad C_{L_{\alpha_t}} = 0.073/\text{deg}$$

Solution. The contribution of the horizontal tail to C_{m_0} and C_{m_α} can be calculated by subtracting the wing-fuselage contribution from the wing-fuselage-horizontal tail contribution, respectively:

$$\begin{aligned} C_{m_{0_t}} &= C_{m_{0_{wft}}} - C_{m_{0_{wff}}} \\ &= 0.15 - (-0.05) = 0.20 \end{aligned}$$

$$\begin{aligned} C_{m_{\alpha_t}} &= C_{m_{\alpha_{wft}}} - C_{m_{\alpha_{wff}}} \\ &= -0.025 - (-0.0035) = -0.0215/\text{deg} \end{aligned}$$

The horizontal tail area is found by determining the horizontal tail volume ratio required to satisfy the required static stability that needs to be created by the tail. Recall the $C_{m_{\alpha_t}}$ was developed earlier and is rewritten here:

$$C_{m_{\alpha_t}} = -\eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\varepsilon}{d\alpha} \right)$$

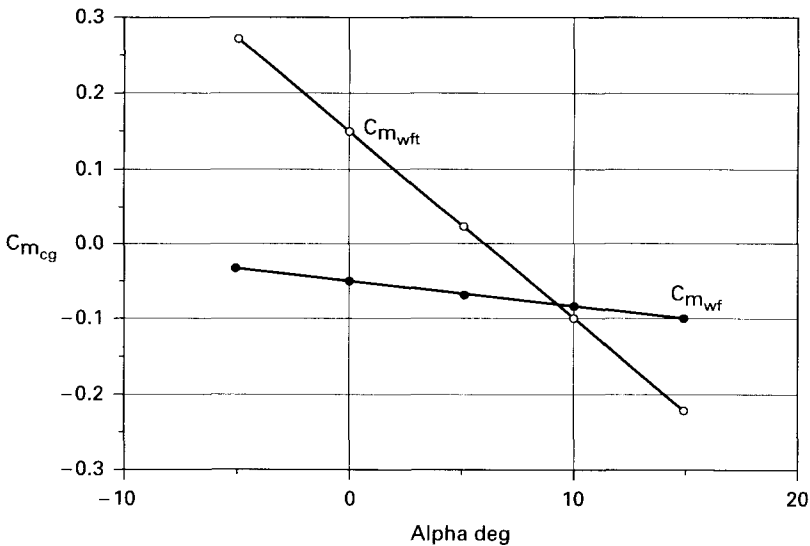


FIGURE 2.10

Pitching moment characteristic for airplane in Example Problem 2.1.

Solving this equation for the volume ratio yields

$$V_H = - \frac{C_{m_{\alpha_i}}}{\eta C_{L_{\alpha_i}} \left(1 - \frac{d\varepsilon}{d\alpha} \right)}$$

The only quantity we do not know in this equation is the rate of change of the downwash angle with respect to the angle of attack, $d\varepsilon/d\alpha$. However, this can be estimated from the wing characteristics as follows:

$$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L_{\alpha_w}}}{\pi AR_w}$$

Using the wing-fuselage $C_{L_{\alpha_{wf}}}$ as an approximation to $C_{L_{\alpha_w}}$ we can obtain an estimate of $d\varepsilon/d\alpha$:

$$\begin{aligned} \frac{d\varepsilon}{d\alpha} &= \frac{2(0.07/\text{deg})(57.3 \text{ deg/rad})}{\pi(7.3)} \\ \frac{d\varepsilon}{d\alpha} &= 0.35 \end{aligned}$$

Substituting $d\varepsilon/d\alpha$ and the other quantities into the expression for V_H yields

$$\begin{aligned} V_H &= - \frac{(-0.0215/\text{deg})}{(1.0)(0.073/\text{deg})(1 - 0.35)} \\ &= 0.453 \end{aligned}$$

The horizontal tail volume ratio is expressed as

$$V_H = \frac{l_t S_t}{\overline{SC}}$$

and solving for the horizontal tail area yields

$$\begin{aligned} S_t &= \frac{(0.453)(178 \text{ ft}^2)(5 \text{ ft})}{(14.75 \text{ ft})} \\ &= 27.3 \text{ ft}^2 \end{aligned}$$

This is the tail area needed to provide the required tail contribution to $C_{m_{\alpha}}$. Next we can determine the tail incidence angle, i_t , from the requirement for C_{m_0} . The equation for C_{m_0} due to the horizontal tail was shown to be

$$C_{m_0} = V_H \eta C_{L_{\alpha_i}} (i_w + \varepsilon_0 - i_t)$$

The tail incidence angle, i_t , can be obtained by rearranging the preceding equation:

$$i_t = - \left(\frac{C_{m_0}}{V_H \eta C_{L_{\alpha_i}}} - i_w - \varepsilon_0 \right)$$

The only quantity that we do not know in this equation is ε_0 ; that is, the downwash angle at the tail when the wing is at zero angle of attack. This can be estimated using

the following expression:

$$\begin{aligned}\varepsilon_0 &= \frac{2C_{L_0}}{\pi AR_w} \\ &= \frac{2[0.26]}{\pi[7.3]} = 0.0226 \text{ rad}\end{aligned}$$

or $\varepsilon_0 = 1.3^\circ$

Substituting ε_0 and the other quantities into the expression for i_t yields

$$\begin{aligned}i_t &= -\left[\frac{0.20}{(0.453)(1.0)(0.073/\text{deg})} - 2.0 - 1.3\right] \\ &= -2.7 \text{ deg.}\end{aligned}$$

The horizontal tail is mounted to the fuselage at a negative 2.7° .

In summary we have shown that the level of static stability can be controlled by the designer by proper selection of the horizontal tail volume ratio. In practice the only parameter making up the volume ratio that can be varied by the stability and control designer is the horizontal tail surface area. The other parameters, such as the tail moment arm, wing area, and mean wing chord, are determined by the fuselage and wing requirements, which are related to the internal volume and performance specifications of the airplane, respectively.

The horizontal tail incidence angle, i_t , is determined by trim angle of attack or lift coefficient. For a given level of static stability, that is, slope of the pitching moment curve, the trim angle depends on the moment coefficient at zero angle of attack, C_{m_0} . The tail incidence angle, i_t , can be adjusted to yield whatever C_{m_0} is needed to achieve the desired trim condition.

2.3.5 Canard—Forward Tail Surface

A canard is a tail surface located ahead of the wing. The canard surface has several attractive features. The canard, if properly positioned, can be relatively free from wing or propulsive flow interference. Canard control is more attractive for trimming the large nose-down moment produced by high-lift devices. To counteract the nose-down pitching moment, the canard must produce lift that will add to the lift being produced by the wing. An aft tail must produce a down load to counteract the pitching moment and thus reduce the airplane's overall lift force. The major disadvantage of the canard is that it produces a destabilizing contribution to the aircraft's static stability. However, this is not a severe limitation. By proper location of the center of gravity, one can ensure the airplane is statically stable.

2.3.6 Fuselage Contribution

The primary function of the fuselage is to provide room for the flight crew and payload such as passengers and cargo. The optimum shape for the internal volume at minimum drag is a body for which the length is larger than the width or height.

For most fuselage shapes used in airplane designs, the width and height are on the same order of magnitude and for many designs a circular cross-section is used.

The aerodynamic characteristics of long, slender bodies were studied by Max Munk [2.8] in the earlier 1920s. Munk was interested in the pitching moment characteristics of airship hulls. In his analysis, he neglected viscosity and treated the flow around the body as an ideal fluid. Using momentum and energy relationships, he showed that the rate of change of the pitching moment with angle of attack (per radian) for a body of revolution is proportional to the body volume and dynamic pressure:

$$\frac{dM}{d\alpha} = \text{fn}\left(\text{volume}, \frac{1}{2}\rho V^2\right) \quad (2.28)$$

Multhopp [2.9] extended this analysis to account for the induced flow along the fuselage due to the wings for bodies of arbitrary cross-section. A summary of Multhopp's method for C_{m_0} and C_{m_α} due to the fuselage is presented as follows:

$$C_{m_{0_f}} = \frac{k_2 - k_1}{36.5S\bar{c}} \int_0^{l_f} w_f^2 (\alpha_{0_w} + i_f) dx \quad (2.29)$$

which can be approximated as

$$C_{m_{0_f}} = \frac{k_2 - k_1}{36.5S\bar{c}} \sum_{x=0}^{x=l_f} w_f^2 (\alpha_{0_w} + i_f) \Delta x \quad (2.30)$$

where $k_2 - k_1$ = the correction factor for the body fineness ratio

S = the wing reference area

$\bar{c}$ = the wing mean aerodynamic chord

w_f = the average width of the fuselage sections

α_{0_w} = the wing zero-lift angle relative to the fuselage reference line

i_f = the incidence of the fuselage camber line relative to the fuselage reference line at the center of each fuselage increment. The incidence angle is defined as negative for nose droop and aft upsweep.

Δx = the length of the fuselage increments

Figure 2.11 illustrates how the fuselage can be divided into segments for the calculation of C_{m_0} and also defines the body width w_f for various body cross-sectional shapes. The correction factor $(k_2 - k_1)$ is given in Figure 2.12.

The local angle of attack along the fuselage is greatly affected by the flow field created by the wing, as was illustrated in Figure 2.8. The portion of the fuselage ahead of the wing is in the wing upwash; the aft portion is in the wing downwash flow. The change in pitching moment with angle of attack is given by

$$C_{m_{\alpha_f}} = \frac{1}{36.5S\bar{c}} \int_0^{l_f} w_f^2 \frac{\partial \epsilon_u}{\partial \alpha} dx \quad (\text{deg}^{-1}) \quad (2.31)$$

which can be approximated by

$$C_{m_{\alpha_f}} = \frac{1}{36.5S\bar{c}} \sum_{x=0}^{x=l_f} w_f^2 \frac{\partial \epsilon_u}{\partial \alpha} \Delta x \quad (2.32)$$

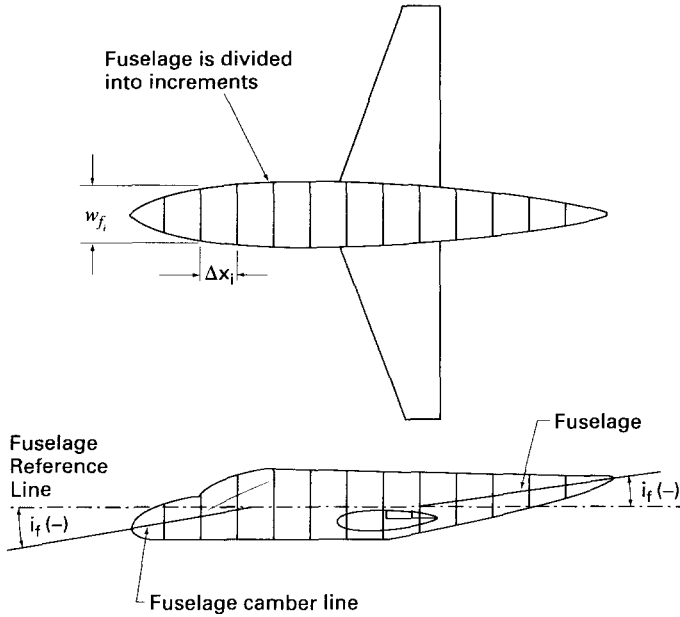


FIGURE 2.11
Procedure for calculating C_{m0} due to the fuselage.

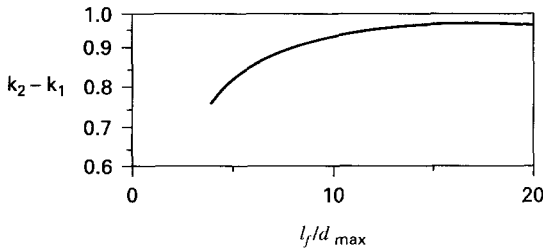


FIGURE 2.12
 $k_2 - k_1$ versus l_f/d .

where S = the wing reference area and $\bar{c}$ = the wing mean aerodynamic chord.

The fuselage again can be divided into segments and the local angle of attack of each section, which is composed of the geometric angle of attack of the section plus the local induced angle due to the wing upwash or downwash for each segment, can be estimated. The change in local flow angle with angle of attack, $\partial \epsilon_u / \partial \alpha$, varies along the fuselage and can be estimated from Figure 2.13. For locations ahead of the wing, the upwash field creates large local angles of attack; therefore, $\partial \epsilon_u / \partial \alpha > 1$. On the other hand, a station behind the wing is in the downwash region of the wing vortex system and the local angle of attack is reduced. For the region behind the wing, $\partial \epsilon_u / \partial \alpha$ is assumed to vary linearly from 0 to $(1 - \partial \epsilon / \partial \alpha)$ at the tail. The region between the wing's leading edge and trailing edge is assumed

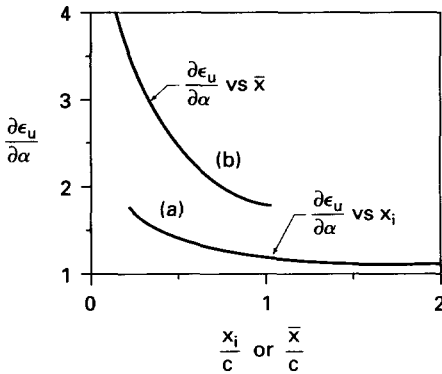


FIGURE 2.13
Variation of local flow angle along the fuselage.

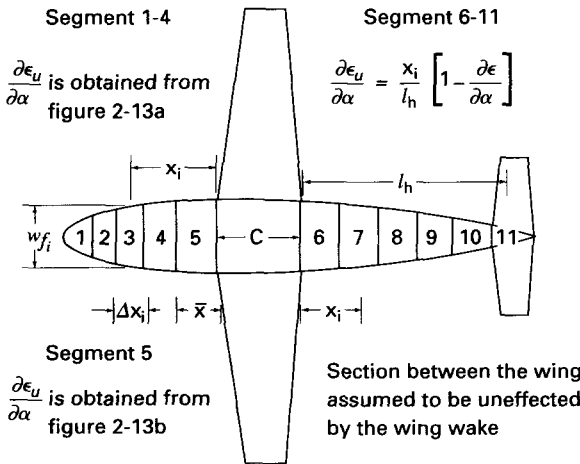


FIGURE 2.14
Procedure for calculating C_{m_α} due to the fuselage.

to be unaffected by the wing's flow field, $\partial \epsilon_u / \partial \alpha = 0$. Figure 2.14 is a sketch showing the application of Equation (2.32).

2.3.7 Power Effects

The propulsion unit can have a significant effect on both the longitudinal trim and static stability of the airplane. If the thrust line is offset from the center of gravity, the propulsive force will create a pitching moment that must be counteracted by the aerodynamic control surface.

The static stability of the airplane also is influenced by the propulsion system. For a propeller driven airplane the propeller will develop a normal force in its plane of rotation when the propeller is at an angle of attack. The propeller's normal force will create a pitching moment about the center of gravity, producing a propulsion

contribution to C_{m_α} . Although one can derive a simple expression for C_{m_α} due to the propeller, the actual contribution of the propulsion system to the static stability is much more difficult to estimate. This is due to the indirect effects that the propulsion system has on the airplane's characteristics. For example, the propeller slipstream can have an effect on the tail efficiency η and the downwash field. Because of these complicated interactions the propulsive effects on airplane stability are commonly estimated from powered wind-tunnel models.

A normal force will be created on the inlet of a jet engine when it is at an angle of attack. As in the case of the propeller powered airplane, the normal force will produce a contribution to C_{m_α} .

2.3.8 Stick Fixed Neutral Point

The total pitching moment for the airplane can now be obtained by summing the wing, fuselage, and tail contributions:

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha} \alpha \quad (2.33)$$

where
$$C_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + \eta V_H C_{L_{\alpha_t}} (\epsilon_0 + i_w - i_t) \quad (2.34)$$

$$C_{m_\alpha} = C_{L_{\alpha_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) + C_{m_{\alpha_f}} - \eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha} \right) \quad (2.35)$$

Notice that the expression for C_{m_α} depends upon the center of gravity position as well as the aerodynamic characteristics of the airplane. The center of gravity of an airplane varies during the course of its operation; therefore, it is important to know if there are any limits to the center of gravity travel. To ensure that the airplane possesses static longitudinal stability, we would like to know at what point $C_{m_\alpha} = 0$. Setting C_{m_α} equal to 0 and solving for the center of gravity position yields

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{C_{m_{\alpha_f}}}{C_{L_{\alpha_w}}} + \eta V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}} \left(1 - \frac{d\epsilon}{d\alpha} \right) \quad (2.36)$$

In obtaining equation 2.36, we have ignored the influence of center of gravity movement on V_H . We call this location the stick fixed neutral point. If the airplane's

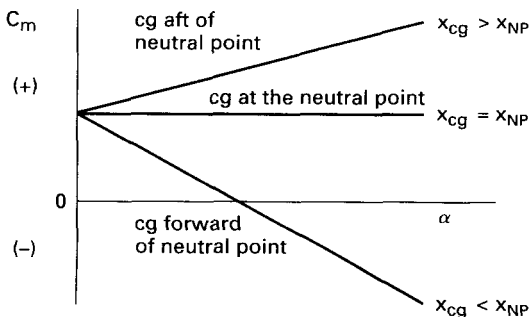


FIGURE 2.15
The influence of center of gravity position on longitudinal static stability.

center of gravity ever reaches this point, the airplane will be neutrally stable. Movement of the center of gravity beyond the neutral point causes the airplane to be statically unstable. The influence of center of gravity position on static stability is shown in Figure 2.15.

EXAMPLE PROBLEM 2.2. Given the general aviation airplane shown in Figure 2.16, determine the contribution of the wing, tail, and fuselage to the C_m versus α curve. Also determine the stick fixed neutral point. For this problem, assume standard sea-level atmospheric conditions.

Solution. The lift curve slopes for the two-dimensional sections making up the wing and tail must be corrected for a finite aspect ratio. This is accomplished using the formula

$$C_{L\alpha} = \frac{C_{l\alpha}}{1 + C_{l\alpha}/(\pi AR)}$$

where $C_{l\alpha}$ is given as per radian.

Substituting the two-dimensional lift curve slope and the appropriate aspect ratio yields

$$\begin{aligned} C_{L\alpha_w} &= \frac{C_{l\alpha_w}}{1 + C_{l\alpha_w}/(\pi AR_w)} \\ &= \frac{(0.097/\text{deg})(57.3 \text{ deg/rad})}{1 + (0.097/\text{deg})(57.3 \text{ deg/rad})/(\pi(6.06))} \\ &= 4.3 \text{ rad}^{-1} \end{aligned}$$

Flight condition

$W = 2750 \text{ lb}$
 $V = 176 \text{ ft/sec}$
 $X_{cg} = 0.295\bar{c}$

Wing airfoil characteristics

$C_{m_{ac}} = -0.116$
 $C_{l\alpha} = 0.097/\text{deg}$
 $\alpha_{0c} = -5^\circ$
 $X_{ac} = 0.25\bar{c}$
 No Twist
 $i_w = 1.0^\circ$

Tail airfoil section

$C_{l\alpha} = 0.01/\text{deg}$
 $C_{m_{ac}} = 0.0$
 $i_t = -1.0^\circ$

Reference geometry

$S = 184 \text{ ft}^2$ $S_H = 43 \text{ ft}^2$
 $b = 33.4 \text{ ft}$ $l_t = 16 \text{ ft}$
 $\bar{c} = 5.7 \text{ ft}$

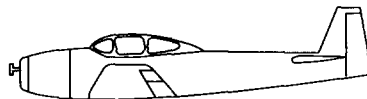
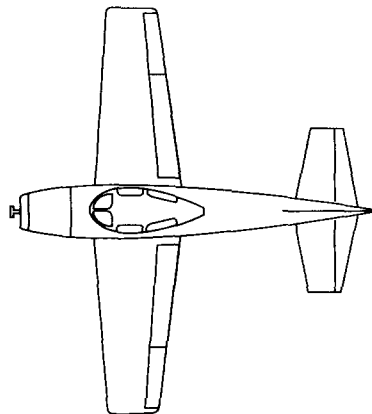


FIGURE 2.16

General aviation airplane.

In a similar manner the lift curve slope for the tail can be found:

$$C_{L_{\alpha_t}} = 3.91 \text{ rad}^{-1}$$

The wing contribution to C_{m_0} and C_{m_α} is found from Equations (2.8) and (2.9):

$$C_{m_{0_w}} = C_{m_{ac_w}} + C_{L_{0_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right)$$

and

$$C_{m_{\alpha_w}} = C_{L_{\alpha_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right)$$

The lift coefficient at zero angle of attack is obtained by multiplying the absolute value of the zero lift angle of attack by the lift curve slope:

$$\begin{aligned} C_{L_{0_w}} &= C_{L_{\alpha_w}} |\alpha_0| \\ &= (4.3 \text{ rad}^{-1})(5 \text{ deg})/(57.3 \text{ deg/rad}) \\ &= 0.375 \end{aligned}$$

Substituting the approximate information into the equations for $C_{m_{0_w}}$ and $C_{m_{\alpha_w}}$ yields

$$\begin{aligned} C_{m_{0_w}} &= C_{m_{ac_w}} + C_{L_{0_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \\ &= -0.116 + (0.375)(0.295 - 0.250) \\ &= -0.099 \\ C_{m_{\alpha_w}} &= C_{L_{\alpha_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) \\ &= (4.3 \text{ rad}^{-1})(0.295 - 0.250) \\ &= 0.1935 \text{ rad}^{-1} \end{aligned}$$

For this particular airplane, the wing contribution to C_{m_α} is destabilizing.

The tail contribution to the intercept and slope can be estimated from Equations (2.26) and (2.27):

$$\begin{aligned} C_{m_{0_t}} &= \eta V_H C_{L_{\alpha_t}} (\epsilon_0 + i_w - i_t) \\ C_{m_{\alpha_t}} &= -\eta V_H C_{L_{\alpha_t}} \left(1 - \frac{d\epsilon}{d\alpha} \right) \end{aligned}$$

The tail volume ratio V_H is given by

$$V_H = \frac{l_t S_t}{S \bar{c}}$$

or

$$V_H = \frac{(16 \text{ ft})(43 \text{ ft}^2)}{(184 \text{ ft}^2)(5.7 \text{ ft})} = 0.66$$

The downwash term is estimated using the expression

$$\epsilon = \frac{2C_{L_w}}{\pi AR}$$

where ε is the downwash angle in radians,

$$\begin{aligned}\varepsilon_0 &= \frac{2C_{L_{w_0}}}{\pi AR} \\ &= \frac{2(0.375)}{\pi(6.06)} = 0.04 \text{ rad} = 2.3^\circ\end{aligned}$$

and
$$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L_{\alpha_w}}}{AR}$$

where $C_{L_{\alpha_w}}$ is in radians,

$$\frac{d\varepsilon}{d\alpha} = \frac{2(4.3)}{\pi(6.06)} = 0.45$$

Substituting the preceding information into the formulas for the intercept and slope yields

$$\begin{aligned}C_{m_{\alpha_i}} &= \eta V_H C_{L_{\alpha_i}} (\varepsilon_0 + i_w - i_t) \\ &= (0.66)(3.91 \text{ rad}^{-1})[2.3^\circ + 1.0^\circ - (-1.0^\circ)]/57.3 \text{ deg/rad} \\ &= 0.194\end{aligned}$$

and
$$\begin{aligned}C_{m_{\alpha_i}} &= -\eta V_H C_{L_{\alpha_i}} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \\ &= -(0.66)(3.91 \text{ rad}^{-1})(1 - 0.45) \\ &= -1.42 \text{ rad}^{-1}\end{aligned}$$

In this example, the ratio η of tail to wing dynamic pressure was assumed to be unity.

The fuselage contribution to C_{m_0} and $C_{m_{\alpha_i}}$ can be estimated from Equations (2.30) and (2.32), respectively. To use these equations, we must divide the fuselage into segments, as indicated in Figure 2.17. The summation in Equation (2.30) easily can be estimated from the geometry and is found by summing the individual contributions as illustrated by the table in Figure 2.17.

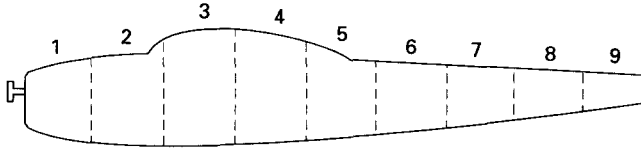
$$\sum_{x=0}^{l_B} w_f^2 (\alpha_{0_w} + i_f) \Delta x = -1665$$

The body fineness ratio is estimated from the geometrical data given in Figure 2.16:

$$\frac{l_f}{d_{\max}} = 6.2$$

and the correction factor $k_2 - k_1$ is found from Figure 2.12, $k_2 - k_1 = 0.86$. Substituting these values into Equation (2.30) yields

$$C_{m_{0f}} = -0.037$$



Station	Δx ft	w_f ft	$\alpha_{0_w} + i_f$	$w_f^2 [\alpha_{0_w} + i_t] \Delta x$
1	3.0	3.6	-5.0	-194
2	3.0	4.6	-5.0	-317
3	3.0	4.6	-5.0	-317
4	3.0	4.6	-5.0	-317
5	3.0	4.1	-5.0	-252
6	3.0	3.1	-5.0	-144
7	3.0	2.3	-5.0	-79
8	3.0	1.5	-5.0	-34
9	3.0	0.8	-5.0	-10
$i_f = 0$ at every station				Sum = -1664

FIGURE 2.17

Sketch of segmented fuselage for calculating C_{m_α} for the example problem.

In a similar manner C_{m_α} can be estimated. A table is included in Figure 2.17 that shows the estimate of the summation. $C_{m_{\alpha f}}$ was estimated to be

$$C_{m_{\alpha f}} = 0.12 \text{ rad}^{-1}$$

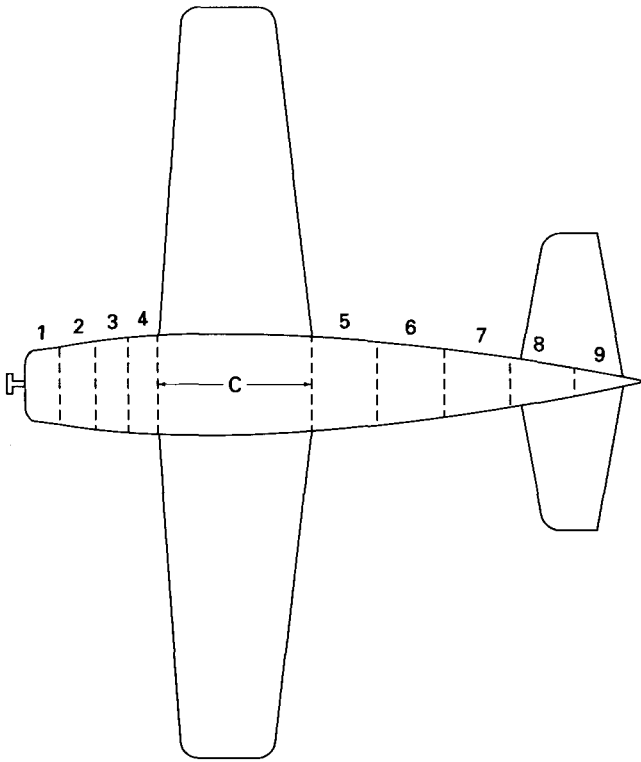
The individual contributions and the total pitching moment curve are shown in Figure 2.18.

The stick fixed neutral point can be estimated from Equation (2.36):

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{C_{m_{\alpha f}}}{C_{L_{\alpha w}}} + \eta V_H \frac{C_{L_{\alpha t}}}{C_{L_{\alpha w}}} \left(1 - \frac{d\varepsilon}{d\alpha} \right)$$

$$\frac{x_{NP}}{\bar{c}} = 0.25 - (0.12/4.3) + 0.66(3.91/4.3)(1 - 0.45)$$

$$\frac{x_{NP}}{\bar{c}} = 0.55$$



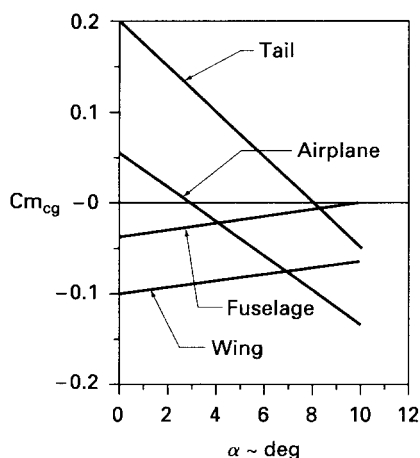
Station	Δx ft	w_f ft	x	$\frac{\partial \epsilon_\mu}{\partial \alpha}$	$w_f^2 \frac{\partial \epsilon_\mu}{\partial \alpha} \Delta x$
1	1.5	3.0	5.25	1.2	16.2
2	1.5	3.4	4.5	1.3	22.5
3	1.5	3.8	3.75	1.4	30.3
4	1.5	4.2	1.5	3.2	84.7
5	2.9	3.8	1.45	0.06	2.5
6	2.9	3.1	4.35	0.18	5.0
7	2.9	2.3	7.25	0.31	4.8
8	2.9	1.5	10.15	0.43	2.8
9	2.9	0.8	13.05	0.55	1.0

$c = 6.5$ ft

$l_h = 13$ ft

Sum = 85.1

FIGURE 2.17
Continued.

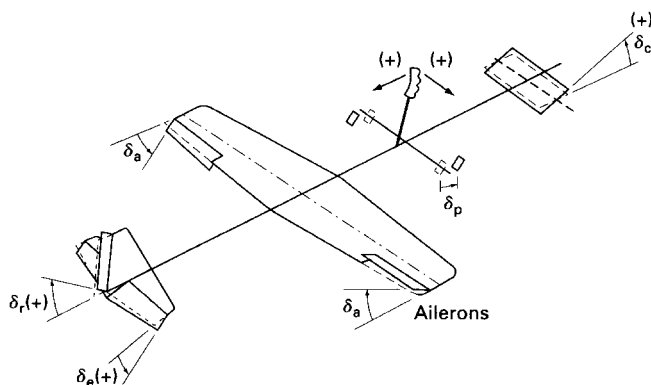
**FIGURE 2.18**

Component contributions to pitching moment for example problem.

2.4

LONGITUDINAL CONTROL

Control of an airplane can be achieved by providing an incremental lift force on one or more of the airplane's lifting surfaces. The incremental lift force can be produced by deflecting the entire lifting surface or by deflecting a flap incorporated in the lifting surface. Because the control flaps or movable lifting surfaces are located at some distance from the center of gravity, the incremental lift force creates a moment about the airplane's center of gravity. Figure 2.19 shows the three primary aerodynamic controls. Pitch control can be achieved by changing the lift on either a forward or aft control surface. If a flap is used, the flapped portion of the tail surface is called an elevator. Yaw control is achieved by deflecting a flap on the vertical tail called the rudder, and roll control can be achieved by deflecting small flaps located outboard toward the wing tips in a differential manner. These flaps are called ailerons. A roll moment can also be produced by deflecting a wing spoiler. As the name implies a spoiler disrupts the lift. This is accomplished by deflecting a section of the upper wing surface so that the flow separates behind the

**FIGURE 2.19**

Primary aerodynamic controls.

spoiler, which causes a reduction in the lifting force. To achieve a roll moment, only one spoiler need be deflected.

In this section we shall be concerned with longitudinal control. Control of the pitch attitude of an airplane can be achieved by deflecting all or a portion of either a forward or aft tail surface. Factors affecting the design of a control surface are control effectiveness, hinge moments, and aerodynamic and mass balancing. Control effectiveness is a measure of how effective the control deflection is in producing the desired control moment. As we shall show shortly, control effectiveness is a function of the size of the flap and tail volume ratio. Hinge moments also are important because they are the aerodynamic moments that must be overcome to rotate the control surface. The hinge moment governs the magnitude of force required of the pilot to move the control surface. Therefore, great care must be used in designing a control surface so that the control forces are within acceptable limits for the pilots. Finally, aerodynamic and mass balancing deal with techniques to vary the hinge moments so that the control stick forces stay within an acceptable range.

2.4.1 Elevator Effectiveness

We need some form of longitudinal control to fly at various trim conditions. As shown earlier, the pitch attitude can be controlled by either an aft tail or forward tail (canard). We shall examine how an elevator on an aft tail provides the required control moments. Although we restrict our discussion to an elevator on an aft tail, the same arguments could be made with regard to a canard surface. Figure 2.20 shows the influence of the elevator on the pitching moment curve. Notice that the elevator does not change the slope of the pitching moment curves but only shifts them so that different trim angles can be achieved.

When the elevator is deflected, it changes the lift and pitching moment of the airplane. The change in lift for the airplane can be expressed as follows:

$$\Delta C_L = C_{L_{\delta_e}} \delta_e \quad \text{where} \quad C_{L_{\delta_e}} = \frac{dC_L}{d\delta_e} \quad (2.37)$$

$$C_L = C_{L_\alpha} \alpha + C_{L_{\delta_e}} \delta_e \quad (2.38)$$

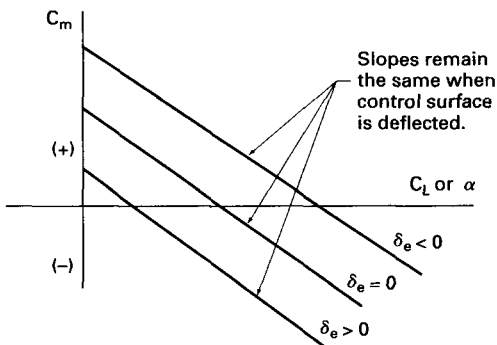


FIGURE 2.20

The influence of the elevator on the C_m versus α curve.

On the other hand, the change in pitching moment acting on the airplane can be written as

$$\Delta C_m = C_{m_{\delta_e}} \delta_e \quad \text{where} \quad C_{m_{\delta_e}} = \frac{dC_m}{d\delta_e} \quad (2.39)$$

The stability derivative $C_{m_{\delta_e}}$ is called the elevator control power. The larger the value of $C_{m_{\delta_e}}$, the more effective the control is in creating the control moment.

Adding ΔC_m to the pitching moment equation yields

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_e}} \delta_e \quad (2.40)$$

The derivatives $C_{L_{\delta_e}}$ and $C_{m_{\delta_e}}$ can be related to the aerodynamic and geometric characteristics of the horizontal tail in the following manner. The change in lift of the airplane due to deflecting the elevator is equal to the change in lift force acting on the tail:

$$\Delta L = \Delta L_t \quad (2.41)$$

$$\Delta C_L = \frac{S_t}{S} \eta \Delta C_{L_t} = \frac{S_t}{S} \eta \frac{dC_{L_t}}{d\delta_e} \delta_e \quad (2.42)$$

where $dC_{L_t}/d\delta_e$ is the elevator effectiveness. The elevator effectiveness is proportional to the size of the flap being used as an elevator and can be estimated from the equation

$$\frac{dC_{L_t}}{d\delta_e} = \frac{dC_{L_t}}{d\alpha_t} \frac{d\alpha_t}{d\delta_e} = C_{L_{\alpha_t}} \tau \quad (2.43)$$

The parameter τ can be determined from Figure 2.21.

$$C_{L_{\delta_e}} = \frac{S_t}{S} \eta \frac{dC_{L_t}}{d\delta_e} \quad (2.44)$$

The increment in airplane pitching moment is

$$\Delta C_m = -V_H \eta \Delta C_{L_t} = -V_H \eta \frac{dC_{L_t}}{d\delta_e} \delta_e \quad (2.45)$$

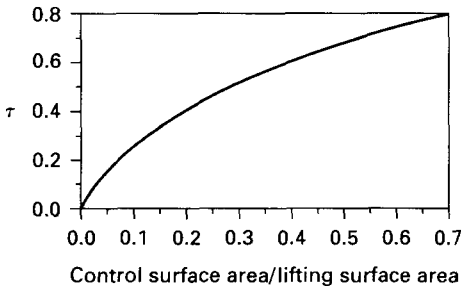


FIGURE 2.21
Flap effectiveness parameter.

or

$$C_{m_{\delta_e}} = -V_H \eta \frac{dC_{L_i}}{d\delta_e} = -V_H \eta C_{L_{\alpha}} \tau \quad (2.46)$$

The designer can control the magnitude of the elevator control effectiveness by proper selection of the volume ratio and flap size.

2.4.2 Elevator Angle to Trim

Now let us consider the trim requirements. An airplane is said to be trimmed if the forces and moments acting on the airplane are in equilibrium. Setting the pitching moment equation equal to 0 (the definition of trim) we can solve for the elevator angle required to trim the airplane:

$$C_m = 0 = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_e}} \delta_e \quad (2.47)$$

or

$$\delta_{\text{trim}} = -\frac{C_{m_0} + C_{m_\alpha} \alpha_{\text{trim}}}{C_{m_{\delta_e}}} \quad (2.48)$$

The lift coefficient to trim is

$$C_{L_{\text{trim}}} = C_{L_\alpha} \alpha_{\text{trim}} + C_{L_{\delta_e}} \delta_{\text{trim}} \quad (2.49)$$

We can use this equation to obtain the trim angle of attack:

$$\alpha_{\text{trim}} = \frac{C_{L_{\text{trim}}} - C_{L_{\delta_e}} \delta_{\text{trim}}}{C_{L_\alpha}} \quad (2.50)$$

If we substitute this equation back into Equation (2.48) we get the following equation for the elevator angle to trim:

$$\delta_{\text{trim}} = -\frac{C_{m_0} C_{L_\alpha} + C_{m_\alpha} C_{L_{\text{trim}}}}{C_{m_{\delta_e}} C_{L_\alpha} - C_{m_\alpha} C_{L_{\delta_e}}} \quad (2.51)$$

The elevator angle to trim can also be obtained directly from the pitching moment curves shown in Figure 2.20.

EXAMPLE PROBLEM 2.3. The longitudinal control surface provides a moment that can be used to balance or trim the airplane at different operating angles of attack or lift coefficient. The size of the control surface depends on the magnitude of the pitching moment that needs to be balanced by the control. In general, the largest trim moments occur when an airplane is in the landing configuration (wing flaps and landing gear deployed) and the center of gravity is at its forwardmost location. This can be explained in the following manner. In the landing configuration we fly the airplane at a high angle of attack or lift coefficient so that the airplane's approach speed can be kept as low as possible. Therefore the airplane must be trimmed at a high lift coefficient. Deployment of the wing flaps and landing gear create a nose-down pitching moment increment that must be added to the clean configuration pitching moment curve. The additional nose-down or negative pitching moment increment due to the flaps and landing gear shifts the pitching moment curve. As the center of gravity moves forward the slope of

the pitching moment curve becomes more negative (the airplane is more stable). This results in a large trim moment at high lift coefficients. The largest pitching moment that must be balanced by the elevator therefore occurs when the flaps and gear are deployed and the center of gravity is at its most forward position.

Assume that the pitching moment curve for the landing configuration for the airplane analyzed in Example Problem 2.2 at its forwardmost center of gravity position is as follows:

$$C_{m_{cg}} = -0.20 - 0.035\alpha$$

where α is in degrees. Estimate the size of the elevator to trim the airplane at the landing angle of attack of 10° . Assume that the elevator angle is constrained to $+20^\circ$ and -25° .

Solution. The increment in moment created by the control surface, $\Delta C_{m_{cg}}$, is both a function of the elevator control power, $C_{m_{\delta_e}}$, and the elevator deflection angle δ_e .

$$\Delta C_{m_{cg}} = C_{m_{\delta_e}} \delta_e$$

For a 10° approach angle of attack, the pitching moment acting on the airplane can be estimated as follows:

$$\Delta C_{m_{cg}} = -0.20 - 0.035 (10^\circ) = -0.55$$

This moment must be balanced by an equal and opposite moment created by deflecting the elevator. The change in moment coefficient created by the elevator was shown to be

$$\Delta C_{m_{cg}} = C_{m_{\delta_e}} \delta_e$$

where $C_{m_{\delta_e}}$ is referred to as the elevator control power. The elevator control power is a function of the horizontal tail volume ratio, V_H , and the flap effectiveness factor, τ :

$$C_{m_{\delta_e}} = -V_H \eta \tau C_{L_{\alpha_i}}$$

The horizontal tail volume ratio, V_H , is set by the static longitudinal stability requirements; therefore, the designer can change only the flap effectiveness parameter, τ , to achieve the appropriate control effectiveness $C_{m_{\delta_e}}$. The flap effectiveness factor is a function of the area of the control flap to the total area of the lift surface on which it is attached. By proper selection of the elevator area the necessary control power can be achieved.

For a positive moment, the control deflection angle must be negative; that is, trailing edge of the elevator is up:

$$\Delta C_{m_{trim}}^{(+)} = C_{m_{\delta_e}}^{(-)} \delta_e^{(-)}$$

$$\text{or} \quad C_{m_{\delta_e}} = \frac{\Delta C_{m_{trim}}}{\delta_e} = \frac{0.55}{-25^\circ} = -0.022/\text{deg}$$

Solving for the flap effectiveness parameter, τ ,

$$\tau = -\frac{C_{m_{\delta_e}}}{V_H \eta C_{L_{\alpha_i}}}$$

Using the values of V_H , η , and $C_{L_{\alpha_i}}$ from Example Problem 2.2 we can estimate τ :

$$\begin{aligned} \tau &= \frac{(-0.022/\text{deg})(57.3 \text{ deg/rad})}{(0.66)(1.0)(3.9/\text{rad})} \\ &= 0.49 \end{aligned}$$

Knowing τ we can use Figure 2.21 to estimate the area of the elevator to the area of the horizontal tail:

$$S_e/S_t = 0.30$$

The elevator area required to balance the largest trim moment is

$$\begin{aligned} S_e &= 0.30S_t \\ &= (0.3)(43 \text{ ft}^2) \\ S_e &= 13 \text{ ft}^2 \end{aligned}$$

This represents the minimum elevator area needed to balance the airplane. In practice the designer probably would increase this area to provide a margin of safety.

This example also points out the importance of proper weight and balance for an airplane. If the airplane is improperly loaded, so that the center of gravity moves forward of the manufacturers specification, the pilot may be unable to trim the airplane at the desired approach C_L . The pilot would be forced to trim the airplane at a lower lift coefficient, which means a higher landing speed.

2.4.3 Flight Measurement of X_{NP}

The equation developed for estimating the elevator angle to trim the airplane can be used to determine the stick fixed neutral point from flight test data. Suppose we conducted a flight test experiment in which we measured the elevator angle of trim at various air speeds for different positions of the center of gravity. If we did this, we could develop curves as shown in Figure 2.22.

Now, differentiating Equation (2.51) with respect to $C_{L_{\text{trim}}}$ yields

$$\frac{d\delta_{\text{trim}}}{dC_{L_{\text{trim}}}} = - \frac{C_{m_\alpha}}{C_{m_{\delta_e}} C_{L_\alpha} - C_{m_\alpha} C_{L_{\delta_e}}} \quad (2.52)$$

Note that when $C_{m_\alpha} = 0$ (i.e., the center of gravity is at the neutral point) Equation (2.53) equals 0. Therefore, if we measure the slopes of the curves in

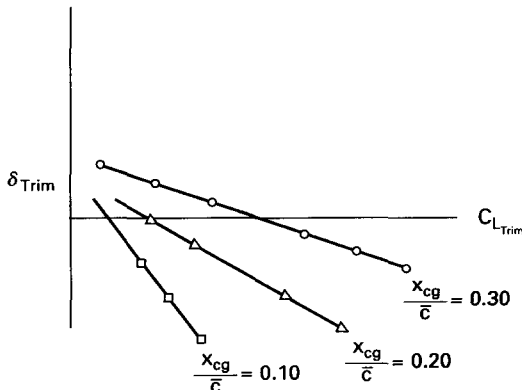


FIGURE 2.22
 $\delta_{e_{\text{trim}}}$ versus $C_{L_{\text{trim}}}$.

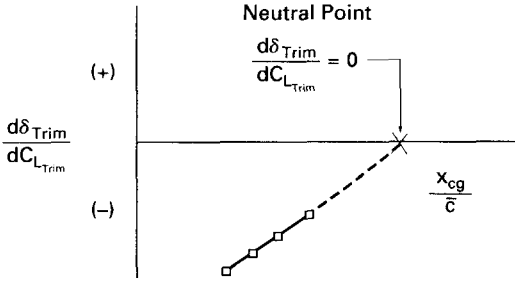


FIGURE 2.23

Figure 2.22 and plot them as a function of center of gravity location, we can estimate the stick fixed neutral point as illustrated in Figure 2.23 by extrapolating to find the center of gravity position that makes $d\delta_{trim}/dC_{L_{trim}}$ equal to 0.

2.4.4 Elevator Hinge Moment

It is important to know the moment acting at the hinge line of the elevator (or other type of control surface). The hinge moment, of course, is the moment the pilot must overcome by exerting a force on the control stick. Therefore to design the control system properly we must know the hinge moment characteristics. The hinge moment is defined as shown in Figure 2.24. If we assume that the hinge moment can be expressed as the addition of the effects of angle of attack, elevator deflection angle, and tab angle taken separately, then we can express the hinge moment coefficient in the following manner:

$$C_{h_e} = C_{h_0} + C_{h_{\alpha_i}} \alpha_i + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_t}} \delta_t \quad (2.53)$$

where C_{h_0} is the residual moment and

$$C_{h_{\alpha_i}} = \frac{dC_h}{d\alpha_i} \quad C_{h_{\delta_e}} = \frac{dC_h}{d\delta_e} \quad C_{h_{\delta_t}} = \frac{dC_h}{d\delta_t} \quad (2.54)$$

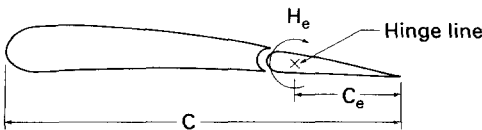
The hinge moment parameters just defined are very difficult to predict analytically with great precision. Wind-tunnel tests usually are required to provide the control system designer with the information needed to design the control system properly.

$$H_e = C_{h_e} \frac{1}{2} \rho V^2 S_e C_e$$

S_e = Area aft of the hinge line
 C_e = Chord measured from hinge line to trailing edge of the flap

FIGURE 2.24

Definition of hinge moments.



When the elevator is set free, that is, the control stick is released, the stability and control characteristics of the airplane are affected. For simplicity, we shall assume that both δ_i and C_{h_0} are equal to 0. Then, for the case when the elevator is allowed to be free,

$$C_{h_e} = 0 = C_{h_{\alpha_i}} \alpha_i + C_{h_{\delta_e}} \delta_e \quad (2.55)$$

Solving for δ_e yields

$$(\delta_e)_{\text{free}} = -\frac{C_{h_{\alpha_i}}}{C_{h_{\delta_e}}} \alpha_i \quad (2.56)$$

Usually, the coefficients $C_{h_{\alpha_i}}$ and $C_{h_{\delta_e}}$ are negative. If this indeed is the case, then Equation (2.56) tells us that the elevator will float upwards as the angle of attack is increased. The lift coefficient for a tail with a free elevator is given by

$$C_{L_t} = C_{L_{\alpha_i}} \alpha_i + C_{L_{\delta_e}} \delta_{e_{\text{free}}} \quad (2.57)$$

$$C_{L_t} = C_{L_{\alpha_i}} \alpha_i - C_{L_{\delta_e}} \frac{C_{h_{\alpha_i}}}{C_{h_{\delta_e}}} \alpha_i \quad (2.58)$$

which simplifies to

$$C_{L_t} = C_{L_{\alpha_i}} \alpha_i \left(1 - \frac{C_{L_{\delta_e}}}{C_{L_{\alpha_i}}} \frac{C_{h_{\alpha_i}}}{C_{h_{\delta_e}}} \right) = C'_{L_{\alpha_i}} \alpha_i \quad (2.59)$$

where

$$C'_{L_{\alpha_i}} = C_{L_{\alpha_i}} \left(1 - \frac{C_{L_{\delta_e}}}{C_{L_{\alpha_i}}} \frac{C_{h_{\alpha_i}}}{C_{h_{\delta_e}}} \right) = C_{L_{\alpha_i}} f \quad (2.60)$$

The slope of the tail lift curve is modified by the term in the parentheses. The factor f can be greater or less than unity, depending on the sign of the hinge parameters $C_{h_{\alpha_i}}$ and $C_{h_{\delta_e}}$. Now, if we were to develop the equations for the total pitching moment for the free elevator case, we would obtain an equation similar to Equations (2.34) and (2.35). The only difference would be that the term $C_{L_{\alpha_i}}$ would be replaced by $C'_{L_{\alpha_i}}$. Substituting $C'_{L_{\alpha_i}}$ into Equations (2.34) and (2.35) yields

$$C'_{m_0} = C_{m_{0w}} + C_{m_{0f}} + C'_{L_{\alpha_i}} \eta V_H (\epsilon_0 + i_w - i_t) \quad (2.61)$$

$$C'_{m_\alpha} = C_{L_{\alpha_w}} \left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}} \right) + C_{m_{\alpha f}} - C'_{L_{\alpha_i}} \eta V_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \quad (2.62)$$

where the prime indicates elevator-free values. To determine the influence of a free elevator on the static longitudinal stability, we again examine the condition in which $C_{m_\alpha} = 0$. Setting C'_{m_α} equal to 0 in Equation (2.62) and solving for $x/\bar{c}$ yields the stick-free neutral point:

$$\frac{x'_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} + V_H \eta \frac{C'_{L_{\alpha_i}}}{C_{L_{\alpha_w}}} \left(1 - \frac{d\epsilon}{d\alpha} \right) - \frac{C_{m_{\alpha f}}}{C_{L_{\alpha_w}}} \quad (2.63)$$

The difference between the stick fixed neutral point and the stick-free neutral point can be expressed as follows:

$$\frac{x_{NP}}{\bar{c}} - \frac{x'_{NP}}{\bar{c}} = (1 - f)V_H\eta\frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad (2.64)$$

The factor f determines whether the stick-free neutral point lies forward or aft of the stick fixed neutral point.

Static margin is a term that appears frequently in the literature. The static margin is simply the distance between the neutral point and the actual center of gravity position

$$\text{Stick fixed static margin} = \frac{x_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} \quad (2.65)$$

$$\text{Stick-free static margin} = \frac{x'_{NP}}{\bar{c}} - \frac{x_{cg}}{\bar{c}} \quad (2.66)$$

For most aircraft designs, it is desirable to have a stick fixed static margin of approximately 5 percent of the mean chord. The stick fixed or stick-free static neutral points represent an aft limit on the center of gravity travel for the airplane.

2.5

STICK FORCES

To deflect a control surface the pilot must move the control stick or rudder pedals. The forces exerted by the pilot to move the control surface is called the stick force or pedal force, depending which control is being used. The stick force is proportional to the hinge moment acting on the control surface:

$$F = \text{fn}(H_e) \quad (2.67)$$

Figure 2.25 is a sketch of a simple mechanical system used for deflecting the elevator. The work of displacing the control stick is equal to the work in moving the control surface to the desired deflection angle. From Figure 2.25 we can write the expression for the work performed at the stick and elevator:

$$Fl_s \delta_s = H_e \delta_e \quad (2.68)$$

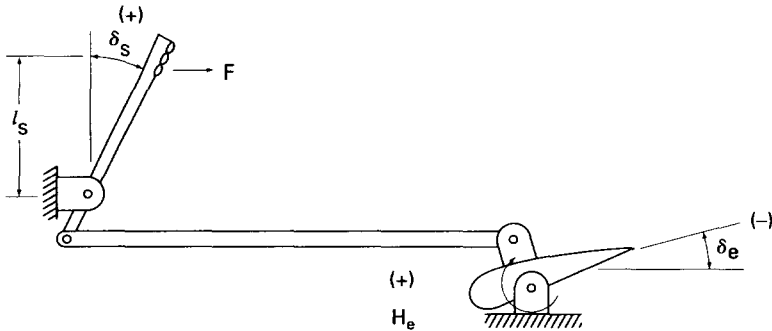
$$\text{or} \quad F = \frac{\delta_e}{l_s \delta_s} H_e \quad (2.69)$$

$$\text{or} \quad F = GH_e \quad (2.70)$$

where $G = \delta_e/(l_s \delta_s)$ called the gearing ratio, is a measure of the mechanical advantage provided by the control system.

Substituting the expression for the hinge moment defined earlier into the stick force equation yields

$$F = GC_{h_e} \frac{1}{2} \rho V^2 S_e \bar{c}_e \quad (2.71)$$

**FIGURE 2.25**

Relationship between stick force and hinge moment.

From this expression we see that the magnitude of the stick force increases with the size of the airplane and the square of the airplane's speed. Similar expressions can be obtained for the rudder pedal force and aileron stick force.

The control system is designed to convert the stick and pedal movements into control surface deflections. Although this may seem to be a relatively easy task, it in fact is quite complicated. The control system must be designed so that the control forces are within acceptable limits. On the other hand, the control forces required in normal maneuvers must not be too small; otherwise, it might be possible to overstress the airplane. Proper control system design will provide stick force magnitudes that give the pilot a feel for the commanded maneuver. The magnitude of the stick force provides the pilot with an indication of the severity of the motion that will result from the stick movement.

The convention for longitudinal control is that a pull force should always rotate the nose upward, which causes the airplane to slow down. A push force will have the opposite effect; that is, the nose will rotate downward and the airplane will speed up. The control system designer must also be sure that the airplane does not experience control reversals due to aerodynamic or aeroelastic phenomena.

2.5.1 Trim Tabs

In addition to making sure that the stick and rudder pedal forces required to maneuver or trim the airplane are within acceptable limits, it is important that some means be provided to zero out the stick force at the trimmed flight speed. If such a provision is not made, the pilot will become fatigued by trying to maintain the necessary stick force. The stick force at trim can be made zero by incorporating a tab on either the elevator or the rudder. The tab is a small flap located at the trailing edge of the control surface. The trim tab can be used to zero out the hinge moment and thereby eliminate the stick or pedal forces. Figure 2.26 illustrates the concept of a trim tab. Although the trim tab has a great influence over the hinge moment, it has only a slight effect on the lift produced by the control surface.

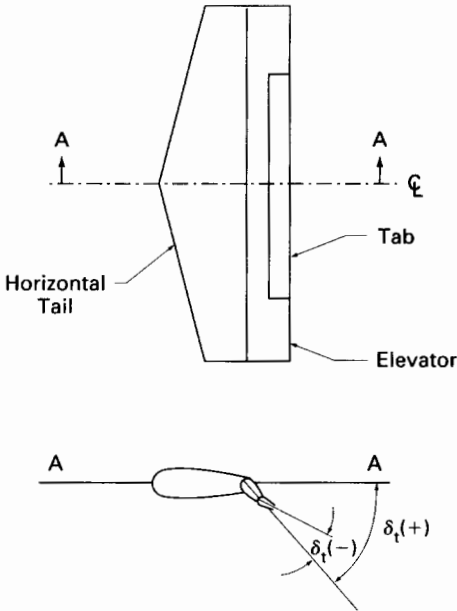


FIGURE 2.26
Trim tabs.

2.5.2 Stick Force Gradients

Another important parameter in the design of a control system is the stick force gradient. Figure 2.27 shows the variation of the stick force with speed. The stick force gradient is a measure of the change in stick force needed to change the speed of the airplane. To provide the airplane with speed stability, the stick force gradient must be negative; that is,

$$\frac{dF}{dV} < 0 \quad (2.72)$$

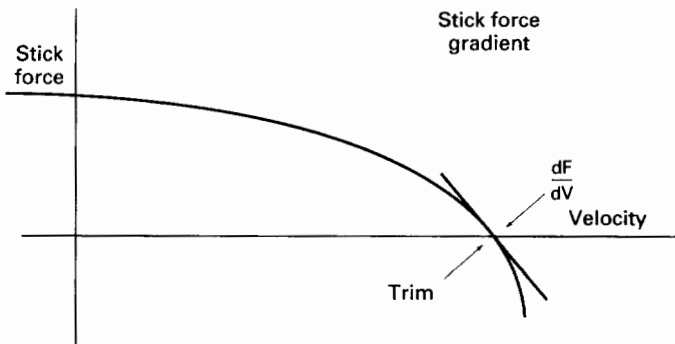


FIGURE 2.27
Stick force versus velocity.

The need for a negative stick-force gradient can be appreciated by examining the trim point in Figure 2.27. If the airplane slows down, a positive stick force occurs that rotates the nose of the airplane downward, which causes the airplane to increase its speed back toward the trim velocity. If the airplane exceeds the trim velocity, a negative (pull) stick force causes the airplane's nose to pitch up, which causes the airplane to slow down. The negative stick force gradient provides the pilot and airplane with speed stability. The larger the gradient, the more resistant the airplane will be to disturbances in the flight speed. If an airplane did not have speed stability the pilot would have to continuously monitor and control the airplane's speed. This would be highly undesirable from the pilot's point of view.

2.6

DEFINITION OF DIRECTIONAL STABILITY

Directional, or weathercock, stability is concerned with the static stability of the airplane about the z axis. Just as in the case of longitudinal static stability, it is desirable that the airplane should tend to return to an equilibrium condition when subjected to some form of yawing disturbance. Figure 2.28 shows the yawing

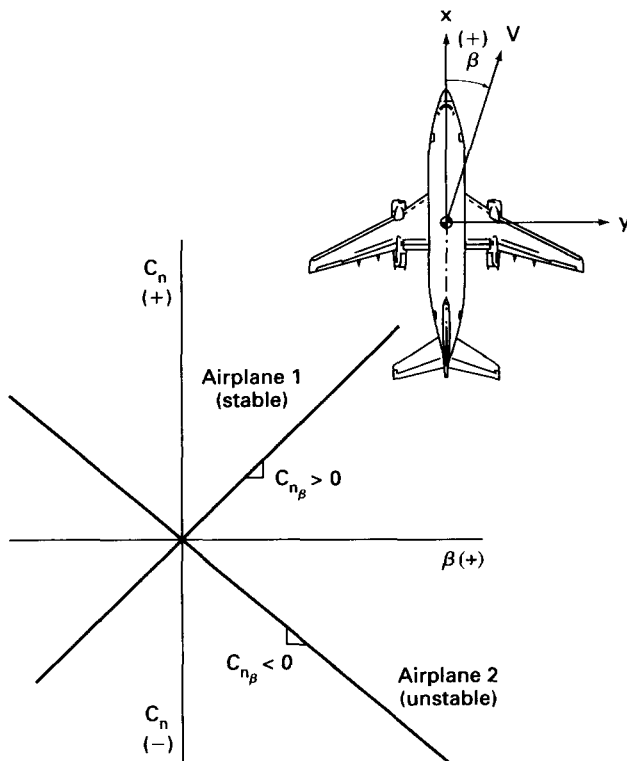


FIGURE 2.28
Static directional stability.

moment coefficient versus sideslip angle β for two airplane configurations. To have static directional stability, the airplane must develop a yawing moment that will restore the airplane to its equilibrium state. Assume that both airplanes are disturbed from their equilibrium condition, so that the airplanes are flying with a positive sideslip angle β . Airplane 1 will develop a restoring moment that will tend to rotate the airplane back to its equilibrium condition; that is, a zero sideslip angle. Airplane 2 will develop a yawing moment that will tend to increase the sideslip angle. Examining these curves, we see that to have static directional stability the slope of the yawing moment curve must be positive ($C_{n_\beta} > 0$). Note that an airplane possessing static directional stability will always point into the relative wind, hence the name weathercock stability.

2.6.1 Contribution of Aircraft Components

The contribution of the wing to directional stability usually is quite small in comparison to the fuselage, provided the angle of attack is not large. The fuselage and engine nacelles, in general, create a destabilizing contribution to directional stability. The wing fuselage contribution can be calculated from the following empirical expression taken from [2.7]:

$$C_{n_{\beta_{wf}}} = -k_n k_{Rl} \frac{S_{fs} l_f}{S_w b} \quad (\text{per deg}) \quad (2.73)$$

where k_n = an empirical wing-body interference factor that is a function of the fuselage geometry

k_{Rl} = an empirical correction factor that is a function of the fuselage Reynolds number

S_{fs} = the projected side area of the fuselage

l_f = the length of the fuselage

The empirical factors k_n and k_{Rl} are determined from Figures 2.29 and 2.30 respectively.

Since the wing-fuselage contribution to directional stability is destabilizing, the vertical tail must be properly sized to ensure that the airplane has directional stability. The mechanism by which the vertical tail produces directional stability is shown in Figure 2.31. If we consider the vertical tail surface in Figure 2.31, we see that when the airplane is flying at a positive sideslip angle the vertical tail produces a side force (lift force in the xy plane) that tends to rotate the airplane about its center of gravity. The moment produced is a restoring moment. The side force acting on the vertical tail can be expressed as

$$Y_v = -C_{L_{\alpha_v}} \alpha_v Q_v S_v \quad (2.74)$$

where the subscript v refers to properties of the vertical tail. The angle of attack α_v that the vertical tail plane will experience can be written as

$$\alpha_v = \beta + \sigma \quad (2.75)$$

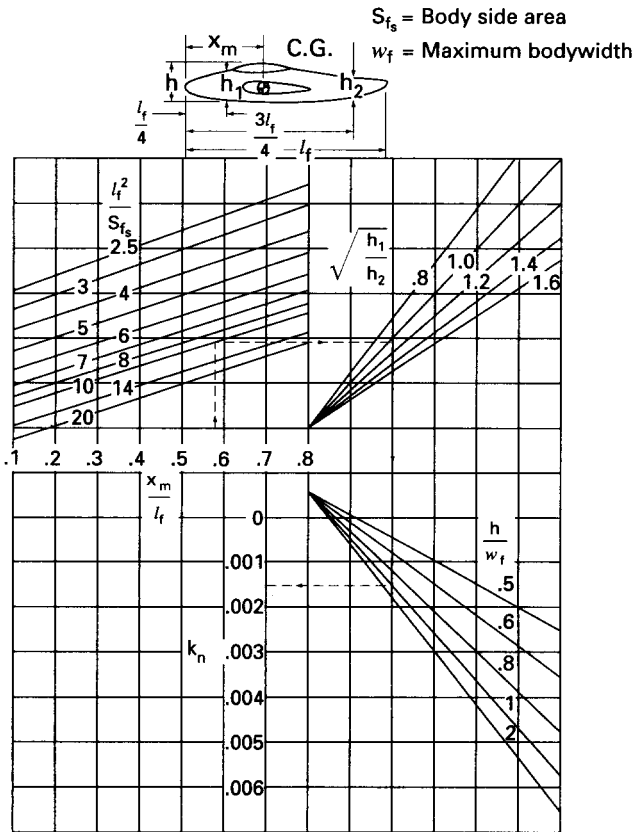


FIGURE 2.29
Wing body interference factor.

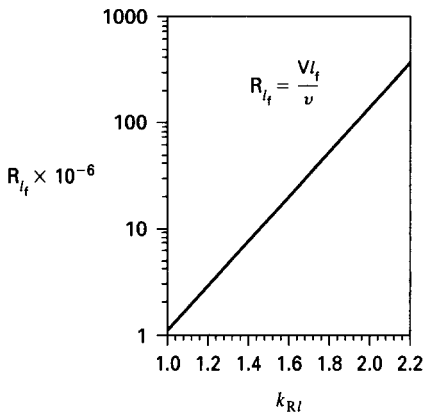


FIGURE 2.30
Reynolds number correction factor.

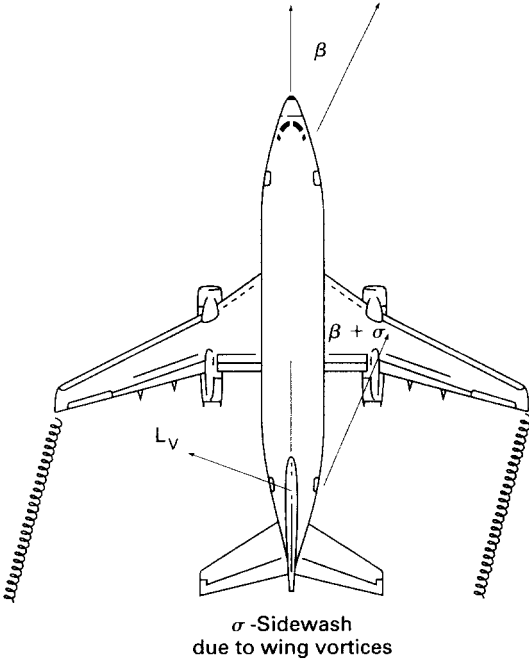


FIGURE 2.31
Vertical tail contribution to
directional stability.

where σ is the sidewash angle. The sidewash angle is analogous to the downwash angle ε for the horizontal tail plane. The sidewash is caused by the flow field distortion due to the wings and fuselage. The moment produced by the vertical tail can be written as a function of the side force acting on it:

$$N_v = l_v Y_v = l_v C_{L_{a_v}} (\beta + \sigma) Q_v S_v \quad (2.76)$$

or in coefficient form

$$C_n = \frac{N_v}{Q_w S b} = \frac{l_v S_v}{S b} \frac{Q_v}{Q_w} C_{L_{a_v}} (\beta + \sigma) \quad (2.77)$$

$$= V_v \eta_v C_{L_{a_v}} (\beta + \sigma) \quad (2.78)$$

where $V_v = l_v S_v / (S b)$ is the vertical tail volume ratio and $\eta_v = Q_v / Q_w$ is the ratio of the dynamic pressure at the vertical tail to the dynamic pressure at the wing.

The contribution of the vertical tail to directional stability now can be obtained by taking the derivative of Equation (2.78) with respect to β :

$$C_{n_{\beta_v}} = V_v \eta_v C_{L_{a_v}} \left(1 + \frac{d\sigma}{d\beta} \right) \quad (2.79)$$

A simple algebraic equation for estimating the combined sidewash and tail efficiency factor η_v is presented in [2.7] and reproduced here:

$$\eta_v \left(1 + \frac{d\sigma}{d\beta} \right) = 0.724 + 3.06 \frac{S_v/S}{1 + \cos \Lambda_{c/4w}} + 0.4 \frac{z_w}{d} + 0.009 AR_w \quad (2.80)$$

where S = the wing area
 S_v = the vertical tail area, including the submerged area to the fuselage centerline
 z_w = the distance, parallel to the z axis, from wing root quarter chord point to fuselage centerline
 d = the maximum fuselage depth
 AR_w = the aspect ratio of the wing
 $\Lambda_{c/4w}$ = sweep of wing quarter chord.

2.7 DIRECTIONAL CONTROL

Directional control is achieved by a control surface, called a rudder, located on the vertical tail, as shown in Figure 2.32. The rudder is a hinged flap that forms the aft portion of the vertical tail. By rotating the flap, the lift force (side force) on the fixed vertical surface can be varied to create a yawing moment about the center of gravity. The size of the rudder is determined by the directional control requirements. The rudder control power must be sufficient to accomplish the requirements listed in Table 2.1.

The yawing moment produced by the rudder depends on the change in lift on the vertical tail due to the deflection of the rudder times its distance from the center of gravity. For a positive rudder deflection, a positive side force is created on the vertical tail. A positive side force will produce a negative yawing moment:

$$N = -l_v Y_v \quad (2.81)$$

where the side force is given by

$$Y_v = C_{L_v} Q_v S_v$$

Rewriting this equation in terms of a yawing moment coefficient yields

$$C_n = \frac{N}{Q_w S b} = -\frac{Q_v}{Q_w} \frac{l_v S_v}{S b} \frac{dC_{L_v}}{d\delta_r} \delta_r \quad (2.82)$$

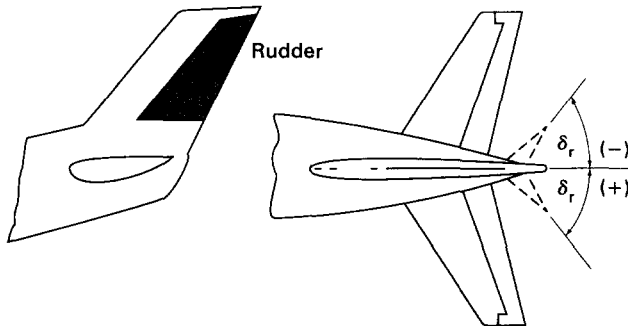


FIGURE 2.32
Directional control by means of the rudder.

TABLE 2.1

Requirements for directional control

Rudder requirements	Implication for rudder design
Adverse yaw	When an airplane is banked to execute a turning maneuver the ailerons may create a yawing moment that opposes the turn (i.e., adverse yaw). The rudder must be able to overcome the adverse yaw so that a coordinated turn can be achieved. The critical condition for adverse yaw occurs when the airplane is flying slow (i.e., high C_{L_v})
Crosswind landings	To maintain alignment with the runway during a crosswind landing requires the pilot to fly the airplane at a sideslip angle. The rudder must be powerful enough to permit the pilot to trim the airplane for the specified crosswinds. For transport airplanes, landing may be carried out for crosswinds up to 15.5 m/s or 51 ft/s.
Asymmetric power condition	The critical asymmetric power condition occurs for a multiengine airplane when one engine fails at low flight speeds. The rudder must be able to overcome the yawing moment produced by the asymmetric thrust arrangement.
Spin recovery	The primary control for spin recovery in many airplanes is a powerful rudder. The rudder must be powerful enough to oppose the spin rotation.

$$\text{or} \quad C_n = -\eta_v V_v \frac{dC_{L_v}}{d\delta_r} \delta_r \quad (2.83)$$

The rudder control effectiveness is the rate of change of yawing moment with rudder deflection angle:

$$C_n = C_{n_{\delta_r}} \delta_r = -\eta_v V_v \frac{dC_{L_v}}{d\delta_r} \delta_r \quad (2.84)$$

$$\text{or} \quad C_{n_{\delta_r}} = -\eta_v V_v \frac{dC_{L_v}}{d\delta_r} \quad (2.85)$$

$$\text{where} \quad \frac{dC_{L_v}}{d\delta_r} = \frac{dC_{L_v}}{d\alpha_v} \frac{d\alpha_v}{d\delta_r} = C_{L_{\alpha_v}} \tau \quad (2.86)$$

and the factor τ can be estimated from Figure 2.21.

2.8

ROLL STABILITY

An airplane possesses static roll stability if a restoring moment is developed when it is disturbed from a wings-level attitude. The restoring rolling moment can be shown to be a function of the sideslip angle β as illustrated in Figure 2.33. The requirement for stability is that $C_{l_\beta} < 0$. The roll moment created on an airplane

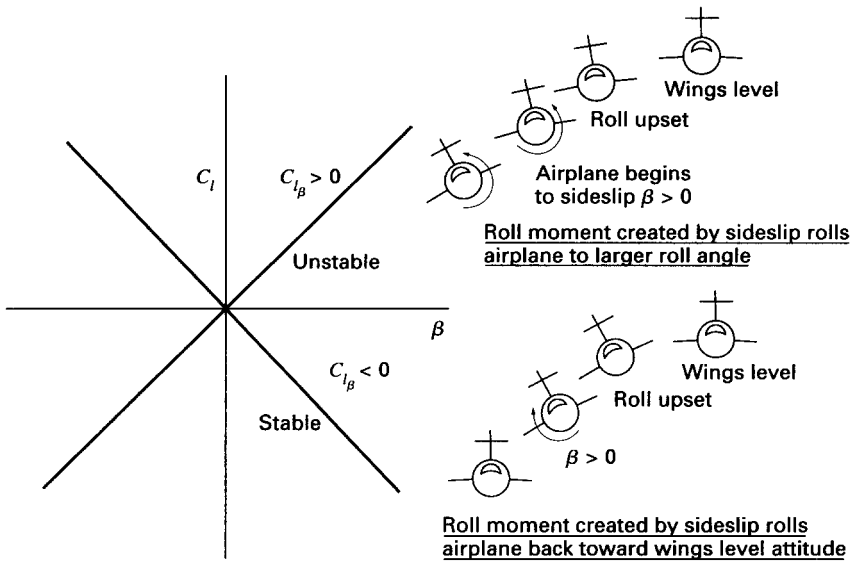


FIGURE 2.33
Static roll stability.

when it starts to sideslip depends on the wing dihedral, wing sweep, position of the wing on the fuselage, and the vertical tail. Each of these contributions will be discussed qualitatively in the following paragraphs.

The major contributor to $C_{l_{\beta}}$ is the wing dihedral angle Γ . The dihedral angle is defined as the spanwise inclination of the wing with respect to the horizontal. If the wing tip is higher than the root section, then the dihedral angle is positive; if the wing tip is lower than the root section, then the dihedral angle is negative. A negative dihedral angle is commonly called anhedral.

When an airplane is disturbed from a wings-level attitude, it will begin to sideslip as shown in Figure 2.34. Once the airplane starts to sideslip a component of the relative wind is directed toward the side of the airplane. The leading wing experiences an increased angle of attack and consequently an increase in lift. The trailing wing experiences the opposite effect. The net result is a rolling moment that tries to bring the wing back to a wings-level attitude. This restoring moment is often referred to as the dihedral effect.

The additional lift created on the downward-moving wing is created by the change in angle of attack produced by the sideslipping motion. If we resolve the sideward velocity component into components along and normal to the wing span the local change in angle of attack can be estimated as

$$\Delta\alpha = \frac{v_n}{u} \quad (2.87)$$

where

$$v_n = V \sin \Gamma \quad (2.88)$$

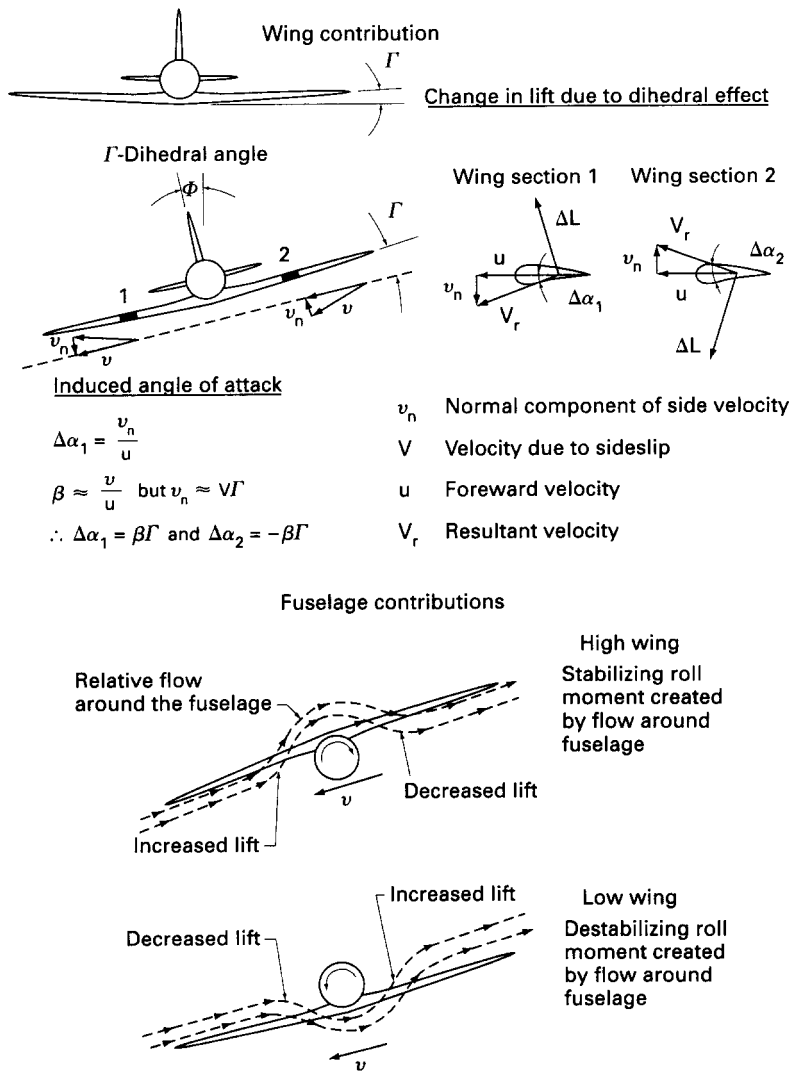


FIGURE 2.34
Wing and fuselage contribution to the dihedral

By approximating the sideslip angle as

$$\beta = \frac{v}{u} \tag{2.89}$$

and assuming that Γ is a small angle, the change of attack can be written as

$$\Delta \alpha \cong \beta \Gamma \tag{2.90}$$

The angle of attack on the upward-moving wing will be decreased by the same amount. Methods for estimating the wing contribution to C_{l_β} can again be found in [2.7].

Wing sweep also contributes to the dihedral effect. In a sweptback wing, the windward wing has an effective decrease in sweep angle and the trailing wing experiences an effective increase in sweep angle. For a given angle of attack, a decrease in sweepback angle will result in a higher lift coefficient. Therefore, the windward wing (with a less effective sweep) will experience more lift than the trailing wing. It can be concluded that sweepback adds to the dihedral effect. On the other hand, sweep forward will decrease the effective dihedral effect.

The fuselage contribution to dihedral effect is illustrated in Figure 2.34. The sideward flow turns in the vicinity of the fuselage and creates a local change in wing angle of attack at the inboard wing stations. For a low wing position, the fuselage contributes a negative dihedral effect; the high wing produces a positive dihedral effect. To maintain the same C_{l_β} , a low-wing aircraft will require a considerably greater wing dihedral angle than a high-wing configuration.

The horizontal tail also can contribute to the dihedral effect in a manner similar to the wing. However, owing to the size of the horizontal tail with respect to the wing, its contribution is usually small. The contribution to dihedral effect from the vertical tail is produced by the side force on the tail due to sideslip. The side force on the vertical tail produces both a yawing moment and a rolling moment. The rolling moment occurs because the center of pressure for the vertical tail is located above the aircraft's center of gravity. The rolling moment produced by the vertical tail tends to bring the aircraft back to a wings-level attitude.

2.9 ROLL CONTROL

Roll control is achieved by the differential deflection of small flaps called ailerons which are located outboard on the wings, or by the use of spoilers. Figure 2.35 is a sketch showing both types of roll control devices. The basic principle behind these devices is to modify the spanwise lift distribution so that a moment is created about the x axis. An estimate of the roll control power for an aileron can be obtained by a simple strip integration method as illustrated in Figure 2.36 and the equations that follow. The incremental change in roll moment due to a change in aileron angle can be expressed as

$$\Delta L = (\Delta \text{Lift})y \quad (2.91)$$

which can be written in coefficient form as

$$\Delta C_l = \frac{\Delta L}{Q S b} = \frac{C_l Q c y \, dy}{Q S b} \quad (2.92)$$

$$= \frac{C_l c y \, dy}{S b} \quad (2.93)$$

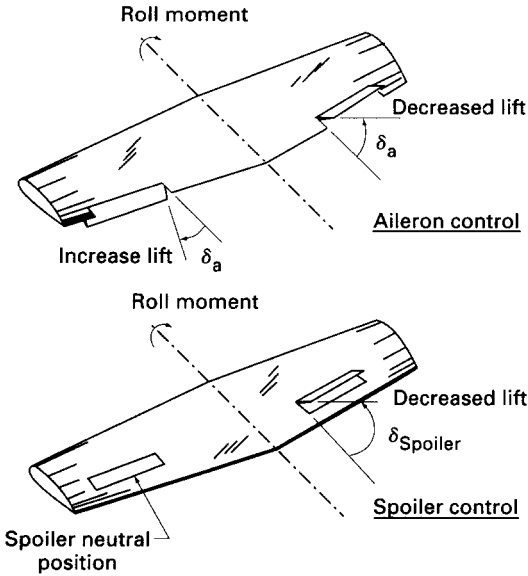


FIGURE 2.35
Aileron and spoilers for roll control.

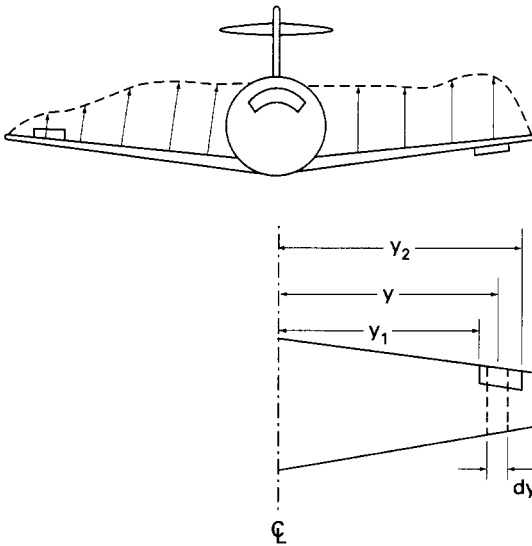


FIGURE 2.36
Strip theory approximation
of roll control effectiveness.

The section lift coefficient C_l on the stations containing the aileron can be written as

$$C_l = C_{l_a} \frac{d\alpha}{d\delta_a} \delta_a = C_{l_a} \tau \delta_a \quad (2.94)$$

which is similar to the technique used to estimate the control effectiveness of an elevator and rudder. Substituting Equation (2.93) into Equation (2.94) and

integrating over the region containing the aileron yields

$$C_l = \frac{2C_{L_{\alpha_w}} \tau \delta_a}{Sb} \int_{y_1}^{y_2} cy \, dy \quad (2.95)$$

where $C_{L_{\alpha_w}}$ and τ have been corrected for three-dimensional flow and the factor of 2 has been introduced to account for the other aileron. The control power $C_{l_{\delta_a}}$ can be obtained by taking the derivative with respect to δ_a :

$$C_{l_{\delta_a}} = \frac{2C_{L_{\alpha_w}} \tau}{Sb} \int_{y_1}^{y_2} cy \, dy \quad (2.96)$$

EXAMPLE PROBLEM 2.4. For the NAVION airplane described in Appendix B, estimate the roll control power, $C_{l_{\delta_a}}$. Assume that the wing and aileron geometry are as shown in Figure 2.37.

Solution. Equation (2.96) can be used to estimate the roll control power, $C_{l_{\delta_a}}$.

$$C_{l_{\delta_a}} = \frac{2C_{L_{\alpha_w}} \tau}{Sb} \int_{y_1}^{y_2} cy \, dy$$

$b/2 = 16.7 \text{ ft.}$	$\lambda = 0.54$	$c_r = 7.2 \text{ ft.}$
$c_t = 3.9 \text{ ft.}$	$y_1 = 11.1 \text{ ft.}$	$y_2 = 16 \text{ ft.}$
$S = 184 \text{ ft.}^2$	$C_{L_{\alpha_w}} = 4.44/\text{rad.}$	$c_a/c = 0.18 \text{ ft.}$

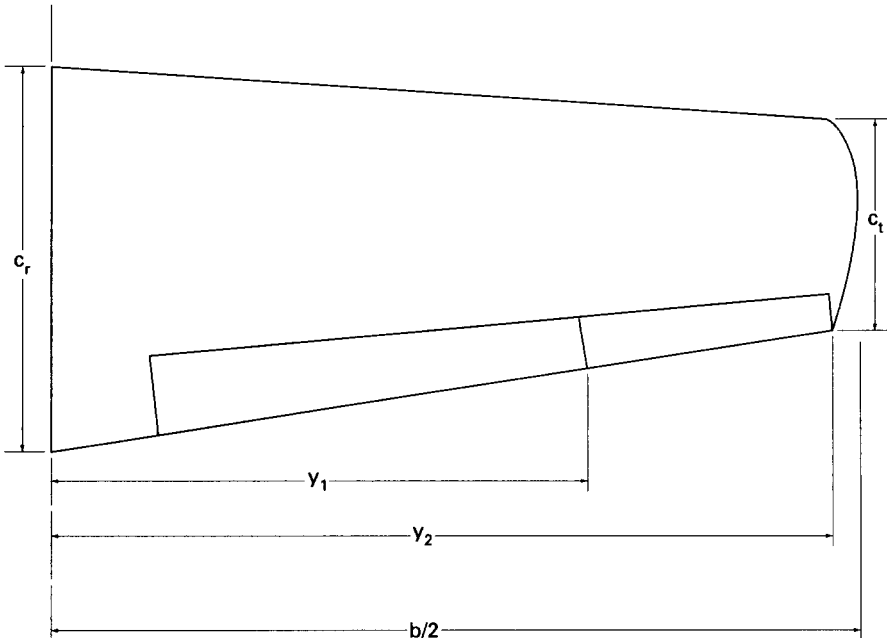


FIGURE 2.37
Approximate wing geometry of the NAVION airplane.

For a tapered wing the chord can be expressed as a function of y by the following relationship:

$$c = c_r \left[1 + \left(\frac{\lambda - 1}{b/2} \right) y \right]$$

Substituting the relationship for the chord back into the expression for $C_{l_{\delta a}}$ yields

$$C_{l_{\delta a}} = \frac{2C_{L_{\alpha_w}} \tau C_r}{Sb} \int_{y_1}^{y_2} \left[1 + \left(\frac{\lambda - 1}{b/2} \right) y \right] y \, dy$$

or

$$C_{l_{\delta a}} = \frac{2C_{L_{\alpha_w}} \tau C_r}{Sb} \left[\frac{y^2}{2} + \left(\frac{\lambda - 1}{b/2} \right) \frac{y^3}{3} \right] \Big|_{y_1}^{y_2}$$

This equation can be used to estimate $C_{l_{\delta a}}$ using the data in Figure 2.37 and estimating τ from Figure 2.21. Because the chord ratio is the same as the area ratio used in Figure 2.21, we can use $c_a/c = 0.18$ to estimate the flap effectiveness parameter, τ .

$$\begin{aligned} C_{l_{\delta a}} &= \frac{2(4.3/\text{rad})(0.36)(7.2 \text{ ft})}{(184 \text{ ft}^2)(33.4 \text{ ft})} (90.4 \text{ ft}^2 - 49 \text{ ft}^2) \\ &= 0.155/\text{rad} \end{aligned}$$

The control derivative $C_{l_{\delta a}}$ is a measure of the power of the aileron control; it represents the change in moment per unit of aileron deflection. The larger $C_{l_{\delta a}}$, the more effective the control is at producing a roll moment.

2.10 SUMMARY

The requirements for static stability were developed for longitudinal, lateral directional, and rolling motions. It is easy to see why a pilot would require the airplane that he or she is flying to possess some degree of static stability. Without static stability the pilot would have to continuously control the airplane to maintain a desired flight path, which would be quite fatiguing. The degree of static stability desired by the pilot has been determined through flying quality studies and will be discussed in a later chapter. The important point at this time is to recognize that the airplane must be made statically stable, either through inherent aerodynamic characteristics or by artificial means through the use of an automatic control system.

The inherent static stability tendencies of the airplane were shown to be a function of its geometric and aerodynamic properties. The designer can control the degree of longitudinal and lateral directional stability by proper sizing of the horizontal and vertical tail surfaces, whereas roll stability was shown to be a consequence of dihedral effect, which is controlled by the wing's placement or dihedral angle.

In addition to static stability, the pilot wants sufficient control to keep the airplane in equilibrium (i.e., trim) and to maneuver. Aircraft response to control input and control force requirements are important flying quality characteristics

determined by the control surface size. The stick force and stick force gradient are important parameters that influence how the pilot feels about the flying characteristics of the airplane. Stick forces must provide the pilot a feel for the maneuver initiated. In addition, we show that the stick force gradient provides the airplane with speed stability. If the longitudinal stick force gradient is negative at the trim flight speed, then the airplane will resist disturbances in speed and fly at a constant speed.

Finally, the relationship between static stability and control was examined. An airplane that is very stable statically will not be very maneuverable; if the airplane has very little static stability, it will be very maneuverable. The degree of maneuverability or static stability is determined by the designer on the basis of the airplane's mission requirements.

PROBLEMS

- 2.1. If the slope of the C_m versus C_L curve is -0.15 and the pitching moment at zero lift is equal to 0.08 , determine the trim lift coefficient. If the center of gravity of the airplane is located at $X_{cg}/\bar{c} = 0.3$, determine the stick fixed neutral point.
- 2.2. For the data shown in Figure P2.2, determine the following:
- The stick fixed neutral point.
 - If we wish to fly the airplane at a velocity of 125 ft/s at sea level, what would be the trim lift coefficient and what would be the elevator angle for trim?

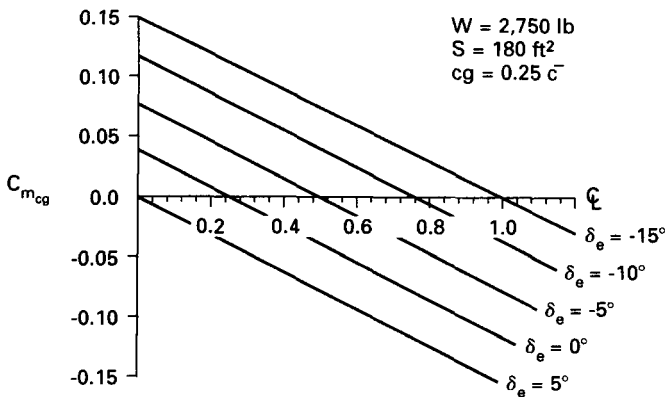


FIGURE P2.2

- 2.3. Analyze the canard-wing combination shown in Figure P2.3. The canard and wing are geometrically similar and are made from the same airfoil section.

$$AR_c = AR_w \quad S_c = 0.2S_w \quad \bar{c}_c = 0.45\bar{c}_w$$

- Develop an expression for the moment coefficient about the center of gravity. You may simplify the problem by neglecting the upwash (downwash) effects between

the lifting surfaces and the drag contribution to the moment. Also assume small angle approximations.

(b) Find the neutral point for this airplane.

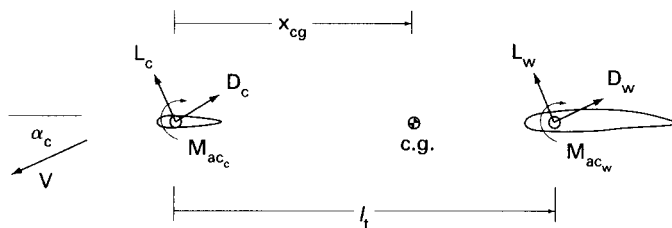


FIGURE P2.3

2.4. The C_m versus, α curve for a large jet transport can be seen in Figure P2.4. Use the figure and the following information to answer questions (a) to (c).

$$C_L = 0.03 + 0.08\alpha \text{ (deg.)}$$

$$-15^\circ \leq \delta_e \leq 20^\circ$$

- (a) Estimate the stick fixed neutral point.
 (b) Estimate the control power $C_{m\delta_e}$.
 (c) Find the forward center of gravity limit. Hint:

$$\frac{dC_{m_{cg}}}{dC_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{NP}}{\bar{c}}$$

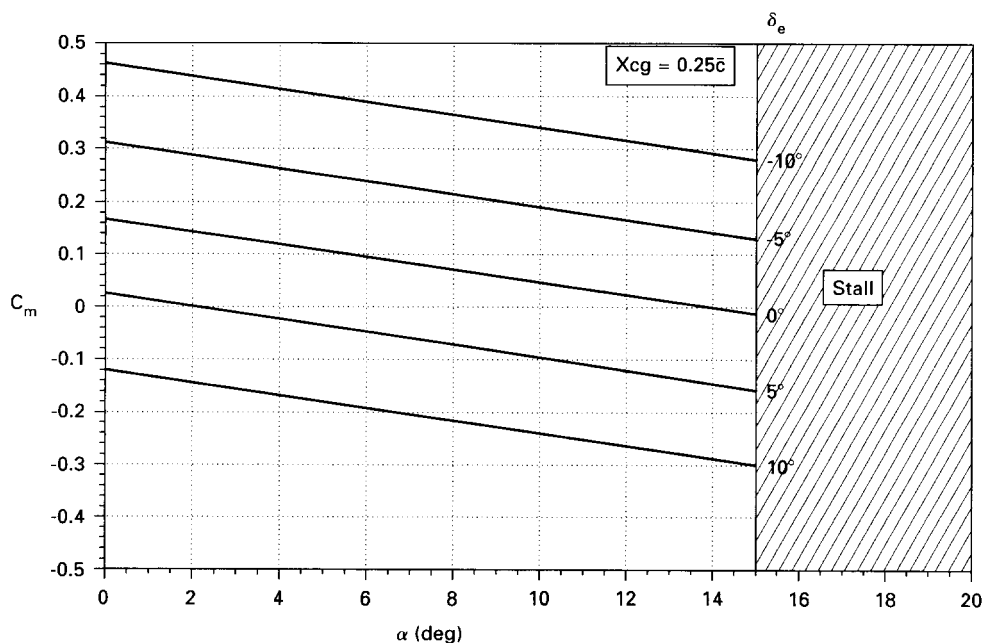


FIGURE P2.4

2.5. Using the data for the business jet aircraft included in Appendix B, determine the following longitudinal stability information at subsonic speeds:

- Wing contribution to the pitching moment
- Tail contribution to the pitching moment
- Fuselage contribution to the pitching moment
- Total pitching moment
- Plot the various contributions
- Estimate the stick fixed neutral point

2.6. An airplane has the following pitching moment characteristics at the center of gravity position:

$$x_{cg}/\bar{c} = 0.3.$$

$$C_{m_{cg}} = C_{m_0} + \frac{dC_{m_{cg}}}{dC_L} C_L + C_{m_{\delta_e}} \delta_e$$

where $C_{m_0} = 0.05$ $\frac{dC_{m_{cg}}}{dC_L} = -0.1$ $C_{m_{\delta_e}} = -0.01/\text{deg}$

$$\frac{dC_{m_{cg}}}{dC_L} = \left[\frac{X_{cg}}{\bar{c}} \right] - \left[\frac{X_{NP}}{\bar{c}} \right]$$

If the airplane is loaded so that the center of gravity position moves to $x_{cg}/\bar{c} = 0.10$, can the airplane be trimmed during landing, $C_L = 1.0$? Assume that C_{m_0} and $C_{m_{\delta_e}}$ are unaffected by the center of gravity travel and that $\delta_{e_{\max}} = \pm 20^\circ$.

2.7. The pitching moment characteristics of a general aviation airplane with the landing gear and flaps in their retracted position are given in Figure P2.7.

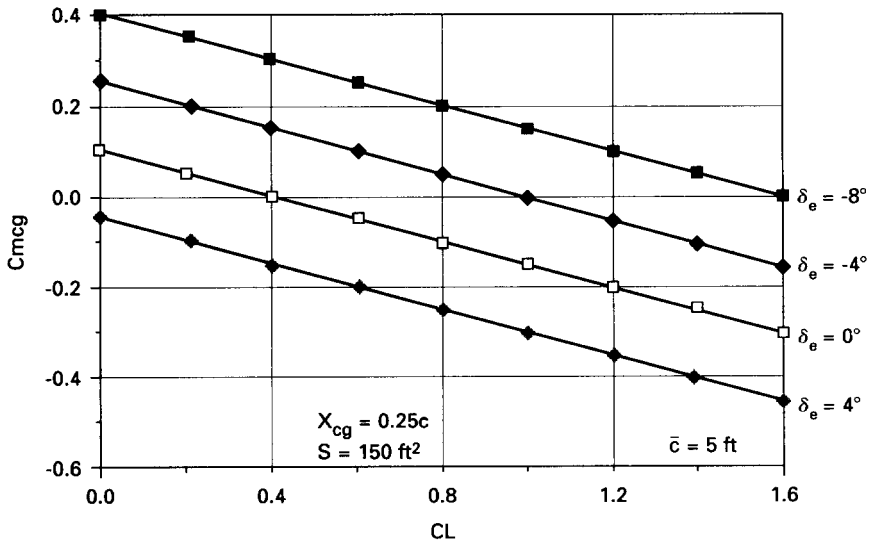


FIGURE P2.7

Pitching moment characteristics of a general aviation airplane.

- (a) Where is the stick fixed neutral point located?
- (b) If the airplane weighs 2500 lbs and is flying at 150 ft/s at sea level, $\rho = 0.002378$ slug/ft³, what is the elevator angle required for trim?
- (c) Discuss what happens to the pitching moment curve when the landing gear is deployed? How does the deflection of the high lift flaps affect the stability of the airplane?

2.8. Estimate the fuselage and engine nacelle contribution to C_{m_α} using the method discussed in section 2.3 for the STOL transport shown in Figure P2.8. The airplane has

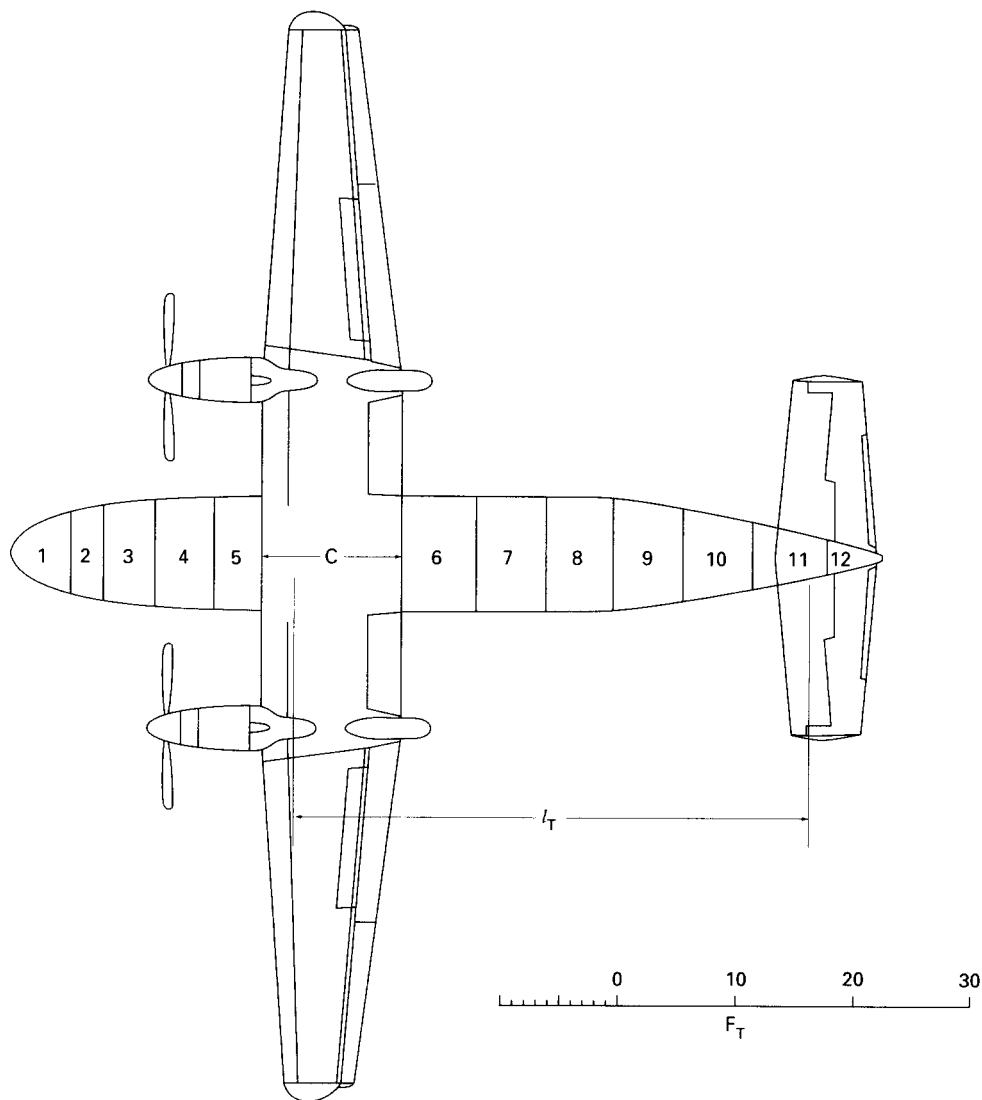


FIGURE P2.8

been divided into 12 sections as indicated in Figure P2.8. The section length, width, and distance from the wing leading or trailing edge to the midpoint of each section is given in the table below. The engine nacelles have been approximated by one section as indicated on the figure.

Station	Fuselage		
	Δx ft	w_f ft	x_i ft
1	4.4	4.4	20.2
2	2.5	6.9	17.0
3	5.0	8.8	13.9
4	5.0	9.5	7.6
5	5.0	10.1	2.5
6	6.3	10.1	2.5
7	6.3	10.1	8.8
8	6.3	10.1	15.1
9	6.3	8.2	21.4
10	6.3	7.6	27.7
11	5.0	5.1	33.4
12	5.0	2.5	39.7

Assume that $c = 12.6$ ft (the fuselage region between the wing leading and trailing edge), $l_h = 34$ ft (the distance from the wing trailing edge to the quarter chord of the horizontal tail), and $d\varepsilon/d\alpha$ at the tail is 0.34.

- 2.9. The downwash angle at zero angle of attack and the rate of change of downwash with angle of attack can be determined experimentally by several techniques. The downwash angle can be measured directly by using a five- or seven-hole pressure probe to determine the flow direction at the position of the tail surface or indirectly from pitching moment data measured from wind-tunnel models. This latter technique will be demonstrated by way of this problem. Suppose that a wind-tunnel test were conducted to measure the pitching moment as a function of the angle of attack for various tail incidence settings as well as for the case when the tail surface is removed. Figure P2.9 plots such information. Notice that the tail-off data intersect the

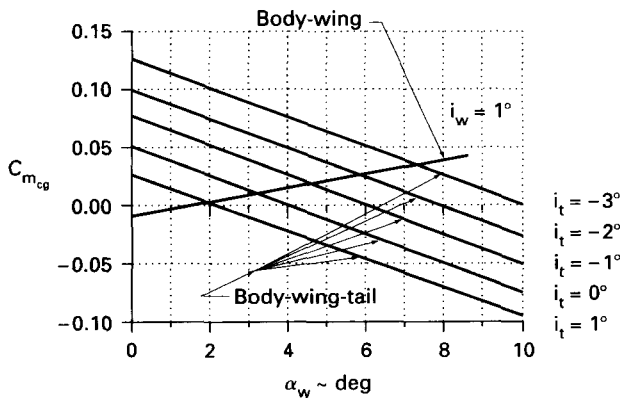


FIGURE P2.9

complete configuration data at several points. At the points of intersection, the contribution of the tail surface to the pitching moment curve must be 0. For this to be the case, the lift on the tail surface is 0, which implies that the tail angle of attack is 0 at these points. From the definition of the tail angle of attack,

$$\alpha_t = \alpha_w - i_w - \varepsilon + i_t$$

we obtain

$$\varepsilon = \alpha_w - i_w + i_t$$

at the interception points. Using the data of Figure P2.9 determine the downwash angle versus the angle of attack of the wing. From this information estimate ε_0 and $d\varepsilon/d\alpha$.

- 2.10.** The airplane in Example Problem 2.2 has the following hinge moment characteristics:

$$\begin{array}{llll} C_{L_{\alpha_w}} = 0.09/\text{deg} & C_{h_{\alpha}} = -0.003/\text{deg} & C_{h_{\delta}} = -0.005/\text{deg} & V_H = 0.4 \\ C_{L_{\alpha_t}} = 0.08/\text{deg} & C_{h_0} = 0.0 & S_e/S_t = 0.35 & de/d\alpha = 0.4 \end{array}$$

What would be the stick-free neutral point location?

- 2.11.** As an airplane nears the ground its aerodynamic characteristics are changed by the presence of the ground plane. This change is called ground effect. A simple model for determining the influence of the ground on the lift drag and pitching moment can be obtained by representing the airplane by a horseshoe vortex system with an image as shown in Figure P2.11. Using this sketch, shown qualitatively, explain the changes that one might expect; that is, whether the forces and moment increase or decrease.

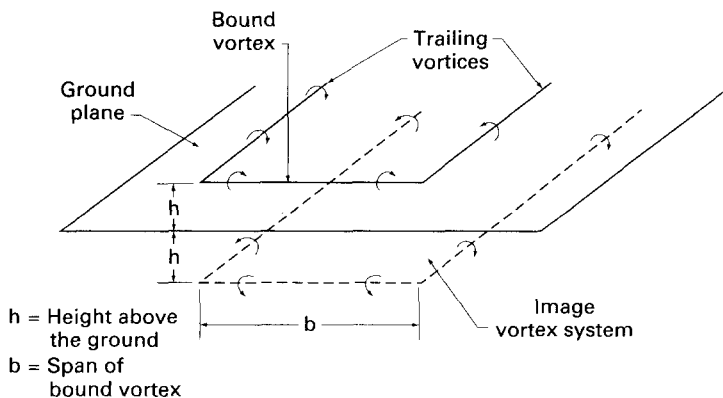


FIGURE P2.11

- 2.12.** If the control characteristics of the elevator used in Example Problem 2.2 are as follows, determine the forwardmost limit on the center of gravity travel so that the airplane can be controlled during landing; that is, at $C_{L_{\max}}$. Neglect ground effects on the airplane's aerodynamic characteristics:

$$C_{m_{\delta_e}} = -1.03/\text{rad} \quad \delta_{e_{\max}} = \begin{cases} +10^\circ \\ -20^\circ \end{cases} \quad C_{L_{\max}} = 1.4$$

- 2.13. Size the vertical tail for the airplane configuration shown in Figure P2.13 so that its weathercock stability has a value of $C_{n_\beta} = +0.1 \text{ rad}^{-1}$. Clearly state your assumptions. Assume $V = 150 \text{ m/s}$ at sea level.

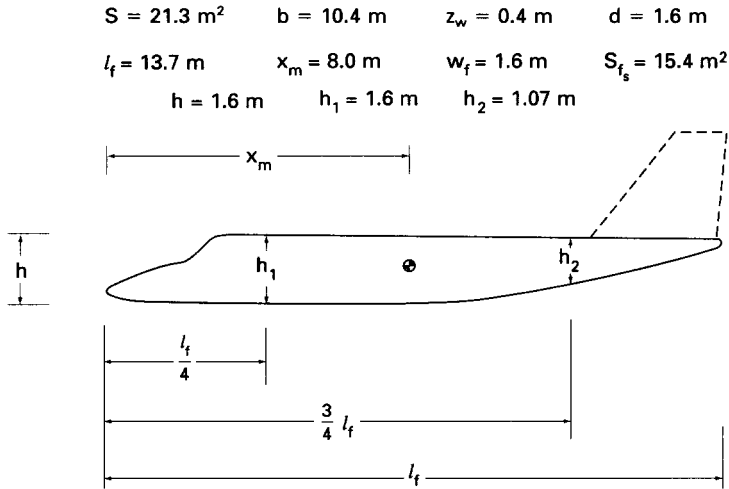


FIGURE P2.13

- 2.14. Figure P2.14 is a sketch of a wing planform for a business aviation airplane.
- Use strip theory to determine the roll control power.
 - Comment on the accuracy of the strip theory integration technique.

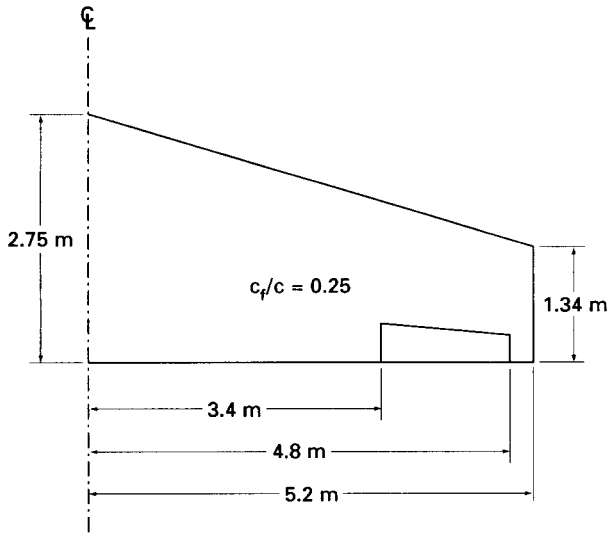


FIGURE P2.14

- 2.15.** Suppose the wing planform in Problem 2.14 is incorporated into a low-wing aircraft design. Find the wing dihedral angle necessary to produce a dihedral effect of $C_{l_\beta} = -0.1 \text{ rad}^{-1}$. Neglect the fuselage interference on the wing dihedral contribution.
- 2.16.** For the twin engine airplane shown in Figure P2.16, determine the rudder size to control the airplane if one engine needs to be shut down. Use the flight information shown in the figure and

Wing: $S = 980 \text{ ft}^2$ $b = 93 \text{ ft}$

Vertical tail: $S_v = 330 \text{ ft}^2$ $AR_v = 4.3$ $l_v = 37 \text{ ft}$ $\eta_v = 1.0$

Rudder: $\delta_r = \pm 15^\circ$

Propulsion: $T = 14,000 \text{ lb each}$ $y_T = 16 \text{ ft}$

Flight condition: $V = 250 \text{ ft/s}$ $\rho = 0.002378 \text{ slug/ft}^3$

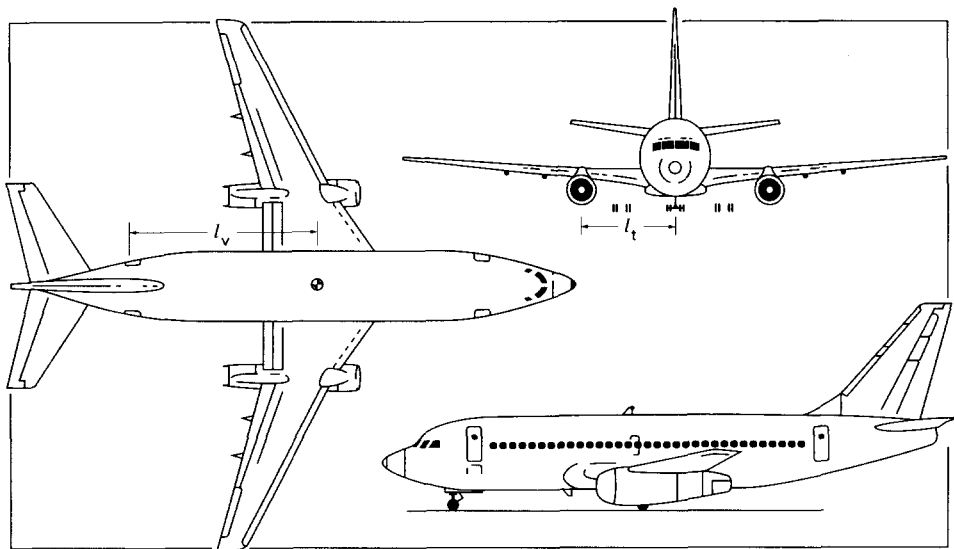


FIGURE P2.16

- 2.17.** The elevator for a business jet aircraft is shown in Figure P2.17. Estimate the elevator's control power $C_{m_{\delta_e}}$ using the geometric information that follows:

$S = 232 \text{ ft}^2$ $AR_t = 4.0$

$\bar{c} = 7.0 \text{ ft}$ $l_t = 21.6 \text{ ft}$

$b_t = 14.7 \text{ ft}$ $C_{l_{\alpha_t}} = 0.1/\text{deg}(2D)$

$S_t = 54 \text{ ft}^2$

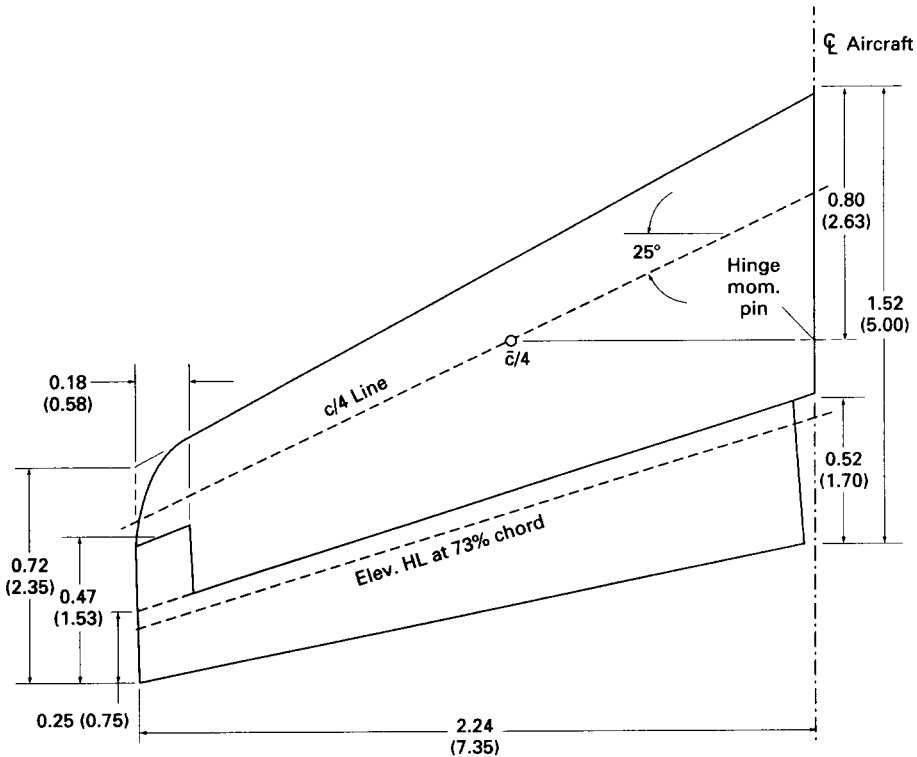


FIGURE P2.17

- 2.18. Develop an expression for the wing dihedral effect C_{l_β} for a wing planform that uses dihedral only for the outboard portion of the wing (see Figure P2.18). Clearly state all of your assumptions.

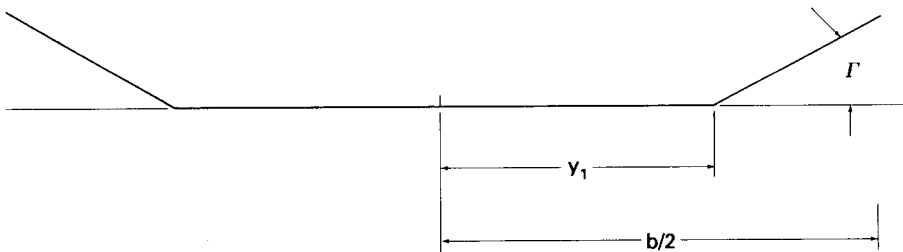
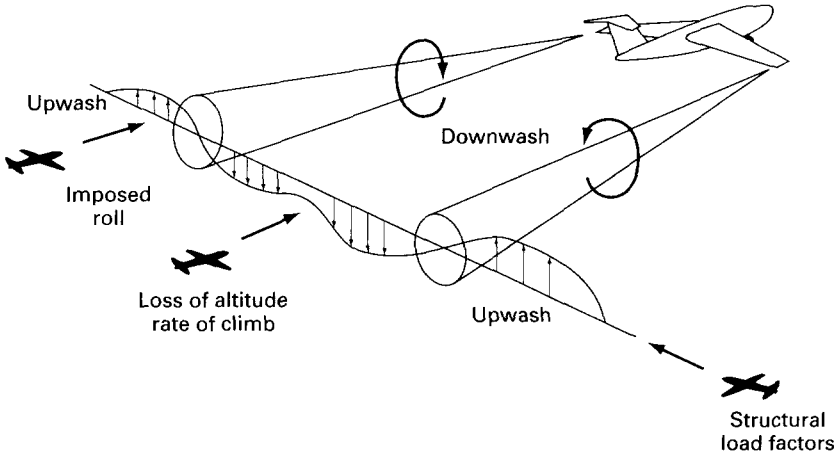


FIGURE P2.18

- 2.19. The trailing vortex wake left behind by an airplane can be a safety hazard to following aircraft as illustrated in Figure P2.19. The most likely place to encounter the wake of another aircraft is in the vicinity of the airport during takeoff or landing. To minimize

**FIGURE P2.19**

the possibility of a wake encounter the FAA has developed a separation criteria between aircraft of different sizes. If an elliptic wing loading is assumed, the strength of the trailing wake can be shown to be related to the size and speed of the generating aircraft.

$$L = W = \rho V \Gamma b'$$

where L = lift
 W = weight
 ρ = air density
 V = velocity of the airplane
 Γ = vortex strength
 b' = effective span of vortices.

The effective span of the wing tip vortices for an elliptic load distribution can be shown to be

$$b' = \frac{\pi}{4} b$$

where b is the wingspan of the generating aircraft. Solving for the circulation (i.e., vortex strength) yields

$$\Gamma = \frac{W}{\rho V b'}$$

The tangential velocity field at some point downstream created by one of the vortices is given by

$$v_\theta = \left(\frac{\Gamma}{2\pi a_c} \right) r \quad r \leq a_c$$

$$v_\theta = \frac{\Gamma}{2\pi r} \quad r \geq a_c$$

From the simple analysis presented here it is clear that the vortex strength is proportional to the weight of the generating aircraft and inversely proportional to its speed. Therefore large heavy transports flying at approach or takeoff speeds will create the strongest wakes and the greatest hazard to following aircraft.

Wake vortices decay slowly in calm atmospheric conditions. Because the wake vortices decay very slowly in a calm atmosphere we will neglect vortex decay in this problem. Develop an expression for estimating the roll moment induced on an airplane wing when the wing is centered in the vortex core of another aircraft's trailing vortex wake.

- 2.20.** Using the expression developed in Problem 2.19, estimate the roll moment induced by the wake of a large jet transport on several smaller aircraft. Use the data in Appendix B. Use the information for the 747 for the generating aircraft and evaluate the roll moment induced on the Convair 880, STOL transport, business jet, and the NAVION. Compare the induced roll moment to the maximum roll moment that could be developed by full aileron deflection. Assume the aileron maximum deflection is $\pm 25^\circ$ for each aircraft.

REFERENCES

- 2.1. Dickman, E. W. *This Aviation Business*. New York: Brentano's, 1926.
- 2.2. Gibbs-Smith, C. H.; and T. D. Crouch. "Wilbur and Orville Wright—A 75th Anniversary Commemoration." *Astronautics and Aeronautics*, December 1978, pp. 8–15.
- 2.3. Perkins, C. D. "Development of Airplane Stability and Control Technology." *AIAA Journal of Aircraft* 7, no. 4 (1970).
- 2.4. Etkin, B. *Dynamics of Flight*. New York: Wiley, 1959.
- 2.5. Perkins, C. D.; and R. E. Hage. *Airplane Performance Stability and Control*. New York: Wiley, 1949.
- 2.6. McRuer, D.; I. Ashkenas; and D. Graham. *Aircraft Dynamics and Automatic Control*. Princeton, NJ: Princeton University Press, 1973.
- 2.7. *USAF Stability and Control Datcom*, Flight Control Division, Air Force Flight Dynamics Laboratory, Wright Patterson Air Force Base, Fairborn, OH.
- 2.8. Munk, M. M. *The Aerodynamic Forces on Airship Hulls*, NACA TR 184, 1924.
- 2.9. Multhopp, H. *Aerodynamics of Fuselage*, NACA TM-1036, 1942.