Lecture 12/9/19

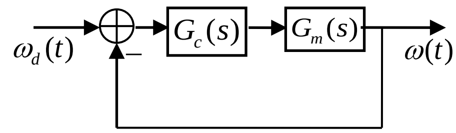


Figure 1 Closed loop block diagram.

In these notes we will use the concept of the root locus to design a unity feedback control system to maintain more accurate speed control of a DC motor. The block diagram is shown at right.

The voltage/speed transfer function of the motor is: $\dfrac{\Omega(s)}{V(s)} = G_m(s) = \dfrac{50}{s+10}\ rpm/volt$.

(a) We desire that the closed loop system have unity static gain. Show that no proportional controller $G_c(s) = K_P$ can achieve this.

<u>*Solution*</u>: The open loop transfer function is $G_c(s)G_m(s) = \dfrac{50K_P}{s+10}$. Hence, the closed loop transfer function is

$W(s) = \dfrac{G_c(s)G_m(s)}{1+G_c(s)G_m(s)} = \dfrac{50K_p}{s+10+50K_p}$, with static gain $W(0) == \dfrac{50K_p}{10+50K_p}$. So for sufficiently large K_p we can get a static gain close to 1.0. But we can never achieve unity static gain.

(b) Show that the integral controller: $G_c(s) = \dfrac{K_I}{s}$ will achieve CL unity static gain.

<u>*Solution*</u>: The open loop transfer function is $G_c(s)G_m(s) = \dfrac{50K_I}{s(s+10)}$. Hence, $W(s) = \dfrac{50K_I}{s^2+10s+50K_I}$ has $W(0) = 1.0$.

(c) Notice that while $W(s) = \dfrac{50K_I}{s^2+10s+50K_I}$ has unity static gain, its dynamics are controlled by the single parameter K_I.

Find the value of K_I so that the system will be critically damped.

<u>*Solution*</u>: This occurs when the roots of $p(s) = s^2+10s+50K_I$ are real and repeated. Setting $s^2+10s+50K_I = (s-s_1)^2 = s^2-2s_1s+s_1^2$, and equating coefficients, gives: $10 = -2s_1 \quad \Rightarrow s_1 = -5$. Hence, $50K_I = s_1^2 = 25 \quad \Rightarrow K_I = 0.5$.

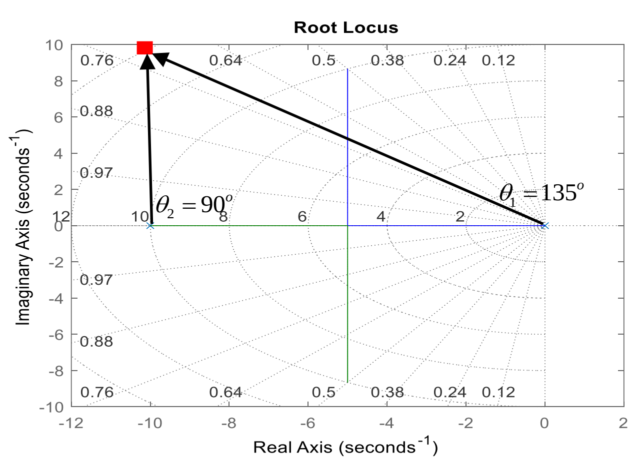
(d) The uncontrolled motor has a time constant $\tau_m = 0.1\,\text{sec}$. The CL system in (c) has a repeated time constant $\tau = 0.2\,\text{sec}$. Hence, even though it has unity static gain, it is half as fast as the uncontrolled motor. To address this deficiency, consider the controller transfer function $G_c(s) = \dfrac{K_I(s-z)}{s}$. The OL is then $G_c(s) = \left(\dfrac{50K_I}{s(s+10)}\right)(s-z)$. Suppose that we desire that $W(s)$ have a pole $s_1 = -10+i10$. This corresponds to a time constant 0.1, as well as 'optimal' damping $\zeta = 0.707$.

(i) Plot the CL root locus associated with the OL $G(s) = \dfrac{50}{s(s+10)}$.

(ii) Find the 'defect angle' associated with $s_1 = -10+i10$.

(iii) Find the value of z that will correct the defect angle.

(iv) Use the data cursor to find the needed value for K_I.



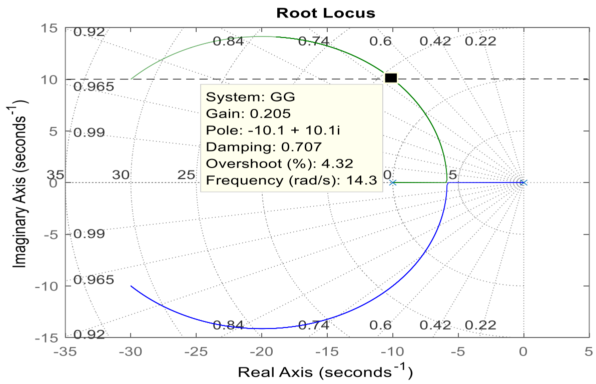
<u>*Solution*</u>:

(i) The commands G=tf(50,[1 10 0]); K=0:.01:2; rlocus(G,K) grid results in the root locus at right.

(ii) since $-(\theta_1+\theta_2) = -225^o = 180^o - 45^o$, the defect angle is -45^o.

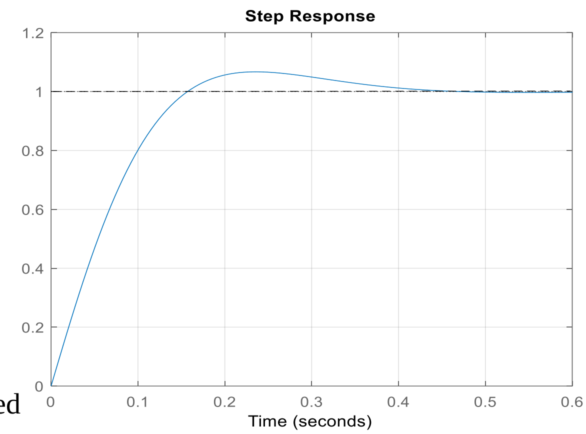
(iii) We need to find the value of z such that the difference vector $s_1 - z$ has an angle of 45^o. This angle is achieved for $z = -20$.

(iv) The resulting OL is $G_c(s) = K_I \dfrac{50(s+20)}{s(s+10)}$. The CL root locus for This OL is shown at right. The required value is $K_I \cong 0.2$



(e) Plot the unit step response of the CL system with OL: $G_c(s) = 0.2\dfrac{50(s+20)}{s(s+10)}$. Then comment.

Solution: The plot at right used the 'feedback' command for convenience.



It has unity static gain, a ~0.5 sec. response time, and minimal overshoot.

Conclusion

The final controller was $G_c(s) = \dfrac{0.2(s+20)}{s} = 0.2 + \dfrac{10}{s} = K_P + \dfrac{K_I}{s}$. This is PI controller. The integral control was needed to achieve unity static gain. The controller zero was needed to place the CL pole at the desired location.

PID Control

The output, $u(t)$, of a PID controller having an input $e(t)$ is given by:

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau)d\tau + K_D \dot{e}(t) \cdot$$

Hence, the controller transfer function is:

$$\frac{U(s)}{E(s)} = G_c(c) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} \cdot$$

Even though the PID controller is one of the oldest controllers used in feedback control systems, it is still the most popular. It is used in many autopilots for hobby drones, and for machinery control.

The controller gains (K_P, K_I, K_D) allow the controller to be 'tuned' to satisfy different sets of performance specifications. There are scores of materials available on Google related to tuning the controller. Here is one that I found to be particularly engaging at this point in the course: https://www.youtube.com/watch?v=4sjXJ5HoU_c

A key thing to point out is how the author uses 'punches' to get an idea of the quadcopter dynamics prior to and after tuning. A second thing to point out is that he uses the three PID gains to control both the longitudinal and lateral dynamics of the quadcopter. To date, we have addressed each set of dynamics independently. Hopefully, you now have a better idea of the need for a solid understanding of the dynamics of a UAV (or any other system), prior to addressing how to best control it. □