APPENDIX B

Geometric, Mass, and Aerodynamic Characteristics of Selected Airplanes

Data on the geometric, mass, and aerodynamic stability and control characteristics are presented for seven airplanes. The airplanes include a general aviation airplane, two jet fighters, an executive business jet, two jet transports, and a STOL transport. The stability coefficients are presented in tabular form for each airplane. Coefficients that were unavailable have been presented with a numerical value of 0 in the following tables. The stability coefficients for the A-4D are presented in graphical form as a function of the Mach number and altitude. These plots show the large variations in the coefficients due to compressibility effects. The definitions of the stability coefficients and geometric data presented in the figures are given in the following nomenclature list. The information presented in this appendix was taken from [B.1], [B.2] and [B.3] given after the nomenclature list.

NOMENCLATURE

b Wing span

$\bar{c}$ Mean chord

$$C_L = \frac{L}{QS}$$

$$C_{L_M} = \frac{\partial C_L}{\partial M}$$

$$C_{L_{\delta_e}} = \frac{\partial C_L}{\partial \delta_e} \text{ (rad}^{-1}\text{)}$$

$$C_D = \frac{D}{QS}$$

$$C_{D_\alpha} = \frac{\partial C_D}{\partial \alpha} \text{ (rad}^{-1}\text{)}$$

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha} \text{ (rad}^{-1}\text{)}$$

$$C_{L_{\dot{\alpha}}} = \frac{\partial C_L}{\partial \left(\frac{\dot{\alpha}\bar{c}}{2u_0}\right)} \text{ (rad}^{-1}\text{)}$$

$$C_{l_{\delta_a}} = \frac{\partial C_l}{\partial \delta_r} \text{ (rad}^{-1}\text{)}$$

$$C_{l_{\delta_r}} = \frac{\partial C_l}{\partial \delta_r} \text{ (rad}^{-1}\text{)}$$

$$C_n = \frac{N}{QSb}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta} \text{ (rad}^{-1}\text{)}$$

$C_{D_M} = \dfrac{\partial C_D}{\partial M}$

$C_{D_{\delta_e}} = \dfrac{\partial C_D}{\partial \delta_e}$ (rad^{-1})

$C_m = \dfrac{M}{QS\bar{c}}$

$C_{m_\alpha} = \dfrac{\partial C_m}{\partial \alpha}$ (rad^{-1})

$C_y = \dfrac{Y}{QS}$

$C_{y_\beta} = \dfrac{\partial C_y}{\partial \beta}$ (rad^{-1})

$C_{y_{\delta_r}} = \dfrac{\partial C_y}{\partial \delta_r}$ (rad^{-1})

$C_l = \dfrac{L}{QSb}$

$C_{l_\beta} = \dfrac{\partial C_l}{\partial \beta}$ (rad^{-1})

$C_{l_p} = \dfrac{\partial C_l}{\partial (pb/2u_0)}$ (rad^{-1})

$C_{l_r} = \dfrac{\partial C_l}{\partial (rb/2u_0)}$ (rad^{-1})

$C_{n_p} = \dfrac{\partial C_n}{\partial (pb/2u_0)}$ (rad^{-1})

$C_{n_r} = \dfrac{\partial C_n}{\partial (rb/2u_0)}$ (rad^{-1})

$C_{m_{\dot\alpha}} = \dfrac{\partial C_m}{\partial (\dot\alpha\bar{c}/2u_0)}$ (rad^{-1})

$C_{m_M} = \dfrac{\partial C_m}{\partial M}$

$C_{m_q} = \dfrac{\partial C_m}{\partial (q\bar{c}/2u_0)}$ (rad^{-1})

$C_{n_{\delta_a}} = \dfrac{\partial C_n}{\partial \delta_a}$ (rad^{-1})

$C_{n_{\delta_r}} = \dfrac{\partial C_n}{\partial \delta_r}$ (rad^{-1})

I_x Rolling moment of inertia
I_y Pitching moment of intertia
I_z Yawing moment of inertia
I_{xz} Product of inertia about xz axis
M Mach number
Q Dynamic pressure
S Wing planform area
u_0 Reference flight speed

REFERENCES

B.1. Teper, G. L. *Aircraft Stability and Control Data.* Hawthorne, CA: System Technology, Technical Report 176-1, April 1969.

B.2. Heffley, R. K.; and W. F. Jewell. *Aircraft Handling Qualities Data.* NASA CR-2144, December 1972.

B.3. Mac Donald, R.A.; M. Garelick; and J.O'Grady. "Linearized Mathematical Models for De Havilland Canada 'Buffalo and Twin Otter' STOL Transports." U.S. Department of Transportation – Transportation System Center Report No. DOT-TSC-FAA-71-8, June 1971.

TABLE B.1

General aviation airplane: NAVION

Longitudinal M = 0.158	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
Sea level	0.41	0.05	4.44	0.33	−0.683	0.0	−4.36	3.8	−9.96	0.0	0.0	0.0	0.355	−0.923
Lateral **M = 0.158**	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$		
Sea level	−0.564	−0.074	−0.071	−0.410	−0.0575	0.107	−0.125	−0.134	−0.0035	0.157	0.107	−0.072		

Note: All derivatives are per radian.

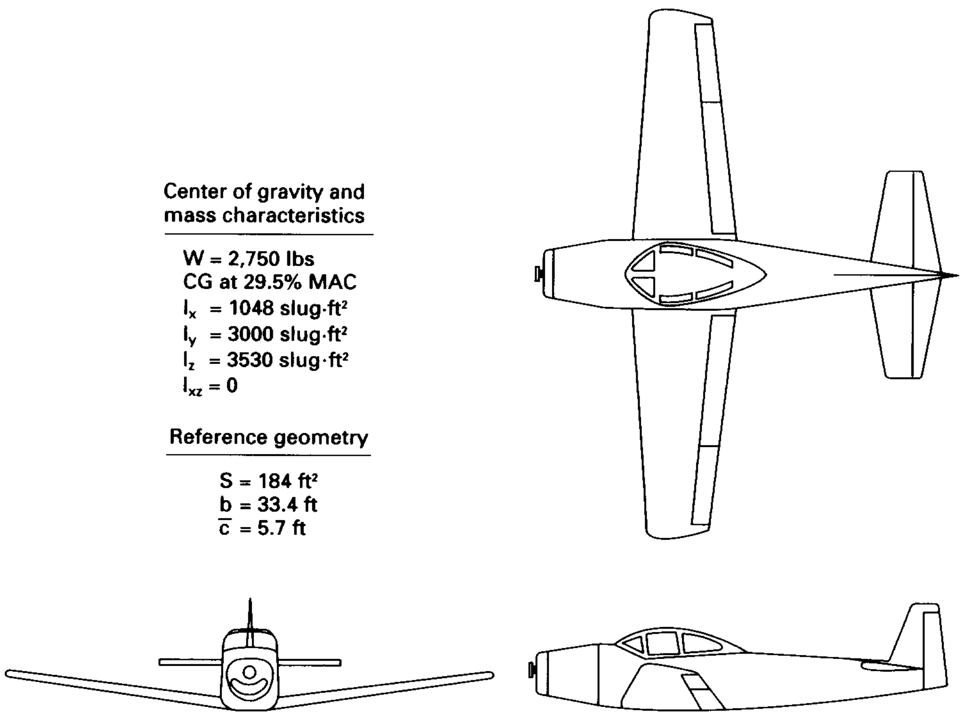

FIGURE B.1
Three-view sketch and stability data for a general aviation airplane.

TABLE B.2

Fighter aircraft: F104-A

Longitudinal	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot{\alpha}}}$	$C_{m_{\dot{\alpha}}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
M = 0.257 Sea level	0.735	0.263	3.44	0.45	−0.64	0.0	−1.6	0.0	−5.8	0.0	0.0	0.0	0.68	−1.46
M = 1.8 55,000 ft	0.2	0.055	2.0	0.38	−1.30	0.0	−2.0	0.0	−4.8	−0.2	0.0	−0.01	0.52	−0.10

Lateral	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
M = 0.257 Sea level	−1.17	−0.175	0.50	−0.285	−0.14	0.265	−0.75	0.039	0.0042	0.208	0.045	−0.16
M = 1.8 55,000 ft	−1.0	−0.09	0.24	−0.27	−0.09	0.15	−0.65	0.017	0.0025	0.05	0.008	−0.04

Note: All derivatives are per radian.

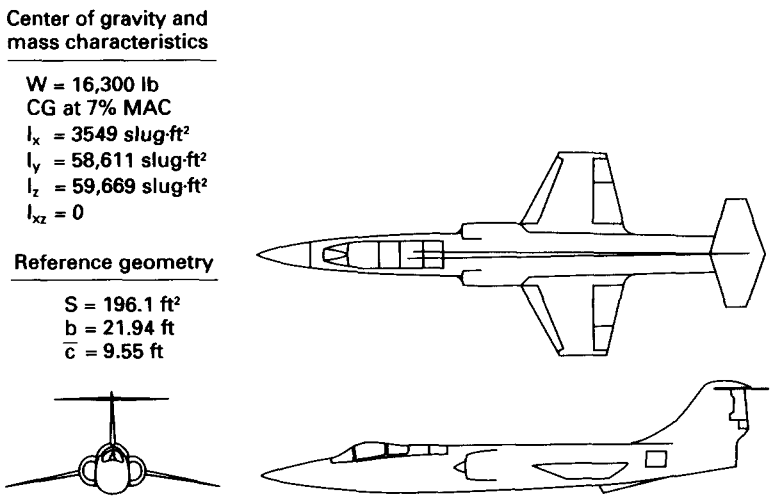

FIGURE B.2
Three-view sketch and stability data for the F-104-A fighter.

TABLE B.3

Fighter aircraft: A-4D

Longitudinal	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
M = 0.4 Sea level	0.28	0.03	3.45	0.30	−0.38	0.72	−1.1	0.0	−3.6	0.0	0.0	0.0	0.36	−0.50
M = 0.8 35,000 ft	0.30	0.038	4.0	0.56	−0.41	1.12	−1.65	0.0	−4.3	0.15	0.03	−0.05	0.4	−0.60

Lateral	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
M = 0.4 Sea level	−0.98	−0.12	0.25	−0.26	0.022	0.14	−0.35	0.08	0.06	0.17	−0.105	0.032
M = 0.8 35,000 ft	−1.04	−0.14	0.27	−0.24	0.029	0.17	−0.39	0.072	0.04	0.17	−0.105	0.032

Note: All derivatives are per radian.

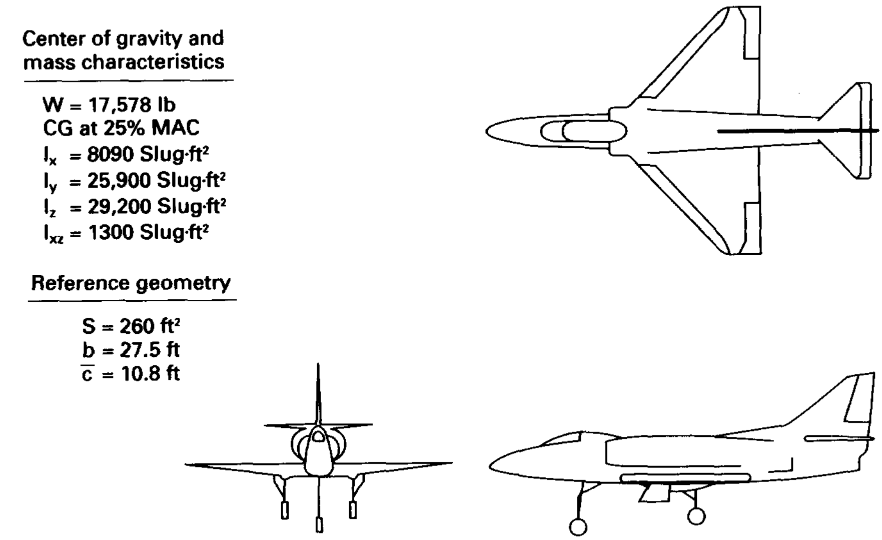

FIGURE B.3
Three-view sketch and stability data for the A-4D fighter.

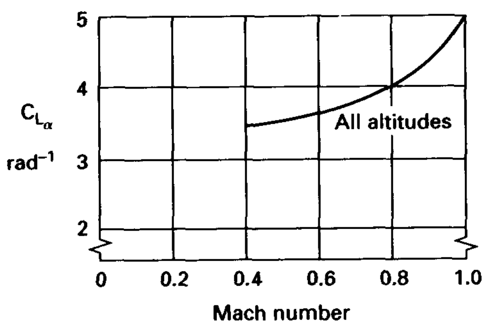

FIGURE B.4
C_{L_α} versus the Mach number.

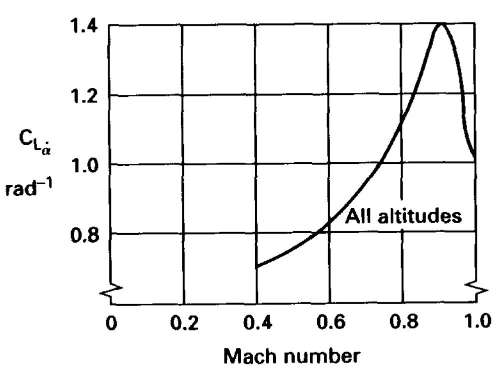

FIGURE B.5
$C_{L_{\dot\alpha}}$ versus the Mach number.

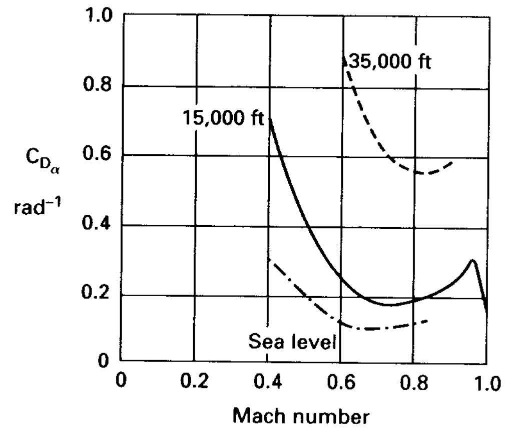

FIGURE B.6
C_{D_α} versus the Mach number.

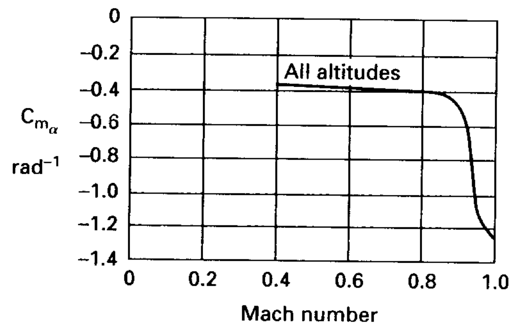

FIGURE B.7
C_{m_α} versus the Mach number.

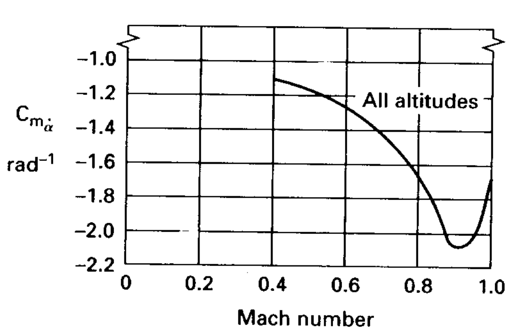

FIGURE B.8
$C_{m_{\dot\alpha}}$ versus the Mach number.

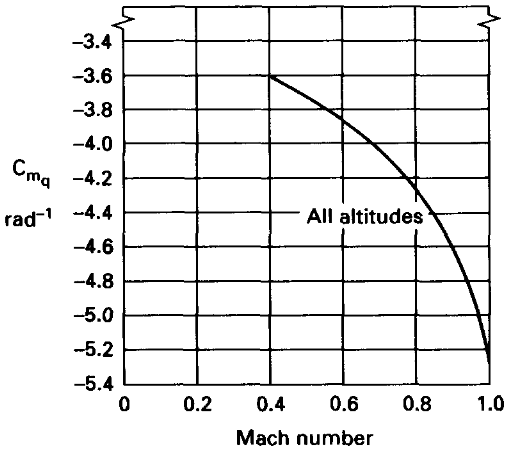

FIGURE B.9
C_{m_q} versus the Mach number.

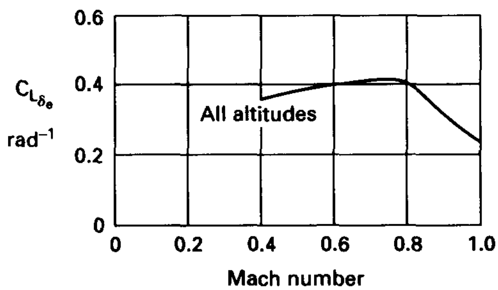

FIGURE B.10
$C_{L_{\delta_e}}$ versus the Mach number.

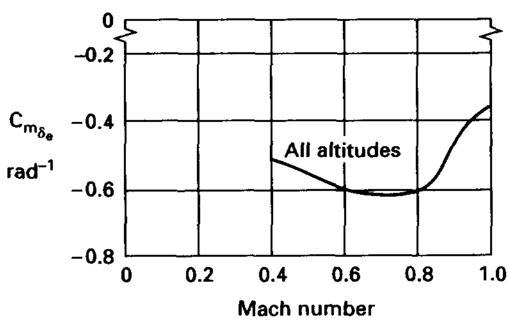

FIGURE B.11
$C_{m_{\delta_e}}$ versus the Mach number.

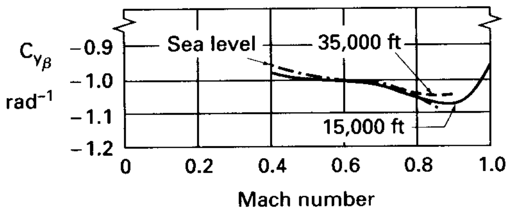

FIGURE B.12
C_{y_β} versus the Mach number.

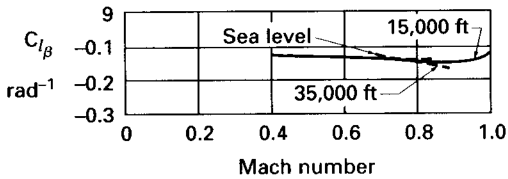

FIGURE B.13
C_{l_β} versus the Mach number.

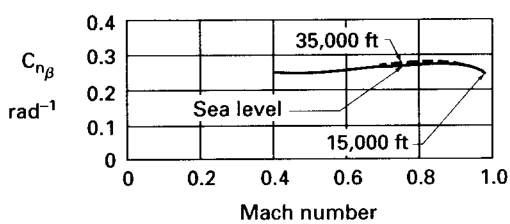

FIGURE B.14
C_{n_β} versus the Mach number.

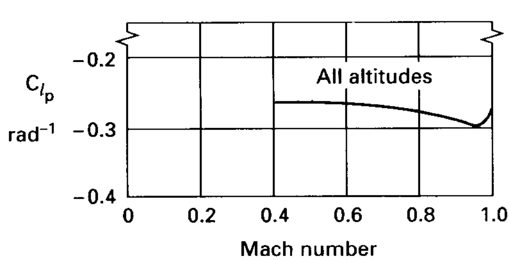

FIGURE B.15
C_{l_p} versus the Mach number.

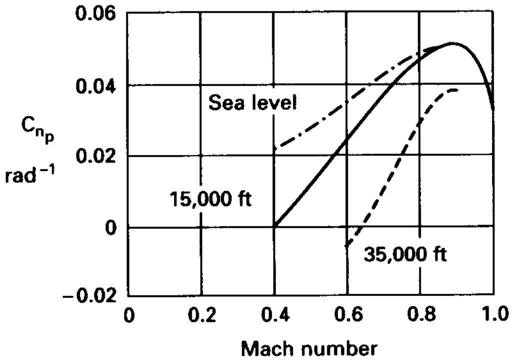

FIGURE B.16
C_{n_p} versus the Mach number.

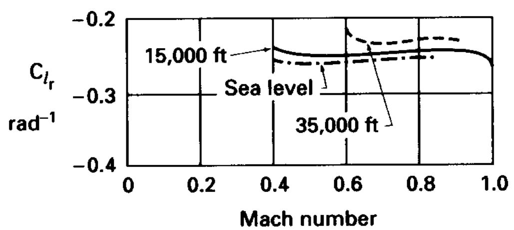

FIGURE B.17
C_{l_r} versus the Mach number.

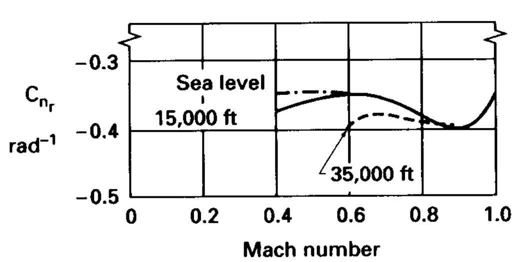

FIGURE B.18
C_{n_r} versus the Mach number.

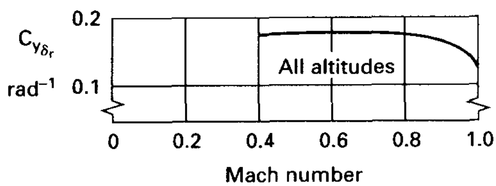

FIGURE B.19
$C_{y_{\delta_r}}$ versus the Mach number.

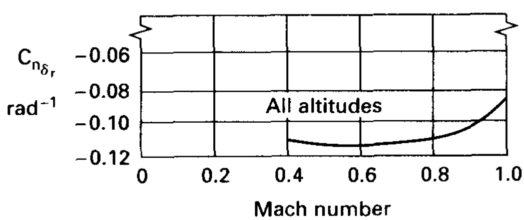

FIGURE B.20
$C_{n_{\delta_r}}$ versus the Mach number.

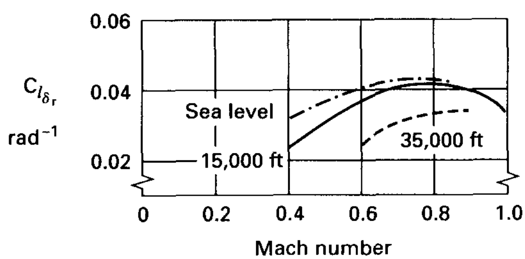

FIGURE B.21
$C_{l_{\delta_r}}$ versus the Mach number.

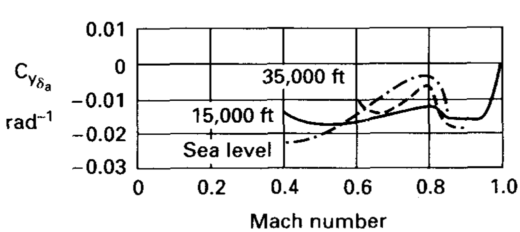

FIGURE B.22
$C_{y_{\delta_a}}$ versus the Mach number.

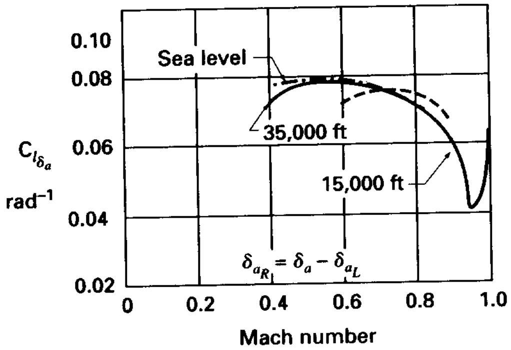

FIGURE B.23
$C_{l_{\delta a}}$ versus the Mach number.

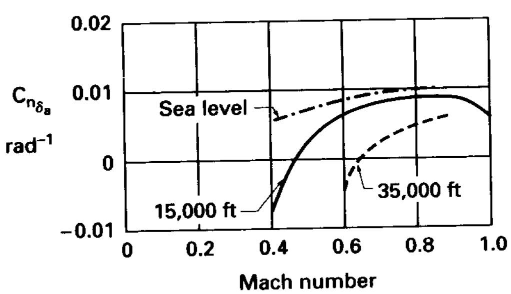

FIGURE B.24
$C_{n_{\delta a}}$ versus the Mach number.

TABLE B.25

Business Jet: Jetstar

Longitudinal	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{l_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
M = 0.20 Sea level	0.737	0.095	5.0	0.75	−0.80	0.0	−3.0	0.0	−8.0	0.0	0.0	−0.05	0.4	−0.81
M = 0.80 40,000 ft	0.4	0.04	6.5	0.60	−0.72	0.0	−0.4	0.0	−0.92	0.0	−0.6	−0.60	0.44	−0.88

Lateral	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
M = 0.20 Sea level	−0.72	−0.103	0.137	−0.37	−0.14	0.11	−0.16	0.054	0.0075	0.175	0.029	−0.063
M = 0.80 40,000 ft	−0.75	−0.06	0.13	−0.42	−0.756	0.04	−0.16	0.060	−0.06	0.16	0.029	−0.057

Note: All derivatives are per radian.

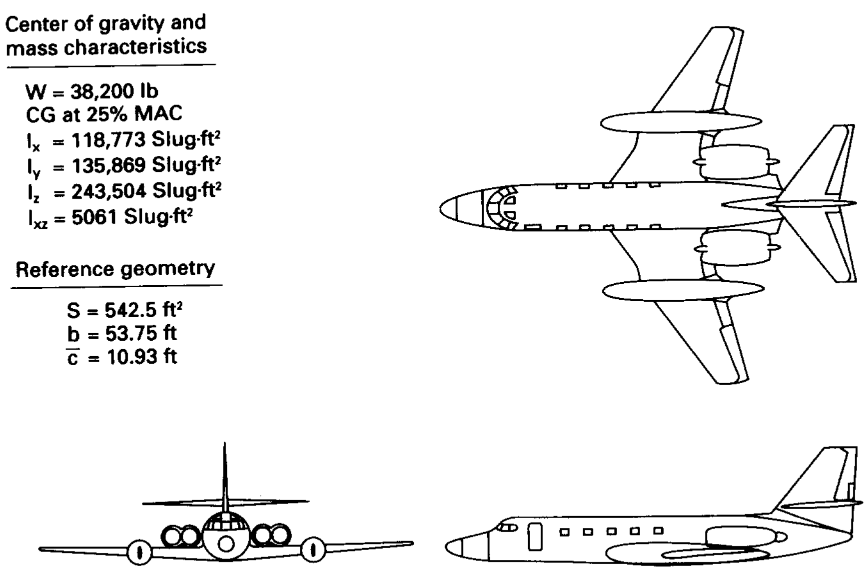

FIGURE B.25
Three-view sketch and stability data for a Jetstar executive business jet.

TABLE B.26

Transport aircraft: Convair 880

Longitudinal M = 0.25	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
Sea level	0.68	0.08	4.52	0.27	−0.903	2.7	−4.13	7.72	−12.1	0.0	0.0	0.0	0.213	−0.637
M = 0.8 35,000 ft	0.347	0.024	4.8	0.15	−0.65	2.7	−4.5	7.5	−4.5	0.0	0.0	0.0	0.190	−0.57

Lateral M = 0.25	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
Sea level	−0.877	−0.196	0.139	−0.381	−0.049	0.198	−0.185	−0.038	0.017	0.216	0.0226	−0.096
M = 0.8 35,000 ft	−0.812	−0.177	0.129	−0.312	−0.011	0.153	−0.165	−0.050	0.008	0.184	0.019	−0.076

Note: All derivatives are per radian

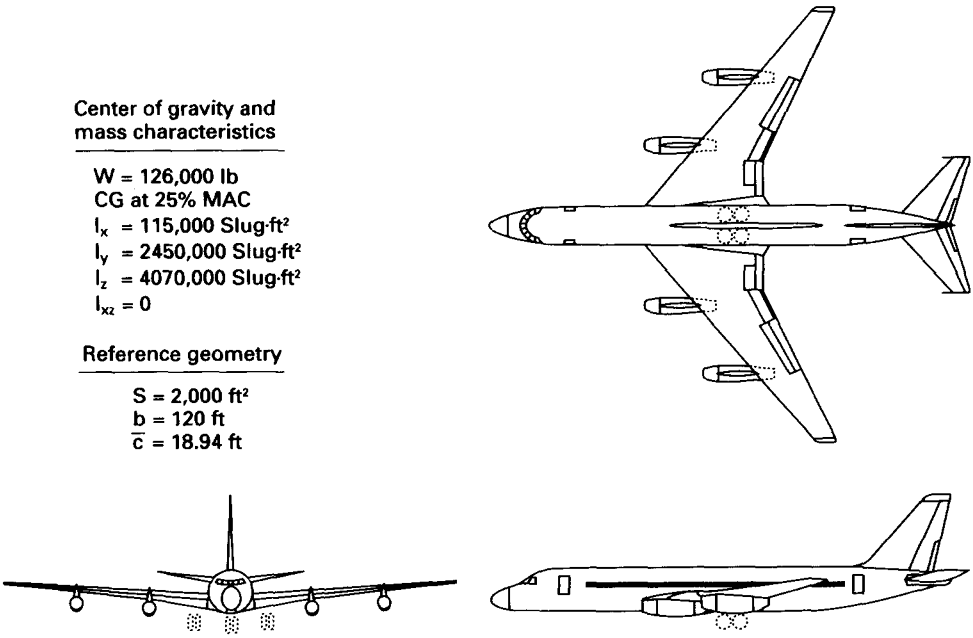

FIGURE B.26
Three-view sketch and stability data for a Convair 880 jet transport.

TABLE B.27

Transport aircraft: Boeing 747

Longitudinal	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_M}	C_{m_M}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
M = 0.25 Sea level	1.11	0.102	5.70	0.66	−1.26	6.7	−3.2	5.4	−20.8	−0.81	0.0	0.27	0.338	−1.34
M = 0.90 40,000 ft	0.5	0.042	5.5	0.47	−1.6	0.006	−9.0	6.58	−25.0	0.2	0.25	−0.10	0.3	−1.2

Lateral	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$
M = 0.25 Sea level	−0.96	−0.221	0.150	−0.45	−0.121	0.101	−0.30	0.0461	0.0064	0.175	0.007	−0.109
M = 0.90 40,000 ft	−0.85	−0.10	0.20	−0.30	0.20	0.20	−0.325	0.014	0.003	0.075	0.005	−0.09

Note: All derivatives are per radian.

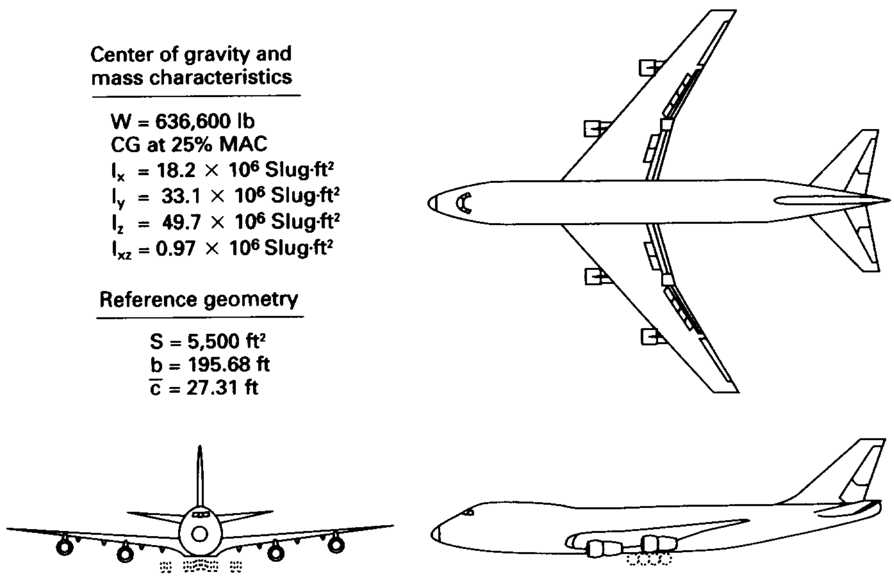

FIGURE B.27
Three-view sketch and stability data for a large Boeing 747 jet transport.

TABLE B.28
STOL Transport

Longitudinal M = 0.14	C_L	C_D	C_{L_α}	C_{D_α}	C_{m_α}	$C_{L_{\dot\alpha}}$	$C_{m_{\dot\alpha}}$	C_{L_q}	C_{m_q}	C_{L_M}	C_{D_m}	C_{M_m}	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$
Sea level	1.5	0.127	5.24	0.67	−0.78	1.33	−6.05	7.83	−35.6	0	0	0	0.465	−2.12
M = 0.37 10,000 ft	0.3	0.036	5.24	0.67	−0.78	1.33	−6.05	7.83	−35.6	0	0	0	0.465	−2.12
Lateral M = 0.14	C_{y_β}	C_{l_β}	C_{n_β}	C_{l_p}	C_{n_p}	C_{l_r}	C_{n_r}	$C_{l_{\delta_a}}$	$C_{n_{\delta_a}}$	$C_{y_{\delta_a}}$	$C_{l_{\delta_r}}$	$C_{n_{\delta_r}}$		
Sea level	−0.362	−0.125	0.101	−0.53	−0.283	0.410	−0.188	0.20	0	−0.233	−0.024	0.107		
M = 0.37 10,000 ft	−0.362	−0.125	0.101	−0.53	−0.037	0.113	−0.171	0.20	0	−0.233	−0.024	0.107		

Note: All derivatives are per radian.

Center of gravity and mass characteristics

W = 40,000 lbs.
CG at 25% MAC
I_x = 273,000 Slug·ft²
I_y = 215,000 Slug·ft²
I_z = 447,000 Slug·ft²
I_{xz} = 0

Reference geometry

S = 945 ft²
b = 96 ft
$\bar{c}$ = 10.1 ft

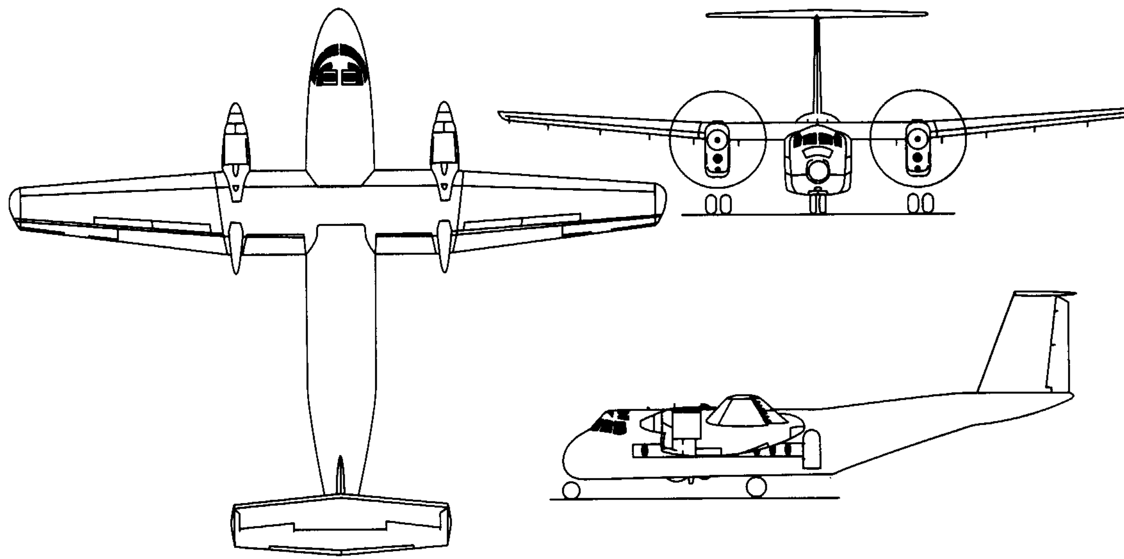

FIGURE B.28
Three-view sketch and stability data for a STOL transport.

APPENDIX C

Mathematical Review of Laplace Transforms and Matrix Algebra

REVIEW OF MATHEMATICAL CONCEPTS

Laplace Transformation

The Laplace transform is a mathematical technique that has been used extensively in control system synthesis. It is a very powerful mathematical tool for solving differential equations. When the Laplace transformation technique is applied to a differential equation it transforms the differential equation to an algebraic equation. The transformed algebraic equation can be solved for the quantity of interest and then inverted back into the time domain to provide the solution to the differential equation.

The Laplace transformation is a mathematical operation defined by

$$\mathscr{L}[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt = F(s) \tag{C.1}$$

where $f(t)$ is a function of time. The operator $\mathscr{L}$ and the complex variable s are the Laplace operator and variable, respectively, and $F(s)$ is the transform of $f(t)$. The Laplace transformation of various functions $f(t)$ can be obtained by evaluating Equation (C.1). The process of obtaining $f(t)$ from the Laplace transform $F(s)$, called the inverse Laplace transformation, is given by

$$f(t) = \mathscr{L}^{-1}[F(s)] \tag{C.2}$$

where the inverse Laplace transformation is given by the following integral relationship:

$$f(t) = \frac{1}{2\pi i}\int_{c-\infty}^{c+\infty} F(s)\, e^{st}\, ds \tag{C.3}$$

Several examples of Laplace transformations follow.

EXAMPLE PROBLEM C.1. Consider the function $f(t) = e^{-at}$.

Solution. The Laplace transform of this expression yields

$$\mathscr{L}[f(t)] = \mathscr{L}[e^{-at}] = \int_0^\infty e^{-at}\, e^{st}\, dt = \int_0^\infty e^{-(a+s)t}\, dt$$