Homework 5 AERE355 Fall 2019 Due 11/11(MON) SOLUTION

Problem 1(40pts) This problem concerns pure yaw. From equations (5.22) and (5.23) on p.189 we have (for $N_{\dot{\beta}} = 0$):

$$\ddot{\beta} - N_r\dot{\beta} + N_\beta \beta = -N_{\delta_r}\delta_r . \qquad (1.1)$$

(a)(3pts) Give the condition for *weathercock* (*static*) *stability*, including a reference to the appropriate page in Nelson.

Answer: The condition is given on p.74: $C_{n_\beta} > 0$.

(b)(8pts) Suppose that the plane has weathercock stability. (i) Identify a second condition related to (1.1) that must hold for (1.1) to be dynamically *strictly* stable. Then (ii) use the appropriate table(s) to ultimately arrive at the conclusion that this second condition does, indeed, hold. You should find that one parameter in particular requires very careful scrutiny.

Answer: (i) (1.1) will be *strictly* stable when, in addition to $C_{n_\beta} > 0$, we also have $C_{n_r} < 0$. (ii) From Table 3.4 we have: $C_{n_r} = -2\frac{l_v V_v \eta_v}{b} C_{L_{\alpha_v}}$. Clearly, all the parameters, with the exception of $C_{L_{\alpha_v}}$, are greater than zero. In relation to $C_{L_{\alpha_v}}$, on p.76 we have $N_v = -l_v Y_v = l_v C_{L_{\alpha_v}}(\beta + \sigma) Q_v S_v$. From Figures 1.10 and 2.31 we have $N_v > 0$. Since the side force $Y_v < 0$, the book contains an error by omitting the minus sign in the leftmost equality in (2.76). In Figure 2.31 one might reasonably assume that $L_v < 0$. However, recall that vertical lift that is associated with a negative force, is referred to as positive lift. The same applies here. To be exact, $L_v = -Y_v\cos(\beta+\sigma) = -Y_v\cos(\alpha_v) = C_{L_{\alpha_v}}\alpha_v$ is positive lift. Hence, $C_{L_{\alpha_v}} > 0$.
This condition is guaranteed to hold. An alternative argument: While $C_{L_{\alpha_v}}$ is not given in any of the plane tables, C_{y_β} is given. In Table 3.5 we have $Y_v \stackrel{\Delta}{=} Y_\beta = (QS/m)C_{y_\beta}$. Hence, $Y_v < 0$ requires that $C_{y_\beta} < 0$. This holds for all planes included in Appendix B.

(c)(12pts) (i)Obtain the transfer function associated with (1.1). Then, from it obtain (ii) the expressions for the system dynamic parameters (ω_n, ζ, τ) and (iii) the system *static gain* g_s. [This assumes the system is underdamped.]

Solution: (i) $L[\ddot{\beta} - N_r\dot{\beta} + N_\beta\beta = -N_{\delta_r}\delta_r] \;\Rightarrow\; (s^2 - N_r s + N_\beta)\beta(s) = -N_{\delta_r}\Delta_r(s)$. So: $\frac{\beta(s)}{\Delta_r(s)} \stackrel{\Delta}{=} H(s) = \frac{-N_{\delta_r}}{s^2 - N_r s + N_\beta}$.

(ii) $\omega_n = \sqrt{N_\beta}$, $2\zeta\omega_n = -N_r \Rightarrow \zeta = -N_r / 2\sqrt{N_\beta}$, and $\zeta\omega_n = -N_r/2 \Rightarrow \tau = -2/N_r$ (iii) $g_s = H(s=0) = \frac{-N_{\delta_r}}{N_\beta}$.

(d)(4pts) (i) Give the definition of the *rudder control effectiveness* (along with the page in Nelson where it is defined). Then (ii) give the numerical value of this quantity for the NAVION plane.

Answer: The *rudder control effectiveness* is given on p.78 as $C_{n_{\delta_r}}$. For the NAVION it is $C_{n_{\delta_r}} = -0.072$.

(e)(5pts) Use the appropriate parameter in (c) to compute the required value of δ_r to achieve a *steady state* sideslip $\beta_{ss} = 10^o$ for the NAVION.

Solution: In the steady state (1.1) becomes $N_\beta\beta_{ss} = -N_{\delta_r}\delta_r$. Hence, $\beta_{ss} = (-N_{\delta_r}/N_\beta)\delta_r = g_s\delta_r$. Since $N_\beta = (QSb/I_z)C_{n_\beta}$ and $N_{\delta_r} = (QSb/I_z)C_{n_{\delta_r}}$, we have $g_s = -N_{\delta_r}/N_\beta = -(-0.072)/0.071 = 1.014$. Hence, $\delta_r = (1/g_s)\beta_{ss} = (1/1.014)10^o = \mathbf{9.86^o}$.

(f)(8pts) Verify your answer in (e) by using the *Matlab* commands 'tf' and 'step' to obtain a plot of the step response of (1.1) for the NAVION. Also, give the transfer function you computed.
<u>*Solution*</u>: [See code @ 1(f).]

The plot verifies (e).
The transfer function is: H = 4.635 / (s^2 + 0.7619 s + 4.571)

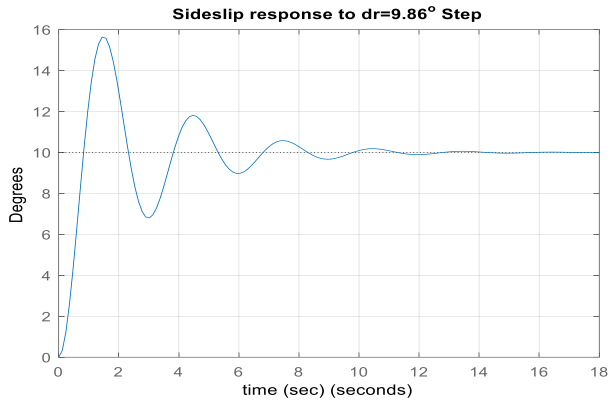

Figure 1(f) Rudder step response for pure yaw.

Problem 2(35pts) As noted on p.198 of Nelson: "If we consider the Dutch roll mode to consist primarily of side-slipping and yaw, then the perturbation lateral dynamics become:

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}\dfrac{Y_\beta}{u_0} & -\left(1-\dfrac{Y_r}{u_0}\right)\\ N_\beta & N_r\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix} \quad + \quad \begin{bmatrix}Y_{\delta_r}/u_0\\ N_{\delta_r}\end{bmatrix}\delta_r . \qquad (5.45^*)$$

[*I have included the input δ_r, resulting from (5.35).]

The characteristic polynomial for (5.45) is: $s^2-(\dfrac{Y_\beta+u_0N_r}{u_0})s+\dfrac{Y_\beta N_r-N_\beta Y_r+u_0N_\beta}{u_0}$. (5.46)

(a)(4pts) From (5.22) and (5.23) on p.189 in relation to pure yaw, arrive at the state space model $\mathbf{\dot{x}}=\mathbf{Ax}+\mathbf{Bu}$ where $\mathbf{x}=[\beta \quad r]^{tr}$. [Note: Assume, as Nelson does, that $N_{\dot{\beta}}=0$.]

Solution: (5.22) is: $\dot{\psi}=r=-\dot{\beta}$. (5.23) is: $\ddot{\psi}-N_r\dot{\psi}+N_\beta\psi=N_{\delta_r}\delta_r$. These give: $\dot{r}-N_r r-N_\beta\beta=N_{\delta_r}\delta_r$. Hence:

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\end{bmatrix}=\begin{bmatrix}0 & -1\\ N_\beta & N_r\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix}+\begin{bmatrix}0\\ N_{\delta_r}\end{bmatrix}\delta_r .$$

(b)(3pts) Give the assumptions needed in relation to (5.45) and in relation to your expression in (a) so that they have exactly the same **A** matrix.
Solution: The two **A** matrices will be the same if $Y_\beta=Y_r=0$.

(c)(3pts) If they have the same **A** matrices, then clearly they have the same eigenvalues. In this case, verify that they have the same characteristic polynomial by comparing (5.46) to the denominator of your transfer function in 1(c).
Solution:
(5.46) becomes: $s^2-N_r s+N_\beta$. The denominator of my transfer function in 1(c) is exactly this polynomial. Verified.

(d)(5pts) Nelson's values $\zeta_{DR}=0.254$ and $\omega_{n\,DR}=2.17r/s$ for the approximate *Dutch Roll* model (5.45) are given at the bottom of p.200. They are given in column 3 below. Use your characteristic polynomials to compute the associated numerical values related to the pure yaw model (column 1 below), and to the numerical values that you arrive at in relation to (5.45) using the information in Table B.1 (column 2 below).
Solution: [See code @ 2(c).]

	pure yaw	Your approx. DR $C_{y_r}=0$	Nelson approx. DR $C_{y_r}=0$	Your approx. DR $C_{y_r}=0.522$
ζ	.178	.233	.254	.2354
ω_n	2.14	2.18	2.17	2.1594

(e)(5pts) Even though it is assumed that $C_{y_r}=0$ (both in Table 5.2 and by its absence in Appendix Table B.1), on p.118 we have: $C_{y_r}=2\dfrac{l_vS_v\eta_v}{Sb}C_{L_{\alpha_v}}$ (3.104). In Table 3.4 we have: $C_{y_\beta}=-\eta_v\dfrac{S_v}{S}C_{L_{\alpha_v}}\left(1+\dfrac{d\sigma}{d\beta}\right)$. Express C_{y_r} as a function of C_{y_β}.

Solution: $\dfrac{S_v\eta_v}{S}C_{L_{\alpha_v}}=\dfrac{-C_{y_\beta}}{\left(1+\dfrac{d\sigma}{d\beta}\right)}$. Hence, $C_{y_r}=2\left(\dfrac{l_v}{b}\right)\dfrac{-C_{y_\beta}}{\left(1+\dfrac{d\sigma}{d\beta}\right)}$.

(f)(5pts) Assume that we have $l_v = 16\,ft$. Use $C_{y_\beta} = -\eta_v \frac{S_v}{S} C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)$ and $C_{n_r} = -2\eta_v V_v (l_v / b) C_{L_{\alpha_v}}$ to show that $\frac{d\sigma}{d\beta} = 1.07$.

Solution: Since $V_v = \frac{S_v l_l}{Sb}$, so $\frac{C_{y_\beta}}{C_{n_r}} = \left(1 + \frac{d\sigma}{d\beta}\right) / 2(l_v / b)^2$. Hence, $\frac{d\sigma}{d\beta} = 2\left(\frac{l_v}{b}\right)^2 \left(\frac{C_{y_\beta}}{C_{n_r}}\right) - 1 = 2\left(\frac{16}{33.4}\right)^2 \left(\frac{0.564}{0.125}\right) - 1 = 1.07$

(g)(5pts) Assume that $\eta_v = 1$. Use (e-f) to show that $C_{y_r} = 0.26$.

Solution: $C_{y_r} = 2\left(\frac{l_v}{b}\right) \frac{-C_{y_\beta}}{\left(1 + \frac{d\sigma}{d\beta}\right)} = 2\left(\frac{16}{33.4}\right) \frac{.564}{\left(1 + \frac{d\sigma}{d\beta}\right)} = \frac{.5404}{2.07} = 0.26$.

(h)(5pts) Use (f) to fill in the rightmost column of the table in (d). Then evaluate the influence of ignoring C_{y_r} in your approximate DR model.

Solution: [See code @ 2(h).] The results are essentially identical.

Problem 3(25pts) Consider the state space model (5.35) on p.195 for the lateral dynamics of a plane. This represents a 2-input/4-output system if we set $\mathbf{C} = \mathbf{I}_{4\times4}$ and $\mathbf{D} = \mathbf{0}_{4\times2}$.

(a)(15pts) Arrive at plots of the four responses to each of an impulse rudder input and an impulse aileron input.
Solution: [See code @ 3(a).]

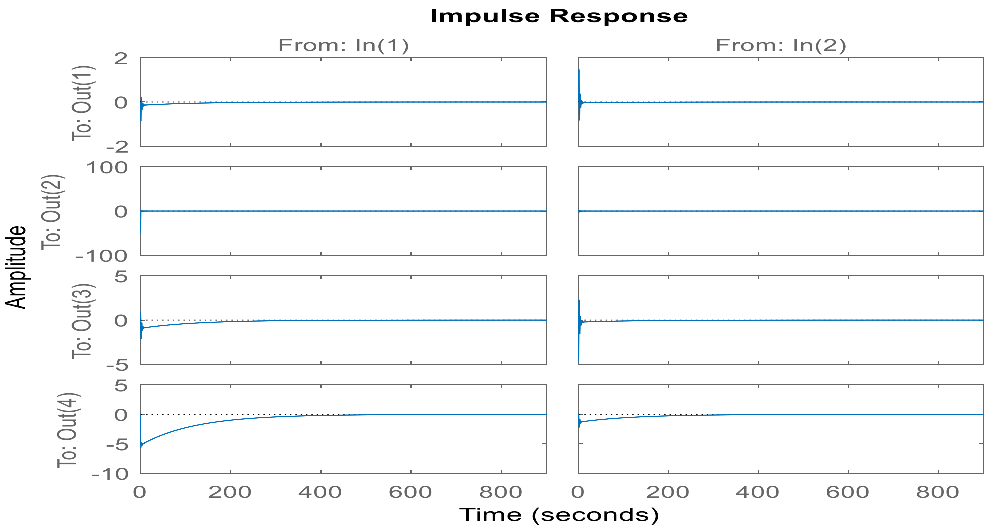

Figure 3(a) State impulse responses.

(b)(10pts) Repeat (a) but for Tfinal=10.
Solution: [See code @ 3(b).]

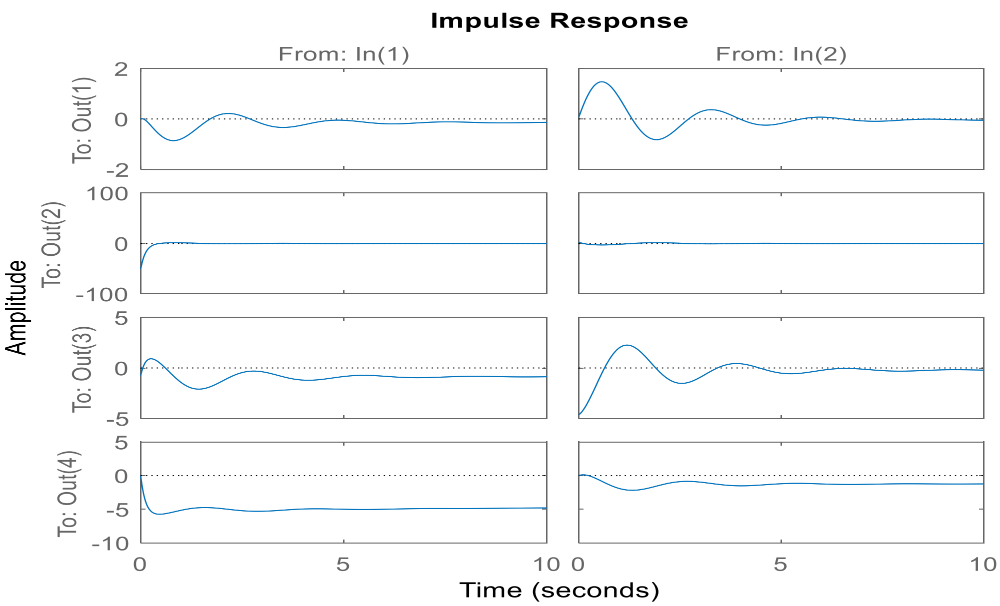

Figure 3(a) State impulse responses for Tfinal = 10 sec.

Matlab Code

```
%PROGRAM NAME: hw5.m  10/30/17
%PROBLEM 1
%(f)
%NAVION Values:
W=2750; Iz=3530; S=184; b=33.4; cbar=5.7;
g=32.174; m=W/g;
M=.158; a=1116.4; u0=M*a;
rho=.002377; Q=0.5*rho*u0^2;
Cnb=.071; Cnr=-.125; Cndr=-.072;
c1=Q*S*b/Iz;
%Transfer Function Coefficients:
Nb=c1*Cnb; Nr=c1*(.5*b/u0)*Cnr; Ndr=-c1*Cndr;
H=tf(-Ndr,[1 , -Nr , Nb]);
dr0=9.86; %Rudder deflection
figure(10)
step(dr0*H)
title(['Sideslip response to dr=',num2str(dr0,3),'^o Step'])
xlabel('time (sec)')
ylabel('Degrees')
grid
%========================================================
%PROBLEM 2
Cyb=-.564; Cydr=.157;
c2=Q*S/m;
Yb=c2*Cyb; Yr=(c2*.5*b/u0)*Cyr; Ydr=c2*Cydr;
A=[Yb/u0 , -(1-Yr/u0) ; Nb , Nr];
B=[Ydr ; Ndr];
C=[1 0]; D=0;
eigs2D=eig(A)
H=ss(A,B,C,D);
%------------------------------------
% 2(c):
%***Column 1***
wnp=sqrt(Nb);
zp=-0.5*Nr/wnp;
[zp wnp]
%***Column 2***
Cyr=0;
Cyr=0.26
Cyb=-.564; Cydr=.157;
c2=Q*S/m;
Yb=c2*Cyb; Yr=(c2*.5*b/u0)*Cyr;
wndr=sqrt((Yb*Nr-Nb*Yr+u0*Nb)/u0);%Nelson (5.46)
zdr=-0.5*(Nr+Yb/u0)/wndr;
[zdr wndr]
%-----------------------------------
%(g): Re-run the above for Cyr=0.522
%========================================================
%PROBLEM 3
%(a) Construction of the 4D Lateral state space matrices:
Ix=1048;
Cyp=0;
Clb=-.074; Clp=-.41; Cnp=-.0575; Clr=.107;
Lb=Clb*(Q*S*b/Ix);
Yp=(c2*0.5*b/u0)*Cyp;
Np=(c1*0.5*b/u0)*Cnp;
Lp=Clp*(Q*S*b^2)/(2*Ix*u0);
Lr=Clr*(Q*S*b^2)/(2*Ix*u0);
A41=[Yb/u0,Yp/u0,-(1-Yr/u0),g/u0];
A42=[Lb,Lp,Lr,0];
A43=[Nb,Np,Nr,0];
A44=[0,1,0,0];
A4=[A41;A42;A43;A44];
%----------------------------
Cydr=.26; Cndr=-.072; Cldr=.012;
Ydr=Cydr*c2; Ndr=Cndr*c1; Ldr=Cldr*(Q*S*b/Ix);
Clda=-.234; Cnda=-.0035;
Lda=(Q*b*S/Ix)*Clda; Nda=(Q*b*S/Ix)*Cnda;
B4=[0,Ydr/u0; Lda,Ldr; Nda,Ndr;0,0];
C4=eye(4); D4=zeros(4,2);
figure(40)
impulse(A4,B4,C4,D4)
```

```
%--------------------------------
%(b):
tfinal=10;
figure(41)
sys=ss(A4,B4,C4,D4);
impulse(sys,tfinal)
```