CHAPTER 1

Introduction

"For some years I have been afflicted with the belief that flight is possible to man."

Wilbur Wright, May 13, 1900

1.1 ATMOSPHERIC FLIGHT MECHANICS

Atmospheric flight mechanics is a broad heading that encompasses three major disciplines; namely, performance, flight dynamics, and aeroelasticity. In the past each of these subjects was treated independently of the others. However, because of the structural flexibility of modern airplanes, the interplay among the disciplines no longer can be ignored. For example, if the flight loads cause significant structural deformation of the aircraft, one can expect changes in the airplane's aerodynamic and stability characteristics that will influence its performance and dynamic behavior.

Airplane performance deals with the determination of performance characteristics such as range, endurance, rate of climb, and takeoff and landing distance as well as flight path optimization. To evaluate these performance characteristics, one normally treats the airplane as a point mass acted on by gravity, lift, drag, and thrust. The accuracy of the performance calculations depends on how accurately the lift, drag, and thrust can be determined.

Flight dynamics is concerned with the motion of an airplane due to internally or externally generated disturbances. We particularly are interested in the vehicle's stability and control capabilities. To describe adequately the rigid-body motion of an airplane one needs to consider the complete equations of motion with six degrees of freedom. Again, this will require accurate estimates of the aerodynamic forces and moments acting on the airplane.

The final subject included under the heading of atmospheric flight mechanics is aeroelasticity. Aeroelasticity deals with both static and dynamic aeroelastic phenomena. Basically, aeroelasticity is concerned with phenomena associated with interactions between inertial, elastic, and aerodynamic forces. Problems that arise for a flexible aircraft include control reversal, wing divergence, and control surface flutter, to name just a few.

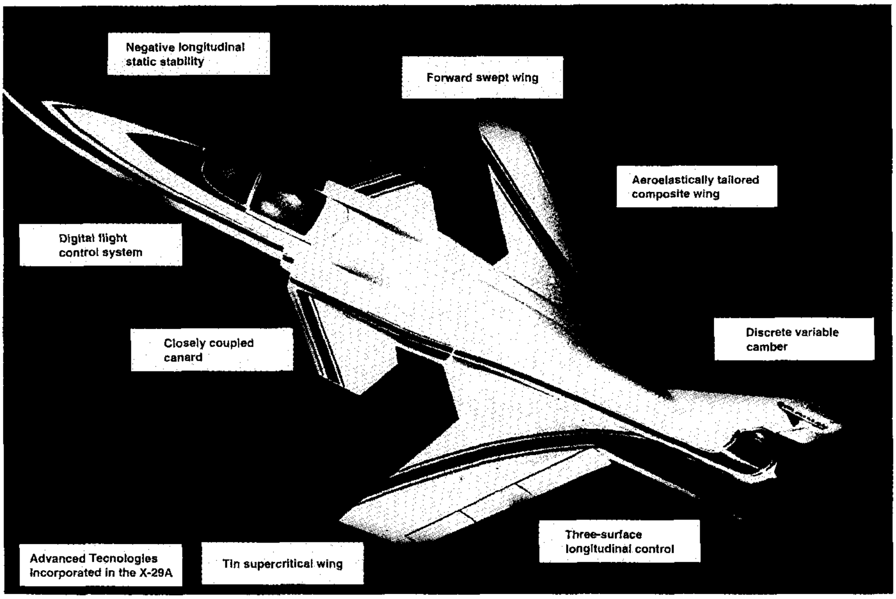


FIGURE 1.1
Advanced technologies incorporated in the X-29A aircraft.

This book is divided into three parts: The first part deals with the properties of the atmosphere, static stability and control concepts, development of aircraft equations of motion, and aerodynamic modeling of the airplane; the second part examines aircraft motions due to control inputs or atmospheric disturbances; the third part is devoted to aircraft autopilots. Although no specific chapters are devoted entirely to performance or aeroelasticity, an effort is made to show the reader, at least in a qualitative way, how performance specifications and aeroelastic phenomena influence aircraft stability and control characteristics.

The interplay among the three disciplines that make up atmospheric flight mechanics is best illustrated by the experimental high-performance airplane shown in Figure 1.1. The X-29A aircraft incorporates the latest advanced technologies in controls, structures, and aerodynamics. These technologies will provide substantial performance improvements over more conventional fighter designs. Such a design could not be developed without paying close attention to the interplay among performance, aeroelasticity, stability, and control. In fact, the evolution of this radical design was developed using trade-off studies between the various disciplines to justify the expected performance improvements.

The forces and moments acting on an airplane depend on the properties of the atmosphere through which it is flying. In the following sections we will review some basic concepts of fluid mechanics that will help us appreciate the atmospheric properties essential to our understanding of airplane flight mechanics. In addition we will discuss some of the important aircraft instruments that provide flight information to the pilot.

1.2 BASIC DEFINITIONS

The aerodynamic forces and moments generated on an airplane are due to its geometric shape, attitude to the flow, airspeed, and the properties of the ambient air mass through which it is flying. Air is a fluid and as such possesses certain fluid properties. The properties we are interested in are the pressure, temperature, density, viscosity, and speed of sound of air at the flight altitude.

1.2.1 Fluid

A fluid can be thought of as any substance that flows. To have such a property, the fluid must deform continuously when acted on by a shearing force. A shear force is a force tangent to the surface of the fluid element. No shear stresses are present in the fluid when it is at rest. A fluid can transmit forces normal to any chosen direction. The normal force and the normal stress are the pressure force and pressure, respectively.

Both liquids and gases can be considered fluids. Liquids under most conditions do not change their weight per unit of volume appreciably and can be considered incompressible for most engineering applications. Gases, on the other hand, change their weight or mass per unit of volume appreciably under the influences of pressure or temperature and therefore must be considered compressible.

1.2.2 Pressure

Pressure is the normal force per unit area acting on the fluid. The average pressure is calculated by dividing the normal force to the surface by the surface area:

$$P = \frac{F}{A} \tag{1.1}$$

The static pressure in the atmosphere is nothing more than the weight per unit of area of the air above the elevation being considered. The ratio of the pressure P at altitude to sea-level standard pressure P_0 is given the symbol δ:

$$\delta = \frac{P}{P_0} \tag{1.2}$$

The relationship between pressure, density ρ, and temperature T is given by the equation of state

$$P = \rho RT \tag{1.3}$$

where R is a constant, the magnitude depending on the gas being considered. For air, R has a value 287 J/(kg°K) or 1718 $\mathrm{ft^2/(s^{2}{}^\circ R)}$. Atmospheric air follows the equation of state provided that the temperature is not too high and that air can be treated as a continuum.

1.2.3 Temperature

In aeronautics the temperature of air is an extremely important parameter in that it affects the properties of air such as density and viscosity. Temperature is an abstract concept but can be thought of as a measure of the motion of molecular particles within a substance. The concept of temperature also serves as a means of determining the direction in which heat energy will flow when two objects of different temperatures come into contact. Heat energy will flow from the higher temperature object to that at lower temperature.

As we will show later the temperature of the atmosphere varies significantly with altitude. The ratio of the ambient temperature at altitude, T, to a sea-level standard value, T_0 is denoted by the symbol θ:

$$\theta = \frac{T}{T_0} \tag{1.4}$$

where the temperatures are measured using the absolute Kelvin or Rankine scales.

1.2.4 Density

The density of a substance is defined as the mass per unit of volume:

$$\rho = \frac{\text{Mass}}{\text{Unit of volume}} \tag{1.5}$$

From the equation of state, it can be seen that the density of a gas is directly proportional to the pressure and inversely proportional to the absolute temperature. The ratio of ambient air density ρ to standard sea-level air density ρ_0 occurs in many aeronautical formulas and is given the designation σ:

$$\sigma = \rho/\rho_0 \tag{1.6}$$

1.2.5 Viscosity

Viscosity can be thought of as the internal friction of a fluid. Both liquids and gases possess viscosity, with liquids being much more viscous than gases. As an aid in visualizing the concept of viscosity, consider the following simple experiment. Consider the motion of the fluid between two parallel plates separated by the distance h. If one plate is held fixed while the other plate is being pulled with a constant velocity u, then the velocity distribution of the fluid between the plates will be linear as shown in Figure 1.2.

To produce the constant velocity motion of the upper plate, a tangential force must be applied to the plate. The magnitude of the force must be equal to the

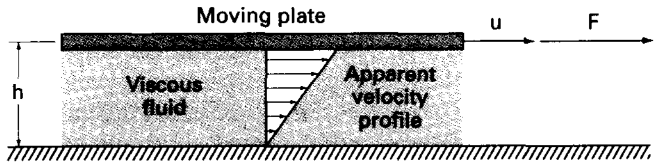


FIGURE 1.2
Shear stress between two plates.

friction forces in the fluid. It has been established from experiments that the force per unit of area of the plate is proportional to the velocity of the moving plate and inversely proportional to the distance between the plates. Expressed mathematically we have

$$\tau \propto \frac{u}{h} \tag{1.7}$$

where τ is the force per unit area, which is called the shear stress.

A more general form of Equation (1.7) can be written by replacing u/h with the derivative du/dy. The proportionality factor is denoted by μ, the coefficient of absolute viscosity, which is obtained experimentally.

$$\tau = \mu \frac{du}{dy} \tag{1.8}$$

Equation (1.8) is known as Newton's law of friction.

For gases, the absolute viscosity depends only on the temperature, with increasing temperature causing an increase in viscosity. To estimate the change in viscosity with the temperature, several empirical formulations commonly are used. The simplest formula is Rayleigh's, which is

$$\frac{\mu_1}{\mu_0} = \left(\frac{T_1}{T_0}\right)^{3/4} \tag{1.9}$$

where the temperatures are on the absolute scale and the subscript 0 denotes the reference condition.

An alternate expression for calculating the variation of absolute viscosity with temperature was developed by Sutherland. The empirical formula developed by Sutherland is valid provided the pressure is greater than 0.1 atmosphere and is

$$\frac{\mu_1}{\mu_0} = \left(\frac{T_1}{T_0}\right)^{3/2} \frac{T_0 + S_1}{T_1 + S_1} \tag{1.10}$$

where S_1 is a constant. When the temperatures are expressed in the Rankine scale, $S_1 = 198°R$; when the temperatures are expressed in the Kelvin scale, $S_1 = 110°K$.

The ratio of the absolute viscosity to the density of the fluid is a parameter that appears frequently and has been identified with the symbol ν; it is called the kinematic viscosity:

$$\nu = \frac{\mu}{\rho} \tag{1.11}$$

An important dimensionless quantity, known as the Reynolds number, is defined as

$$R_e = \frac{\rho V l}{\mu} = \frac{V l}{\nu} \tag{1.12}$$

where l is a characteristic length and V is the fluid velocity.

The Reynolds number can be thought of as the ratio of the inertial to viscous forces of the fluid.

1.2.6 The Mach Number and the Speed of Sound

The ratio of an airplane's speed V to the local speed of sound a is an extremely important parameter, called the Mach number after the Austrian physicist Ernst Mach. The mathematical definition of Mach number is

$$M = \frac{V}{a} \tag{1.13}$$

As an airplane moves through the air, it creates pressure disturbances that propagate away from the airplane in all directions with the speed of sound. If the airplane is flying at a Mach number less than 1, the pressure disturbances travel faster than the airplane and influence the air ahead of the airplane. An example of this phenomenon is the upwash field created in front of a wing. However, for flight at Mach numbers greater than 1 the pressure disturbances move more slowly than the airplane and, therefore, the flow ahead of the airplane has no warning of the oncoming aircraft.

The aerodynamic characteristics of an airplane depend on the flow regime around the airplane. As the flight Mach number is increased, the flow around the airplane can be completely subsonic, a mixture of subsonic and supersonic flow, or completely supersonic. The flight Mach number is used to classify the various flow regimes. An approximate classification of the flow regimes follows:

Incompressible subsonic flow	$0 < M < 0.5$
Compressible subsonic flow	$0.5 < M < 0.8$
Transonic flow	$0.8 < M < 1.2$
Supersonic flow	$1.2 < M < 5$
Hypersonic flow	$5 < M$

To have accurate aerodynamic predictions at $M > 0.5$ compressibility effects must be included.

The local speed of sound must be known to determine the Mach number. The speed of sound can be shown to be related to the absolute ambient temperature by the following expression:

$$a = (\gamma RT)^{1/2} \tag{1.14}$$

where γ is the ratio of specific heats and R is the gas constant. The ambient temperature will be shown in a later section to be a function of altitude.

1.3 AEROSTATICS

Aerostatics deals with the state of a gas at rest. It follows from the definition given for a fluid that all forces acting on the fluid must be normal to any cross-section within the fluid. Unlike a solid, a fluid at rest cannot support a shearing force. A consequence of this is that the pressure in a fluid at rest is independent of direction. That is to say that at any point the pressure is the same in all directions. This fundamental concept owes its origin to Pascal, a French scientist (1623–1662).

1.3.1 Variation of Pressure in a Static Fluid

Consider the small vertical column of fluid shown in Figure 1.3. Because the fluid is at rest, the forces in both the vertical and horizontal directions must sum to 0. The forces in the vertical direction are due to the pressure forces and the weight of the fluid column. The force balance in the vertical direction is given by

$$PA = (P + \mathrm{d}P)A + \rho g A\,\mathrm{d}h \tag{1.15}$$

or

$$\mathrm{d}P = -\rho g\,\mathrm{d}h \tag{1.16}$$

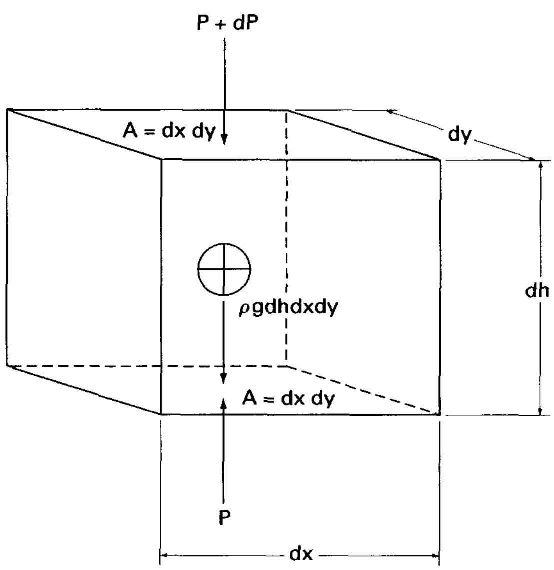


FIGURE 1.3
Element of fluid at rest.

Equation (1.16) tells us how the pressure varies with elevation above some reference level in a fluid. As the elevation is increased, the pressure will decrease. Therefore, the pressure in a static fluid is equal to the weight of the column of fluid above the point of interest.

One of the simplest means of measuring pressure is by a fluid manometer. Figure 1.4 shows two types of manometers. The first manometer consists of a U-shaped tube containing a liquid. When pressures of different magnitudes are applied across the manometer the fluid will rise on the side of the lower pressure and fall on the side of the higher pressure. By writing a force balance for each side, one can show that

$$P_1 A + \rho g x A = P_2 A + \rho g(x + h)A \tag{1.17}$$

which yields a relationship for the pressure difference in terms of the change in height of the liquid column:

$$P_1 - P_2 = \rho g h \tag{1.18}$$

The second sketch shows a simple mercury barometer. The barometer can be thought of as a modified U-tube manometer. One leg of the tube is closed off and evacuated. The pressure at the top of this leg is 0 and atmospheric pressure acts on the open leg. The atmospheric pressure therefore is equal to the height of the mercury column; that is,

$$P_{atm} = \rho g h \tag{1.19}$$

In practice the atmospheric pressure is commonly expressed as so many inches or millimeters of mercury. Remember, however, that neither inches nor millimeters of mercury are units of pressure.

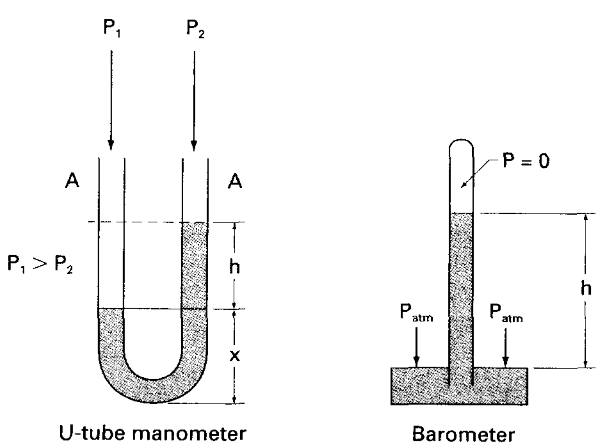


FIGURE 1.4
Sketch of U-tube manometer and barometer.

1.4 DEVELOPMENT OF BERNOULLI'S EQUATION

Bernoulli's equation establishes the relationship between pressure, elevation, and velocity of the flow along a stream tube. For this analysis, the fluid is assumed to be a perfect fluid; that is, we will ignore viscous effects. Consider the element of fluid in the stream tube shown in Figure 1.5. The forces acting on the differential element of fluid are due to pressure and gravitational forces. The pressure force acting in the direction of the motion is given by

$$F_{\text{pressure}} = P\,\mathrm{d}A - \left(P + \frac{\partial P}{\partial s}\,\mathrm{d}s\right) dA \tag{1.20}$$

or

$$= -\mathrm{d}P\,\mathrm{d}A \tag{1.21}$$

The gravitational force can be expressed as

$$F_{\text{gravitational}} = -g\,\mathrm{d}m \sin\alpha \tag{1.22}$$

$$= -g\,\mathrm{d}m\,\frac{\mathrm{d}z}{\mathrm{d}s} \tag{1.23}$$

Applying Newton's second law yields

$$-\mathrm{d}P\,\mathrm{d}A - g\,\mathrm{d}m\,\frac{\mathrm{d}z}{\mathrm{d}s} = \mathrm{d}m\,\frac{\mathrm{d}V}{\mathrm{d}t} \tag{1.24}$$

The differential mass dm can be expressed in terms of the mass density of the fluid element times its respective volume; that is,

$$\mathrm{d}m = \rho\,\mathrm{d}A\,\mathrm{d}s \tag{1.25}$$

Inserting the expression for the differential mass, the acceleration of the fluid can

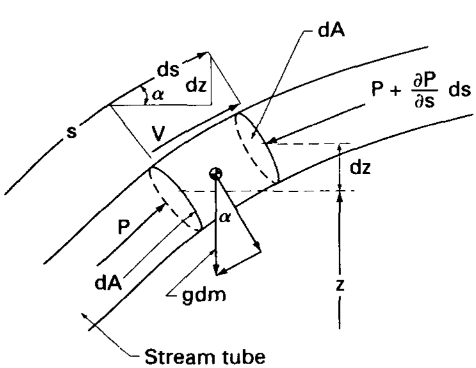


FIGURE 1.5
Forces acting on an element of flow in a stream tube.

be expressed as

$$\frac{dV}{dt} = -\frac{1}{\rho}\frac{dP}{ds} - g\frac{dz}{ds} \tag{1.26}$$

The acceleration can be expressed as

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial s}\frac{ds}{dt} \tag{1.27}$$

The first term on the right-hand side, $\partial V/\partial t$, denotes the change in velocity as a function of time for the entire flow field. The second term denotes the acceleration due to a change in location. If the flow field is steady, the term $\partial V/\partial t = 0$ and Equation (1.27) reduce to

$$\frac{\partial V}{\partial s}\frac{ds}{dt} = -\frac{1}{\rho}\frac{dP}{ds} - g\frac{dz}{ds} \tag{1.28}$$

The changes of pressure as a function of time cannot accelerate a fluid particle. This is because the same pressure would be acting at every instant on all sides of the fluid particles. Therefore, the partial differential can be replaced by the total derivative in Equation (1.28):

$$V\frac{dV}{ds} = -\frac{1}{\rho}\frac{dP}{ds} - g\frac{dz}{ds} \tag{1.29}$$

Integrating Equation (1.29) along a streamline yields

$$\int_1^2 V\,dV = -\int_1^2 \frac{dP}{\rho} - g\int_1^2 dz \tag{1.30}$$

which is known as Bernoulli's equation. Bernoulli's equation establishes the relationship between pressure, elevation, and velocity along a stream tube.

1.4.1 Incompressible Bernoulli Equation

If the fluid is considered to be incompressible, Equation (1.29) readily can be integrated to yield the incompressible Bernoulli equation:

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 \tag{1.31}$$

The differences in elevation usually can be ignored when dealing with the flow of gases such as air. An important application of Bernoulli's equation is the determination of the so-called stagnation pressure of a moving body or a body exposed to a flow. The stagnation point is defined as that point on the body at which the flow comes to rest. At that point the pressure is

$$P_0 = P_\infty + \frac{1}{2}\rho V_\infty^2 \tag{1.32}$$

where P_∞ and V_∞ are the static pressure and velocity far away from the body; that is, the pressures and velocities that would exist if the body were not present. In the case of a moving body, V_∞ is equal to the velocity of the body itself and P_∞ is the static pressure of the medium through which the body is moving.

1.4.2 Bernoulli's Equation for a Compressible Fluid

At higher speeds (on the order of 100 m/s), the assumption that the fluid density of gases is constant becomes invalid. As speed is increased, the air undergoes a compression and, therefore, the density cannot be treated as a constant. If the flow can be assumed to be isentropic, the relationship between pressure and density can be expressed as

$$P = c\rho^\gamma \tag{1.33}$$

where γ is the ratio of specific heats for the gas. For air, γ is approximately 1.4.

Substituting Equation (1.33) into Equation (1.30) and performing the indicated integrations yields the compressible form of Bernoulli's equation:

$$\frac{\gamma}{\gamma - 1}\frac{P}{\rho} + \frac{1}{2}V^2 + gz = \text{constant} \tag{1.34}$$

As noted earlier, the elevation term usually is quite small for most aeronautical applications and therefore can be ignored. The stagnation pressure can be found by letting $V = 0$, in Equation (1.34):

$$\frac{\gamma}{\gamma - 1}\frac{P}{\rho} + \frac{1}{2}V^2 = \frac{\gamma}{\gamma - 1}\frac{P_0}{\rho_0} \tag{1.35}$$

If we rearrange Equation (1.35), we obtain

$$1 + \frac{\gamma - 1}{2}\frac{1}{\gamma P/\rho}V^2 = \frac{P_0/P}{\rho_0/\rho} \tag{1.36}$$

Equation (1.36) can be solved for the velocity by substituting the following expressions,

$$a^2 = \gamma RT = \gamma P/\rho \tag{1.37}$$

and

$$\frac{P_0}{P} = \left(\frac{\rho_0}{\rho}\right)^\gamma \tag{1.38}$$

into Equation (1.36) and rearranging to yield a relationship for the velocity and the Mach number as follows.

$$V = \left[\frac{2a^2}{\gamma - 1}\left[\left(\frac{P_0}{P}\right)^{(\gamma-1)/\gamma} - 1\right]\right]^{1/2} \tag{1.39}$$

$$\mathrm{M} = \left[\frac{2}{\gamma - 1}\left[\left(\frac{P_0}{P}\right)^{(\gamma-1)/\gamma} - 1\right]\right]^{1/2} \tag{1.40}$$

Equations (1.39) and (1.40) can be used to find the velocity and Mach number provided the flow regime is below M = 1.

1.5 THE ATMOSPHERE

The performance characteristics of an airplane depend on the properties of the atmosphere through which it flies. Because the atmosphere is continuously changing with time, it is impossible to determine airplane performance parameters precisely without first defining the state of the atmosphere.

The earth's atmosphere is a gaseous envelope surrounding the planet. The gas that we call air actually is a composition of numerous gases. The composition of dry air at sea level is shown in Table 1.1. The relative percentages of the constituents remains essentially the same up to an altitude of 90 km or 300,000 ft owing primarily to atmospheric mixing caused by winds and turbulence. At altitudes above 90 km the gases begin to settle or separate. The variability of water vapor in the atmosphere must be taken into account by the performance analyst. Water vapor can constitute up to 4 percent by volume of atmospheric air. When the relative humidity is high, the air density is lower than that for dry air for the same conditions of pressure and temperature. Under these conditions the density may be reduced by as much as 3 percent. A change in air density will cause a change in the aerodynamic forces acting on the airplane and therefore influence its performance capabilities. Furthermore, changes in air density created by water vapor will affect engine performance, which again influences the performance of the airplane.

TABLE 1.1
Composition of atmospheric air

	Density		Percentage by volume	Percentage by weight
	kg/m³	slugs/ft³		
Air	1.2250	2.3769×10^{-3}	100	100
Nitrogen			78.03	75.48
Oxygen			20.99	23.18
Argon			0.94	1.29

The remaining small portion of the composition of air is made up of neon, helium, krypton, xenon, CO_2 and water vapor.

The atmosphere can be thought of as composed of various layers, with each layer of the atmosphere having its own distinct characteristics. For this discussion we will divide the atmosphere into four regions. In ascending order the layers are the troposphere, stratosphere, ionosphere, and exosphere. The four layers are illustrated in Figure 1.6. The troposphere and stratosphere are extremely important to aerospace engineers since most aircraft fly in these regions. The troposphere extends from the Earth's surface to an altitude of approximately 6–13 miles or 10–20 km. The air masses in the troposphere are in constant motion and the region is characterized by unsteady or gusting winds and turbulence. The influence of turbulence and wind shear on aircraft structural integrity and flight behavior continues to be an important area of research for the aeronautical community. The structural loads imposed on an aircraft during an encounter with turbulent air can reduce the structural life of the airframe or in an encounter with severe turbulence can cause structural damage to the airframe.

Wind shear is an important atmospheric phenomenon that can be hazardous to aircraft during takeoff or landing. Wind shear is the variation of the wind vector in both magnitude and direction. In vertical wind shear, the wind speed and direction

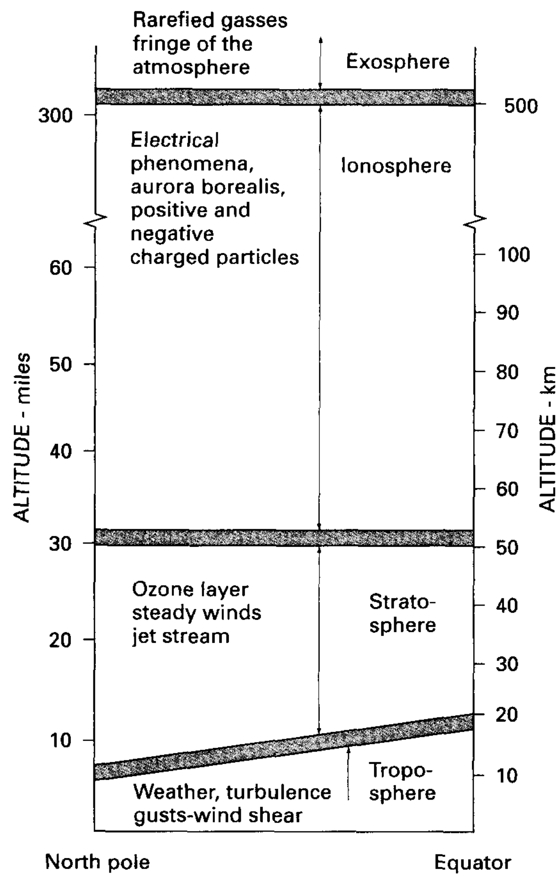


FIGURE 1.6
Layers of earth's atmosphere.

change with altitude. An airplane landing in such a wind shear may be difficult to control; this can cause deviations from the intended touchdown point. Wind shears are created by the movement of air masses relative to one another or to the earth's surface. Thunderstorms, frontal systems, and the earth's boundary layer all produce wind shear profiles that at times are severe enough to be hazardous to aircraft flying at a low altitude.

The next layer above the troposhere is called the stratosphere. The stratosphere extends up to over 30 miles, or 50 km, above the Earth's surface. Unlike the troposphere, the stratosphere is a relatively tranquil region, free of gusts and turbulence, but it is characterized by high, steady winds. Wind speeds of the order of 37 m/s or 120 ft/s have been measured in the stratosphere.

The ionosphere extends from the upper edge of the stratosphere to an altitude of up to 300 miles or 500 km. (The name is derived from the word *ion,* which describes a particle that has either a positive or negative electric charge.) This is the region where the air molecules undergo dissociation and many electrical phenomena occur. The aurora borealis is a visible electrical display that occurs in the ionosphere.

The last layer of the atmsophere is called the exosphere. The exosphere is the outermost region of the atmosphere and is made up of rarefied gas. In effect this is

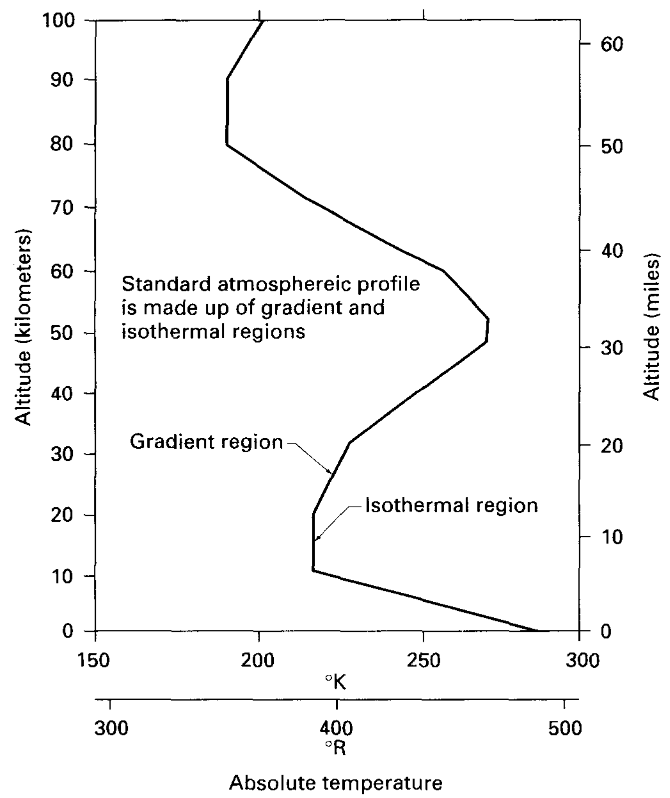


FIGURE 1.7
Temperature profile in the standard atmosphere.

TABLE 1.2
Properties of air at sea level in the standard atmosphere

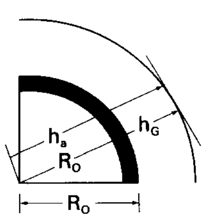
	English units	SI units
Gas constant, R	1718 ft.lb/(slug · °R)	287 $m^2/(°K \cdot s^2)$
Pressure, P	2116.2 lb/ft^2	1.012×10^5 N/m^2
	29.92 in Hg	760 mm Hg
Density, ρ	2.377×10^{-3} $slug/ft^3$	1.225 kg/m^3
Temperature	518.69°R	288.16°K
Absolute viscosity, μ	3.737×10^{-7} lb · s/ft^2	1.789×10^{-3} N · s/m^2
Kinematic viscosity, ν	1.572×10^{-4} ft^2/s	1.460×10^{-5} m^2/s
Speed of sound, a	1116.4 ft/s	340.3 m/s

a transition zone between the earth's atmosphere and interplanetary space. For many applications we can consider air resistance to cease in the exosphere.

As stated previously, the properties of the atmosphere change with time and location on the Earth. To compare the flight performance characteristics of airplanes and flight instruments, a standard atmosphere was needed. The modern standard atmosphere was first developed in the 1920s, independently in the United States and Europe. The National Advisory Committee for Aeronautics (NACA) generated the American Standard Atmosphere. The European standard was developed by the International Commission for Aerial Navigation (ICAN). The two standard atmospheres were essentially the same except for some slight differences. These differences were resolved by an international committee and an international standard atmosphere was adopted by the International Civil Aviation Organization (ICAO) in 1952.

The standard atmosphere assumes a unique temperature profile that was determined by an extensive observation program. The temperature profile consists of regions of linear variations of temperature with altitude and regions of constant temperature (isothermal regions). Figure 1.7 shows the temperature profile through the standard atmosphere. The standard sea-level properties of air are listed in Table 1.2.

The properties of the atmosphere can be expressed analytically as a function of altitude. However, before proceeding with the development of the analytical model of the atmosphere, we must define what we mean by altitude. For the present we will be concerned with three different definitions of altitude: absolute, geometric, and geopotential. Figure 1.8 shows the relationship between absolute and geometric altitude. Absolute altitude is the distance from the center of the Earth to



R_O – Radius of the earth

h_G – Geometric altitude above earth's surface

h_a – Absolute altitude distance from the center of the earth to the point in question

FIGURE 1.8
Definition of geometric and absolute altitudes.

the point in question, whereas the geometric altitude is the height of the point above sea level. The absolute and geometric altitudes are related to each other in the following manner:

$$h_a = h_G + R_0 \tag{1.41}$$

where h_a, h_G, and R_0 are the absolute altitude, geometric altitude, and radius of the earth, respectively.

Historically, measurements of atmospheric properties have been based on the assumption that the acceleration due to gravity is constant. This assumption leads to a fictitious altitude called the geopotential altitude. The relationship between the geometric and geopotential altitudes can be determined from an examination of the hydrostatic equation (Equation (1.16)). Rewriting the hydrostatic equation,

$$\mathrm{d}P = -\rho g \, \mathrm{d}h \tag{1.42}$$

we see that the change in pressure is a function of the fluid density, and if we employ the acceleration due to gravity at sea level, then h is the geopotential altitude. Therefore, we have

$$\mathrm{d}P = -\rho g_0 \, \mathrm{d}h \tag{1.43}$$

when h is the geopotential height and

$$\mathrm{d}P = -\rho g \, \mathrm{d}h_G \tag{1.44}$$

when h_G is the geometric height.

Equations (1.43) and (1.44) can be used to establish the relationship between the geometric and geopotential altitude. On comparing these equations we see that

$$\mathrm{d}h = \frac{g}{g_0} \, \mathrm{d}h_G \tag{1.45}$$

Further it can be shown that

$$g = g_0 \left(\frac{R_0}{R_0 + h_G} \right)^2 \tag{1.46}$$

which when substituted into Equation (1.45) yields

$$\mathrm{d}h = \frac{R_0^2 \, \mathrm{d}h_G}{(R_0 + h_G)^2} \tag{1.47}$$

Equation (1.47) can be integrated to give an expression relating the two altitudes:

$$h = \frac{R_0}{R_0 + h_G} h_G \tag{1.48}$$

or

$$h_G = \frac{R_0}{R_0 - h} h \tag{1.49}$$

In practice, the difference between the geometric and geopotential altitudes is quite small for altitudes below 15.2 km or 50,000 ft. However, for the higher altitudes the difference must be taken into account for accurate performance calculations.

Starting with the relationship for the change in pressure with altitude and the equations of state

$$\mathrm{d}P = -\rho g_0 \,\mathrm{d}h \tag{1.50}$$

and

$$P = \rho R T \tag{1.51}$$

we can obtain the following expression by dividing (1.50) by (1.51):

$$\frac{\mathrm{d}P}{P} = -\frac{g_0}{R}\frac{\mathrm{d}h}{T} \tag{1.52}$$

If the temperature varies with altitude in a linear manner, Equation (1.52) yields

$$\int_{P_1}^{P} \frac{\mathrm{d}P}{P} = -\frac{g_0}{R}\int_{h_1}^{h} \frac{\mathrm{d}h}{T_1 + \lambda(h - h_1)} \tag{1.53}$$

which on integration gives

$$\ln \frac{P}{P_1} = -\frac{g_0}{R\lambda} \ln \frac{T_1 + \lambda(h - h_1)}{T_1} \tag{1.54}$$

where P_1, T_1, and h_1 are the pressure, temperature, and altitude at the start of the linear region and λ is the rate of temperature change with altitude, which is called the lapse rate. Equation (1.54) can be rewritten in a more convenient form as

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-g_0/(R\lambda)} \tag{1.55}$$

Equation (1.55) can be used to calculate the pressure at various altitudes in any one of the linear temperature profile regions, provided the appropriate constants P_1, T_1, h_1, and λ are used.

The density variation can be easily determined as follows:

$$\frac{P}{P_1} = \frac{\rho T}{\rho_1 T_1} \tag{1.56}$$

and therefore

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-[1+g_0/(R\lambda)]} \tag{1.57}$$

In the isothermal regions the temperature remains constant as the altitude varies. Starting again with Equation (1.52) we obtain

$$\ln \frac{P}{P_1} = -\frac{g_0}{RT_1}(h - h_1) \tag{1.58}$$

or

$$\frac{P}{P_1} = e^{-g_0(h-h_1)/(RT_1)} \tag{1.59}$$

TABLE 1.3
Properties of the atmosphere at the isothermal gradient boundaries

Geopotential altitude, H, km	Geometric altitude, Z, km	T, °K	P, N/m²	ρ, kg/m³	dT/dH, °K/km
0	0	288.15	1.01325×10^5	1.225	−6.5
11	11.019	216.65	2.2636×10^4	3.639×10^{-1}	0
20	20.063	216.65	5.474×10^3	8.803×10^{-2}	1
32	32.162	228.65	8.6805×10^2	1.332×10^{-2}	−2.8
47	47.350	270.65	1.1095×10^2	1.427×10^{-3}	0
52	52.429	270.65	5.9002×10^1	7.594×10^{-4}	−2
61	61.591	252.65	1.8208×10^1	2.511×10^{-4}	−4
79	79.994	180.65	1.03757	2.001×10^{-5}	0
88.74	90.0	180.65	0.16435	3.170×10^{-5}	

where P_1, T_1, and h_1 are the values of pressure, temperature, and altitude at the start of the isothermal region. The density variation in the isothermal regions can be obtained as

$$\frac{\rho}{\rho_1} = e^{-g_0(h-h_1)/(RT_1)} \tag{1.60}$$

Equations (1.55), (1.57), (1.59), and (1.60) can be used to predict accurately the pressure and density variation in the standard atmosphere up to approximately 57 miles, or 91 km. Table 1.3 gives the values of temperature, pressure, and density at the boundaries between the various temperature segments. The properties of the standard atmosphere as a function of altitude are presented in tabular form in Appendix A.

EXAMPLE PROBLEM 1.1. The temperature from sea level to 30,000 ft is found to decrease in a linear manner. The temperature and pressure at sea level are measured to be 40°F and 2050 lb/ft², respectively. If the temperature at 30,000 ft is −60°F, find the pressure and density at 20,000 ft.

Solution. The temperature can be represented by the linear equation

$$T = T_1 + \lambda h$$

where

$$T_1 = 499.6°\text{R}$$

and

$$\lambda = \frac{T - T_1}{h} = -0.00333°\ \text{R/ft}$$

The temperature at 20,000 ft can be obtained as

$$T = 499.6 - (0.00333°\ \text{R/ft})h$$

When $h = 20{,}000$ ft, $T = 432.9$°R. The pressure can be calculated from Equation (1.54); that is,

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-g_0/R\lambda} \qquad P = P_1\left(\frac{T}{T_1}\right)^{-g_0/R\lambda} = (2050\ \text{lb/ft}^2)\left(\frac{432.9°\text{R}}{499.6°\text{R}}\right)^{5.63} = 915\ \text{lb/ft}^2$$

The density can be found from either Equation (1.3) or (1.56). Using the equation of state,

$$P = \rho RT \qquad \rho = \frac{P}{RT}$$

$$\rho = \frac{915 \text{ lb/ft}^2}{(1718 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}))(432.9^\circ\text{R})} = 0.00123 \text{ slug/ft}^3$$

1.6 AERODYNAMIC NOMENCLATURE

To describe the motion of an airplane it is necessary to define a suitable coordinate system for the formulation of the equations of motion. For most problems dealing with aircraft motion, two coordinate systems are used. One coordinate system is fixed to the Earth and may be considered for the purpose of aircraft motion analysis to be an inertial coordinate system. The other coordinate system is fixed to the airplane and is referred to as a body coordinate system. Figure 1.9 shows the two right-handed coordinate systems.

The forces acting on an airplane in flight consist of aerodynamic, thrust, and gravitational forces. These forces can be resolved along an axis system fixed to the airplane's center of gravity, as illustrated in Figure 1.10. The force components are denoted X, Y, and Z; T_x, T_y, and T_z; and W_x, W_y, and W_z for the aerodynamic, thrust, and gravitational force components along the x, y, and z axes, respectively. The

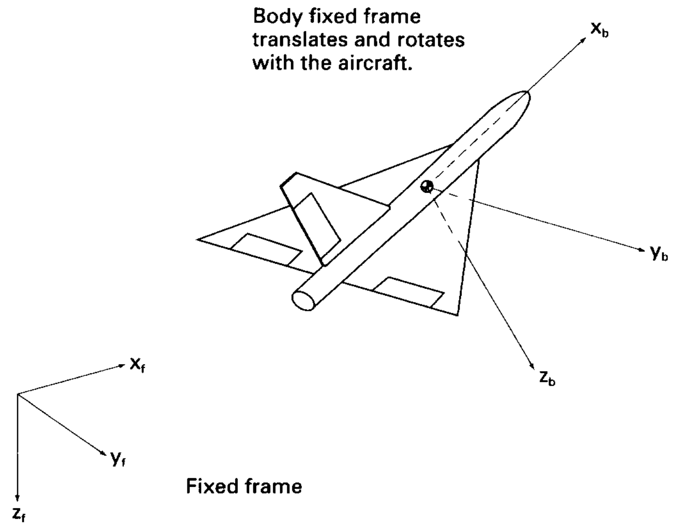
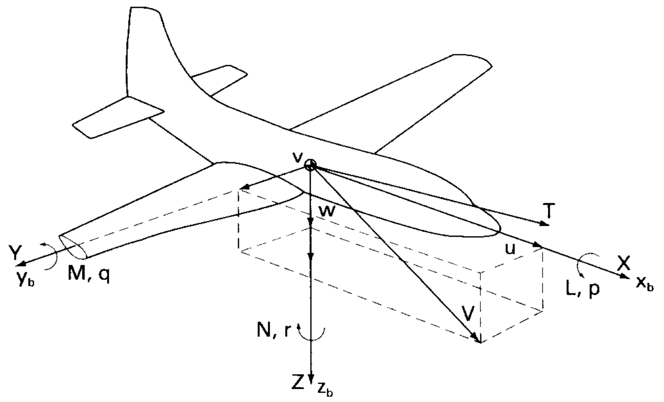


FIGURE 1.9
Body axis coordinate system.



	Roll Axis x_b	Pitch Axis y_b	Yaw Axis z_b
Angular rates	p	q	r
Velocity components	u	v	w
Aerodynamic force components	X	Y	Z
Aerodynamic moment components	L	M	N
Moment of inertia about each axis	I_x	I_y	I_z
Products of inertia	I_{yz}	I_{xz}	I_{xy}

FIGURE 1.10
Definition of forces, moments, and velocity components in a body fixed coordinate

aerodynamic forces are defined in terms of dimensionless coefficients, the flight dynamic pressure Q, and a reference area S as follows:

$$X = C_x QS \qquad \text{Axial force} \tag{1.61}$$

$$Y = C_y QS \qquad \text{Side force} \tag{1.62}$$

$$Z = C_z QS \qquad \text{Normal force} \tag{1.63}$$

In a similar manner, the moments on the airplane can be divided into moments created by the aerodynamic load distribution and the thrust force not acting through the center of gravity. The components of the aerodynamic moment also are expressed in terms of dimensionless coefficients, flight dynamic pressure, reference area, and a characteristic length as follows:

$$L = C_l QSl \qquad \text{Rolling moment} \tag{1.64}$$

$$M = C_m QSl \qquad \text{Pitching moment} \tag{1.65}$$

$$N = C_n QSl \qquad \text{Yawing moment} \tag{1.66}$$

For airplanes, the reference area S is taken as the wing platform area and the characteristic length l is taken as the wing span for the rolling and yawing moment and the mean chord for the pitching moment. For rockets and missiles, the reference area is usually taken as the maximum cross-sectional area, and the characteristic length is taken as the maximum diameter.

The aerodynamic coefficients C_x, C_y, C_z, C_l, C_m, and C_n primarily are a function of the Mach number, Reynolds number, angle of attack, and sideslip angle; they are secondary functions of the time rate of change of angle of attack and sideslip, and the angular velocity of the airplane.

The aerodynamic force and moment acting on the airplane and its angular and translational velocity are illustrated in Figure 1.10. The x and z axes are in the plane of symmetry, with the x axis pointing along the fuselage and the positive y axis along the right wing. The resultant force and moment, as well as the airplane's velocity, can be resolved along these axes.

The angle of attack and sideslip can be defined in terms of the velocity components as illustrated in Figure 1.11. The equations for α and β follow:

$$\alpha = \tan^{-1} \frac{w}{u} \tag{1.67}$$

and

$$\beta = \sin^{-1} \frac{v}{V} \tag{1.68}$$

where

$$V = (u^2 + v^2 + w^2)^{1/2} \tag{1.69}$$

If the angle of attack and sideslip are small, that is, $< 15°$, then Equations (1.67)

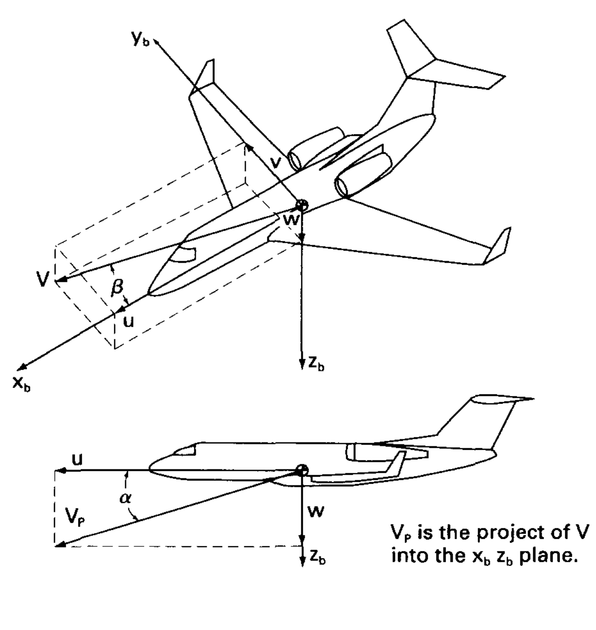


FIGURE 1.11
Definition of angle of attack and sideslip.

and (1.68) can be approximated by

$$\alpha = \frac{w}{u} \tag{1.70}$$

and

$$\beta = \frac{v}{u} \tag{1.71}$$

where α and β are in radians.

1.7 AIRCRAFT INSTRUMENTS

The earliest successful airplanes were generally flown without the aid of aircraft instruments.* The pilots of these early vehicles were preoccupied primarily with maneuvering and controlling their sometimes temperamental aircraft. However, as new designs were developed, the performance, stability, and control steadily improved to the point where the pilot needed more information about the airplane's flight conditions to fly the airplane safely. One major change in aircraft design that led to improved performance was the evolution of the open-air cockpit. Prior to this development, pilots flew their airplanes in either a crouched or inclined position, exposed to the oncoming airstream. In addition to providing the pilot shelter from the airstream, the cockpit also provided a convenient place to locate aircraft instruments. The early open-cockpit pilots were hesitant to fly from a closed cockpit because this eliminated their ability to judge sideslip (or skid) by the wind blowing on one side of their face. They also used the sound of the slipstream to provide an indication of the airspeed.

A chronological development of aircraft instruments is not readily available; however, one can safely guess that some of the earliest instruments to appear on the cockpit instrument panel were a magnetic compass for navigation, airspeed and altitude indicators for flight information, and engine instruments such as rpm and fuel gauges. The flight decks of modern airplanes are equipped with a multitude of instruments that provide the flight crew with information they need to fly their aircraft. The instruments can be categorized according to their primary use as flight, navigation, power plant, environmental, and electrical systems instruments.

Several of the instruments that compose the flight instrument group will be discussed in the following sections. The instruments include the airspeed indicator, altimeter, rate of climb indicator, and the Mach meter. These four instruments, along with angle of attack and sideslip indicators, are extremely important for flight test measurement of performance and stability data.

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*The Wright brothers used several instruments on their historic flight. They had a tachometer to measure engine rpm, an anemometer to measure airspeed, and a stopwatch.

1.7.1 Air Data Systems

The Pitot static system of an airplane is used to measure the total pressure created by the forward motion of the airplane and the static pressure of the ambient atmosphere. The difference between total and static pressures is used to measure airspeed and the Mach number, and the static pressure is used to measure altitude and rate of climb. The Pilot static system is illustrated in Figure 1.12. The Pilot static probe normally consists of two concentric tubes. The inner tube is used to determine the total pressure, and the outer tube is used to determine the static pressure of the surrounding air.

1.7.2 Airspeed Indicator

The pressures measured by the Pitot static probe can be used to determine the airspeed of the airplane. For low flight speeds, when compressibility effects can be safely ignored, we can use the incompressible form of Bernoulli's equation to show that the difference between the total and the static pressure is

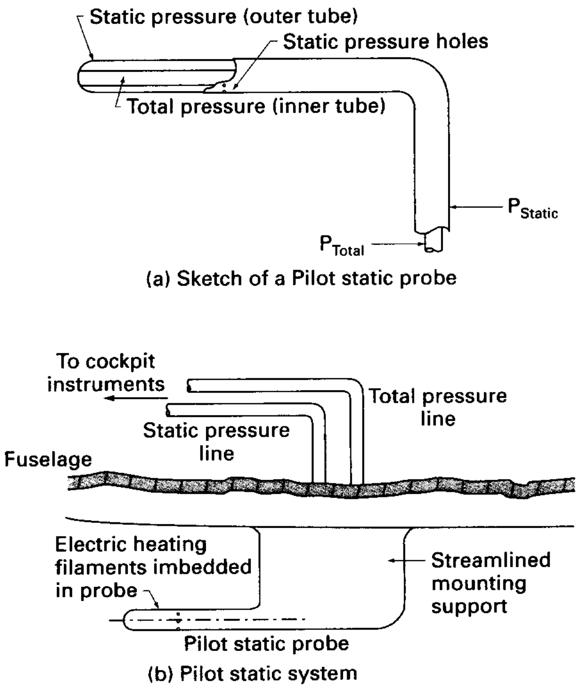

FIGURE 1.12
Pitot static system.

the dynamic pressure:

$$P_0 = P + \frac{1}{2}\rho V_\infty^2 \tag{1.72}$$

$$\frac{1}{2}\rho V_\infty^2 = P_0 - P \tag{1.73}$$

or

$$V_\infty = \left(\frac{2(P_0 - P)}{\rho}\right)^{1/2} \tag{1.74}$$

The airspeed indicator in the cockpit consists of a differential pressure gauge that measures the dynamic pressure and deflects an indicator hand proportionally to the pressure difference. As indicated by Equation (1.74), the airspeed is a function of both the measured pressure difference and the air density ρ. As was shown earlier, the air density is a function of altitude and atmospheric conditions. To obtain the true airspeed, the airspeed indicator would be required to measure the change in both pressure and air density. This is not feasible for a simple instrument and therefore the scale on the airspeed indicator is calibrated using standard sea-level air. The speed measured by the indicator is called the indicated airspeed (IAS).

The speed measured by an airspeed indicator can be used to determine the true flight speed, provided that the indicated airspeed is corrected for instrument error, position error, compressibility effects, and density corrections for altitude variations. Instrument error includes those errors inherent to the instrument itself; for example, pressure losses or mechanical inaccuracies in the system. Position error has to do with the location of the Pitot static probe on the airplane. Ideally, the probe should be located so that it is in the undisturbed freestream; in general this is not possible and so the probe is affected by flow distortion due to the fuselage or wing. The total pressure measured by a Pitot static probe is relatively insensitive to flow inclination. Unfortunately, this is not the case for the static measurement and care must be used to position the probe to minimize the error in the static measurement. If one knows the instrument and position errors, one can correct the indicated airspeed to give what is referred to as the calibrated airspeed (CAS).

At high speeds, the Pitot static probe must be corrected for compressibility effects. This can be demonstrated by examining the compressible form of the Bernoulli equation:

$$\frac{V^2}{2} + \frac{\gamma}{\gamma - 1}\frac{P}{\rho} = \frac{\gamma}{\gamma - 1}\frac{P_0}{\rho_0} \tag{1.75}$$

Equation (1.75) can be expressed in terms of the Mach number as follows:

$$P_0 = P\left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{\gamma/(\gamma-1)} \tag{1.76}$$

Recall that the airspeed indicator measures the difference between the total and static pressure. Equation (1.76) can be rewritten as

$$Q_c = P_0 - P = P\left[\left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{\gamma/(\gamma-1)} - 1\right] \tag{1.77}$$

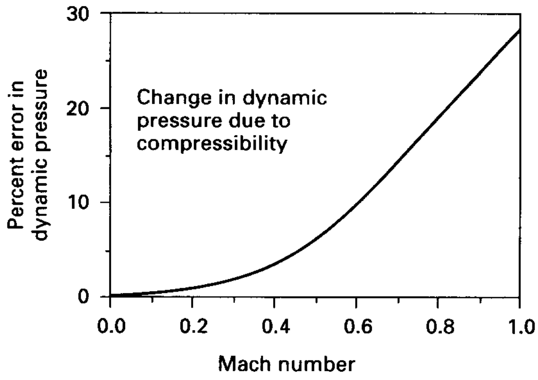

FIGURE 1.13
Percent of error in dynamic pressure if compressibility is neglected.

where Q_c is the compressible equivalent to the dynamic pressure. Figure 1.13 shows the percentage error in dynamic pressure if compressibility is ignored.

The equivalent airspeed (EAS) can be thought of as the flight speed in the standard sea-level air mass that produces the same dynamic pressure as the actual flight speed. To obtain the actual, or true, airspeed (TAS), the equivalent airspeed must be corrected for density variations. Using the fact that the dynamic pressures are the same, one can develop a relationship between the true and equivalent airspeeds as follows:

$$\frac{1}{2}\rho_0 V_{\mathrm{EAS}}^2 = \frac{1}{2}\rho V_{\mathrm{TAS}}^2 \tag{1.78}$$

$$V_{\mathrm{TAS}} = \frac{V_{\mathrm{EAS}}}{\sqrt{\sigma}} \tag{1.79}$$

where $\sigma = \rho/\rho_0$.

The definitions for the various airspeed designations are summarized in Table 1.4.

TABLE 1.4
Airspeed designations

Airspeed*	**Definition**
V_{IAS} Indicated airspeed	Airspeed indicated by the airspeed instrument. The indicated airspeed is affected by altitude, compressibility, instrument, and position error.
V_{CAS} Calibrated airspeed	Indicated airspeed corrected for instrument and position errors.
V_{EAS} Equivalent airspeed	Calibrated airspeed corrected for compressibility.
V_{TAS} True airspeed	Equivalent airspeed corrected for density altitude.

*When the prefix K is used in the subscript, the airspeed is in knots.

1.7.3 Altimeter

An altimeter is a device to measure the altitude of an airplane. The control of an airplane's altitude is very important for safe operation. Pilots use an altimeter to maintain adequate vertical spacing between their aircraft and other airplanes operating in the same area and to establish sufficient distance between the airplane and the ground.

Earlier in this chapter we briefly discussed the mercury barometer. A barometer can be used to measure the atmospheric pressure. As we have shown, the static pressure in the atmosphere varies with altitude, so that if we use a device similar to a barometer we can measure the static pressure outside the airplane, and then relate that pressure to a corresponding altitude in the standard atmosphere. This is the basic idea behind a pressure altimeter.

The mercury barometer of course would be impractical for application in aircraft, because it is both fragile and sensitive to the airplane's motion. To avoid this difficulty, the pressure altimeter uses the same principle as an aneroid* barometer. This type of barometer measures the pressure by magnifying small deflections of an elastic element that deforms as pressure acts on it.

The altimeter is a sensitive pressure transducer that measures the ambient static pressure and displays an altitude value on the instrument dial. The altimeter is calibrated using the standard atmosphere and the altitude indicated by the instrument is referred to as the pressure altitude. The *pressure altitude* is the altitude in the standard atmosphere corresponding to the measured pressure. The pressure altitude and actual or geometric altitude will be the same only when the atmosphere through which the airplane is flying is identical to the standard atmosphere.

In addition to pressure altitude two other altitudes are important for performance analysis: the density and temperature altitudes. The *density altitude* is the altitude in the standard atmosphere corresponding to the ambient density. In general, the ambient density is not measured but rather calculated from the pressure altitude given by the altimeter and the ambient temperature measured by a temperature probe. The *temperature altitude,* as you might guess, is the altitude in the standard atmosphere corresponding to the measured ambient temperature.

As noted earlier the atmosphere is continuously changing; therefore, to compare performance data for an airplane from one test to another or to compare different airplanes the data must be referred to a common atmospheric reference. The density altitude is used for airplane performance data comparisons.

An altimeter is an extremely sophisticated instrument, as illustrated by the drawing in Figure 1.14. This particular altimeter uses two aneroid capsules to increase the sensitivity of the instrument. The deflections of the capsules are magnified and represented by the movement of the pointer with respect to a scale on the surface plate of the meter and a counter. This altimeter is equipped with a

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*Aneroid is derived from the Greek word *aneros,* which means "not wet."

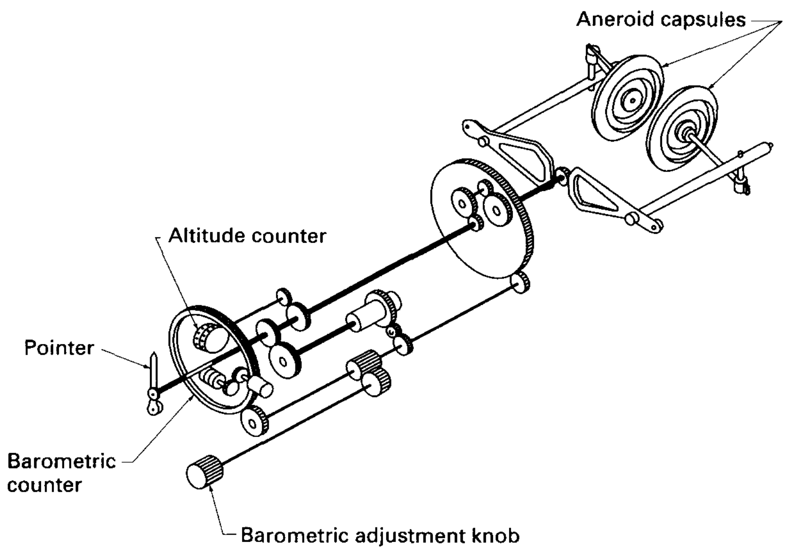

FIGURE 1.14
Cutaway drawing of an altimeter.

barometric pressure-setting mechanism. The adjusting mechanism allows the pilot manually to correct the altimeter for variations in sea-level barometric pressure. With such adjustments, the altimeter will indicate an altitude that closely approaches the true altitude above sea level.

1.7.4 Rate of Climb Indicator

One of the earliest instruments used to measure rate of climb was called a statoscope. This instrument was used by balloonists to detect variation from a desired altitude. The instrument consisted of a closed atmospheric chamber connected by a tube containing a small quantity of liquid to an outer chamber vented to the atmosphere. As the altitude changed, air would flow from one chamber to the other to equalize the pressure. Air passing through the liquid would create bubbles and the direction of the flow of bubbles indicated whether the balloon was ascending or descending. A crude indication of the rate of climb was obtained by observing the frequency of the bubbles passing through the liquid.

Although the statoscope provided the balloonist a means of detecting departure from a constant altitude, it was difficult to use as a rate of climb indicator. A new instrument, called the balloon variometer, was developed for rate of climb measurements. The variometer was similar to the statoscope; however, the flow into the chamber took place through a capillary leak. The pressure difference across the leak was measured with a sensitive liquid manometer that was calibrated to indicate the rate of climb.

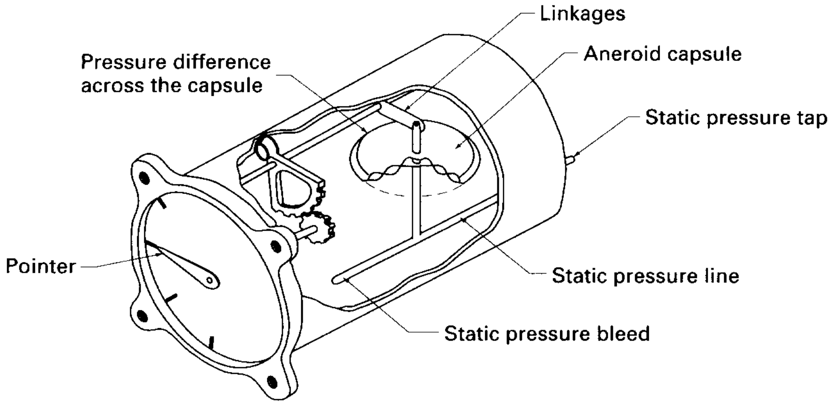

FIGURE 1.15
Sketch of the basic components of a rate of climb indicator.

Present-day rate of climb indicators are similar to the variometer. An example of a leak type rate of climb indicator is shown in Figure 1.15. This instrument consists of an insulated chamber, a diaphragm, a calibrated leak, and an appropriate mechanical linkage to measure the deflection of the diaphragm. The static pressure is applied to the interior of the diaphragm and also allowed to leak into the chamber by way of a capillary or orifice opening. The diaphragm measures the differential pressure across the leak and the deflection of the diaphragm is transmitted to the indicator dial by a mechanical linkage, as illustrated in the sketch in Figure 1.15.

1.7.5 Machmeter

The Pitot static tube can be used to determine the Mach number of an airplane from the measured stagnation and static pressure. If the Mach number is less than 1, Equation (1.40) can be used to find the Mach number of the airplane:

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2}\mathrm{M}^2\right)^{\gamma/(\gamma-1)} \tag{1.80}$$

However, when the Mach number is greater than unity, a bow wave forms ahead of the Pitot probe, as illustrated in Figure 1.16. The bow wave is a curved detached shock wave. In the immediate vicinity of the Pitot orifice, the shock wave can be approximated as a normal shock wave. Using the normal shock relationships, the

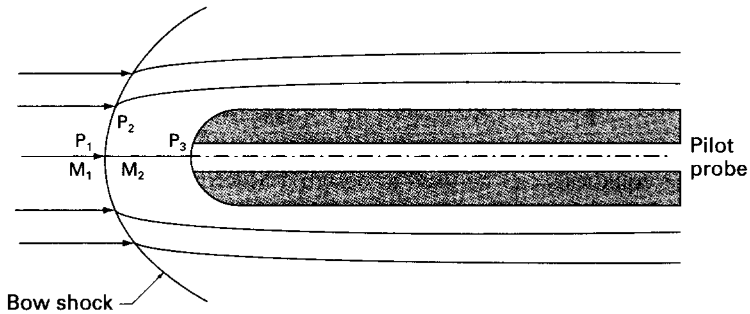

FIGURE 1.16
Detached shock wave ahead of a Pitot static probe.

pressure ratio across the shock can be written as

$$\frac{P_2}{P_1} = \left(\frac{2\gamma}{\gamma - 1}\right)M_1^2 - \left(\frac{\gamma - 1}{\gamma + 1}\right) \tag{1.81}$$

where M_1 is the Mach number ahead of the shock wave. The relationship between the Mach number M_1 ahead of the normal shock and the Mach number M_2 behind the shock is given by Equation (1.82):

$$M_2^2 = \frac{\frac{1}{2}(\gamma - 1)M_1^2 + 1}{\gamma M_1^2 - \frac{1}{2}(\gamma - 1)} \tag{1.82}$$

After passing through the shock wave, the air is slowed adiabatically to zero velocity at the total pressure orifice of the Pitot probe. The pressure ratio behind the shock can be expressed as

$$\frac{P_3}{P_2} = \left(1 + \frac{\gamma - 1}{2}M_2^2\right)^{\gamma/(\gamma-1)} \tag{1.83}$$

On combining the previous equations, the ratio of stagnation pressure to static pressure in terms of the flight Mach number can be written:

$$\begin{aligned}\frac{P_3}{P_1} = &\left[\left(\frac{2\gamma}{\gamma + 1}\right)M_1^2 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\right] \\ &\times \left[1 + \frac{\gamma - 1}{2}\left[\left(\frac{\frac{1}{2}(\gamma - 1)M_1^2 + 1}{\gamma M_1^2 - \frac{1}{2}(\gamma - 1)}\right)\right]\right]^{\gamma/(\gamma-1)}\end{aligned} \tag{1.84}$$

This expression is known as the Rayleigh Pitot tube formula, named after Lord Rayleigh, who first developed this equation in 1910. If we assume that the ratio γ

of specific heats for air is 1.4, the expression can be rewritten as

$$\frac{P_3}{P_1} = \frac{7M_1^2 - 1}{6}\left[1 + 0.2\left(\frac{M_1^2 + 5}{7M_1^2 - 1}\right)\right]^{3.5} \tag{1.85}$$

The preceding equations can be used to design a Mach meter.

The use of Rayleigh's formula is invalid for every high Mach numbers or altitudes. When the Mach number is high, appreciable heat will be exchanged, which violates the assumption of adiabatic flow used in the development of the equation. At very high altitude, air cannot be considered as a continuous medium and again the analysis breaks down.

1.7.6 Angle of Attack Indicators

The measurement of angle of attack is important for cruise control and stall warning. Several devices can be used to measure the angle of attack of an airplane, two of which are the vane and pressure-sensor type indicator. The pivot vane sensor is a mass-balanced wind vane that is free to align itself with the oncoming flow. The vane type angle of attack sensor has been used extensively in airplane flight test programs. For flight test applications the sensor usually is mounted on a nose boom or a boom mounted to the wing tips along with a Pitot static probe, as illustrated in Figure 1.17. Note that a second vane system is mounted on the boom to measure the sideslip angle.

The angle measured by the vane is influenced by the distortion of the flow field created by the airplane. Actually, the sensor measures only the local angle of attack. The difference between the measured and actual angles of attack is called the position error. Position error can be minimized by mounting the sensor on the fuselage, where the flow distortion is small. The deflection of the vane is recorded by means of a potentiometer.

A null-seeking pressure sensor also can be used to measure the angle of attack. Figure 1.18 is a schematic of a null-seeking pressure sensor. The sensor consists of the following components: a rotatable tube containing two orifices spaced at equal angles to the tube axis, a pressure transducer to detect the difference in pressure between the two orifices, a mechanism for rotating the probe until the pressure differential is 0, and a device for measuring the rotation or angle of attack. The device shown in Figure 1.18 consists of a rotable probe that protrudes through the

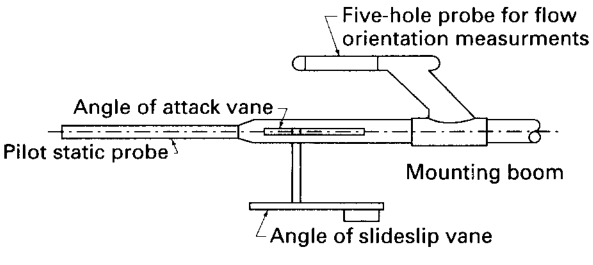

FIGURE 1.17
Flight test instrumentation, Pitot static probe, angle of attack and sideslip vanes, five-hole probe mounted on a nose or wing boom.

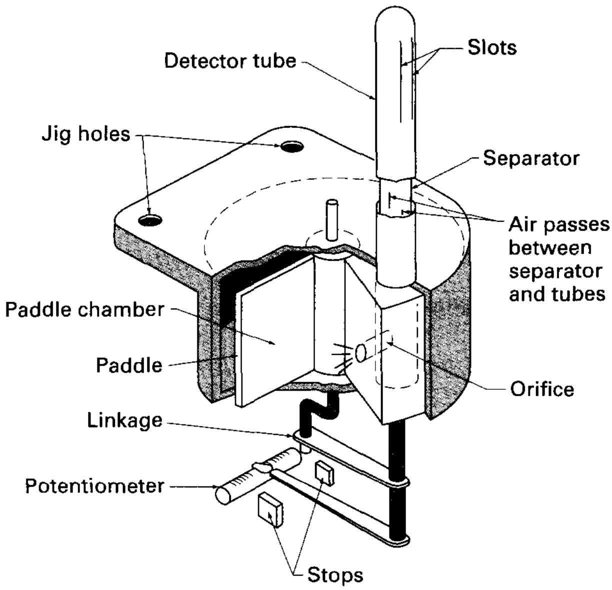

FIGURE 1.18
Null-sensing pressure probe for measuring angle of attack.

fuselage and an air chamber mounted inside the fuselage. The pressures from the two slits are vented to air chambers by a swivel paddle. If a pressure difference exists at the two slots, the swivel paddle will rotate. The paddle is connected by way of linkages so that, as the paddle moves, the pressure tube is rotated until the pressures are equalized. The angular position of the probe is recorded by a potentiometer.

EXAMPLE PROBLEM 1.2. An aircraft altimeter calibrated to the standard atmosphere reads 10,000 ft. The airspeed indicator has been calibrated for both instrument and position errors and reads a velocity of 120 knots. If the outside air temperature is 20°F, determine the true airspeed.

Solution. The altimeter is a pressure gauge calibrated to the standard atmosphere. If the altimeter reads 10,000 ft, the static pressure it senses must correspond to the static pressure at 10,000 ft in the standard atmosphere. Using the standard atmospheric table in the Appendix, the static pressure at 10,000 ft is given as

$$P = 1455.6 \text{ lb/ft}^2$$

The ambient density can be calculated using the equation of state:

$$\rho = \frac{P}{RT}$$

$$\rho = \frac{1455.6 \text{ lb/ft}^2}{(1716 \text{ ft}^2/(s^2 \cdot {}^\circ\text{R}))(479.7^\circ\text{R})}$$

$$\rho = 0.001768 \text{ slug/ft}^3$$

A low-speed airspeed indicator corrected for instrument and position error reads the equivalent airspeed. The true speed and equivalent airspeed are related by

$$V_{\mathrm{TAS}} = \frac{V_{\mathrm{EAS}}}{\sqrt{\sigma}}$$

where σ is the ratio of the density at altitude to the standard sea-level value of density:

$$\sigma = \rho/\rho_0 = (0.001768/0.002739) = 0.7432$$

Now, solving for the true airspeed,

$$V_{\mathrm{KTAS}} = \frac{V_{\mathrm{KEAS}}}{\sqrt{\sigma}} = \frac{120 \text{ knots}}{\sqrt{0.7432}}$$
$$= 139 \text{ knots}$$

1.8 SUMMARY

In this chapter we examined the properties of air and how those properties vary with altitude. For the comparison of flight test data and calibrating aircraft instruments, a standard atmosphere is a necessity: The 1962 U.S. Standard Atmosphere provides the needed reference for the aerospace community. The standard atmosphere was shown to be made up of gradient and isothermal regions.

Finally, we discussed the basic concepts behind several basic flight instruments that play an important role in flight test measurements of aircraft performance, stability and control. In principle these instruments seem to be quite simple; they in fact, are, extremely complicated mechanical devices. Although we have discussed several mechanical instruments, most of the information presented to the flight crew on the newest aircraft designs comes from multifunctional electronic displays. Color cathode ray tubes are used to display air data such as attitude, speed, and altitude. Additional displays include navigation, weather, and engine performance information, to name just a few items. The improvements offered by this new technology can be used to reduce the workload of the flight crew and improve the flight safety of the next generation of airplane designs.

PROBLEMS

1.1. An altimeter set for sea-level standard pressure indicates an altitude of 20,000 ft. If the outside ambient temperature is $-5°$F, find the air density and the density altitude.

1.2. An airplane is flying at an altitude of 5000 m as indicated by the altimeter and the outside air temperature is $-20°$C. If the airplane is flying at a true airspeed of 300 m/s, determine the indicated airspeed.

1.3. A high-altitude, remotely piloted communications platform is flying at a pressure altitude of 60,000 ft and an indicated airspeed of 160 ft/s. The outside ambient temperature is −75°F. Estimate the Reynolds number of the wing based on a mean chord of 3.5 ft.

1.4. An airplane is flying at a pressure altitude of 10,000 ft and the airspeed indicator reads 100 knots. If there is no instrument error and the position error is given by Figure P1.4, find the true airspeed of the airplane.

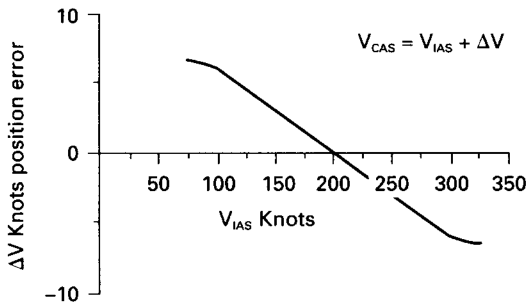

FIGURE P1.4 Position error versus indicated airspeed.

1.5. Under what conditions are following relationships valid?

$$V_{\text{CAS}} = V_{\text{EAS}} = V_{\text{TAS}}$$
$$V_{\text{CAS}} = V_{\text{EAS}} \neq V_{\text{TAS}}$$
$$V_{\text{CAS}} \neq V_{\text{EAS}} = V_{\text{TAS}}$$

1.6. A small right circular cylinder is used to measure the angle of attack of an airplane by measuring the difference in pressure at two port locations that are located at $\theta = \pm 20°$. Assuming that the flow on the forward face of the cylinder can be accurately modeled as an inviscid flow, the velocity along the cylinder surface can be expressed as

$$V_\theta = 2V_\infty \sin \theta$$

If, while flying at 200 ft/s under sea-level standard conditions, the pressure difference is 32.5 lb/ft^2, what is the angle of the airplane?

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