Homework 4 AERE355 Fall 2019 Due in class 10/25(F) SOLUTION
NOTE: Any solution/plot that is not placed directly beneath the typed statement will not be graded.

PROBLEM 1(20pts) Consider the system described by**:** $\ddot{y}+0.2\dot{y}+100y=10x$

(a)(3pts) Give the system *transfer function***:** $G_p(s)=10/(s^2+0.2s+100)$

(b)(2pts) Give the system *static gain***:** $g_s=G_p(0)=0.1$

(c)(2pts) Give the system *undamped natural frequency***:** $\omega_n=\sqrt{100}=10$

(d)(3pts) Give the system *damping ratio***:** ξ: $2\xi\omega_n=0.2\Rightarrow\xi\omega_n=0.1\Rightarrow\xi=0.1/10\Rightarrow\underline{\xi=0.01}$

(e)(2pts) Give the system *damped natural frequency***:** $\omega_d=\omega_n\sqrt{1-\xi^2}=\underline{10\sqrt{0.9999}}\cong 10$

(f)(3pts) The two *poles* of $G_p(s)$ in (a) are $s_{1,2}=-\xi\omega_n\pm i\omega_d$. Express these in *polar coordinates*:

$s_1=\rho e^{i\varphi}$ *where* $\underline{\rho=\sqrt{(-\xi\omega_n)^2+\omega_d^2}=\omega_n}$ & $\underline{\varphi=\pi-\tan^{-1}(\sqrt{1-\xi^2}/\xi}=\underline{\pi-\cos^{-1}(\xi)}$; $s_2=\rho e^{-i\varphi}$.

(g)(5pts) Use the Matlab command 'impulse' to arrive at a plot of the system impulse response. Copy/paste your code HERE.
Solution:

```
>> G=tf(10,[1 0.2 100]);
>> impulse (G)
>> grid
```

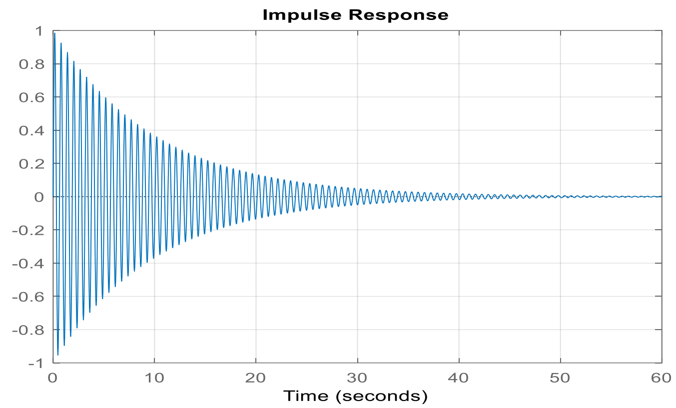

Figure 1(g) System impulse response.

PROBLEM 2(15pts) Consider the transfer function $\frac{Y(s)}{X(s)} \stackrel{\Delta}{=} G(s) = \frac{24s+250}{s^3+12s^2+45s+250}$.

(a)(3pts) Give the corresponding differential equation.

Answer: $\dddot{y} + 12\ddot{y} + 45\dot{y} + 250y = 24\dot{x} + 250x$

(b)(5pts) Use the Matlab command 'roots' to compute the system poles and zeros. Copy/paste your commands/answers HERE.

Solution: poles=roots([1 12 45 250]) = -10.0000 ; -1.0000 +/- 4.8990i ; zeros=roots([24 250]) = -10.4167

(c)(7pts) The system static gain is $G(0) = \frac{250}{250} = 1$. You should have found that the system includes a zero that nearly cancels a system pole. (i)Arrive at the reduced order system $\hat{G}(s)$, obtained by removing this pole/zero pair while retaining $G(0) = 1$. Then (ii) use the Matlab command 'step' to obtain overlaid plots of the unit step responses for $G(s)$ and $\hat{G}(s)$. (iii) Comment on how they compare.

Solution: [See code @ 2(c).]

(i) $\hat{G}(s) = \frac{25}{s^2+2s+25}$.

(iii) They are visually identical.

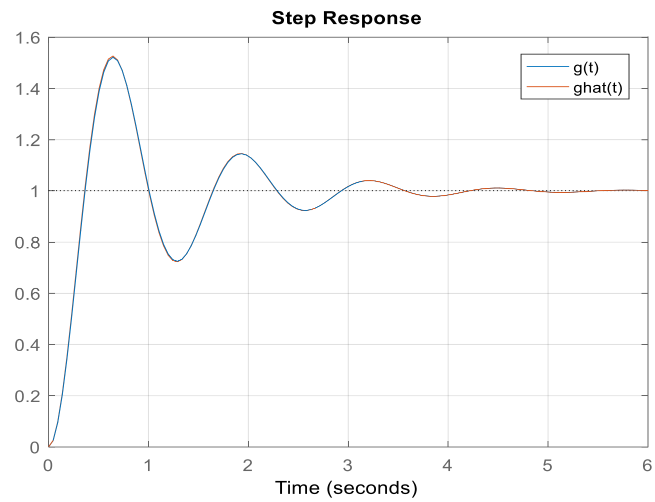

Figure 2(c) Overlaid impulse responses for $G(s)$ and $\hat{G}(s)$.

PROBLEM 3(65pts) The approximate *short period* longitudinal mode response to an elevator input is**

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}\end{bmatrix}=\begin{bmatrix}Z_w & 1\\ M_\alpha+M_{\dot\alpha}Z_w & M_q+M_{\dot\alpha}\end{bmatrix}\begin{bmatrix}\alpha\\ q\end{bmatrix}+\begin{bmatrix}Z_{\delta_e}/u_0\\ M_{\delta_e}+M_{\dot\alpha}Z_{\delta_e}/u_0\end{bmatrix}[\delta_e]. \qquad (8.1)$$

On p.284 the authors derive the transfer functions

$$\frac{\alpha(s)}{\delta_e(s)}\overset{\Delta}{=}G_\alpha(s)=\frac{(Z_{\delta_e}/u_0)s+(M_{\delta_e}-M_qZ_{\delta_e}/u_0)}{s^2-(M_q+M_{\dot\alpha}+Z_w)s+(M_qZ_w-M_\alpha)} \quad \text{and} \quad \frac{q(s)}{\delta_e(s)}\overset{\Delta}{=}G_q(s)=\frac{(M_{\delta_e}+M_{\dot\alpha}Z_{\delta_e}/u_0)s+(M_\alpha Z_{\delta_e}/u_0-M_{\delta_e}Z_w)}{s^2-(M_q+M_{\dot\alpha}+Z_w)s+(M_qZ_w-M_\alpha)}. \qquad (8.7,9)$$

Both of (8.7.9) have the denominator $p(s)=s^2-(M_q+M_{\dot\alpha}+Z_w)s+(M_qZ_w-M_\alpha)$. **I have replaced Z_α/u_0 by Z_w [See Table 3.5.]

(a)(35pts) (i)Use Tables 3.3 and 3.5, along with the information given on pp.400-401 to arrive at the numerical value for each parameter in $p(s)$. (ii) Arrive at the numerical values of the roots of $p(s)$. (iii) Compute the associated *time constant, undamped natural frequency, and damping ratio.*

<u>*Solution*</u>: [See code @ 3(a).]

The term "arrive at" here is a tad ambiguous/vague. Should you give all those equations HERE? Well, this would be a reasonable interpretation of that term. On the other hand, those terms ARE given directly in the tables. Furthermore, they are 'legion'. So here I would expect students to ASK what is meant by the term. Me? I personally found it to be extremely time-consuming, and so I did not include them here; only in the Matlab code.

(i) [Mq Ma Madot Zw] = [-2.0758 -8.7906 -0.9087 -2.0222]

(ii) $s_{1,2}$ = -2.5034 +/- 2.5926i = $-\zeta\,\omega_n \pm i\,\omega_d$.

(iii) tau=-1/real(R1) = **0.3995**; wn=abs(R1)= **3.6039** ; zeta=1/(tau*wn) = **0.6946**.

(b)(20pts) (i) Arrive at a plot of the roots of $p(s)$ for the speed range u_0 = 200: -1 : 50. You should find that they move along straight lines. (ii) Use the angle of the upper line to estimate the (constant) damping ratio, assuming these lines intersect at the origin. (iii) compare your estimate to your answer in (a)

<u>*Solution*</u>: [See code @ 3(b).]

(ii) $\tan\theta = 1.1/1 \Rightarrow \theta = 0.833\,rad$. $\hat{\zeta}=\cos(0.833)=0.673$.

(iii) They compare reasonably well.

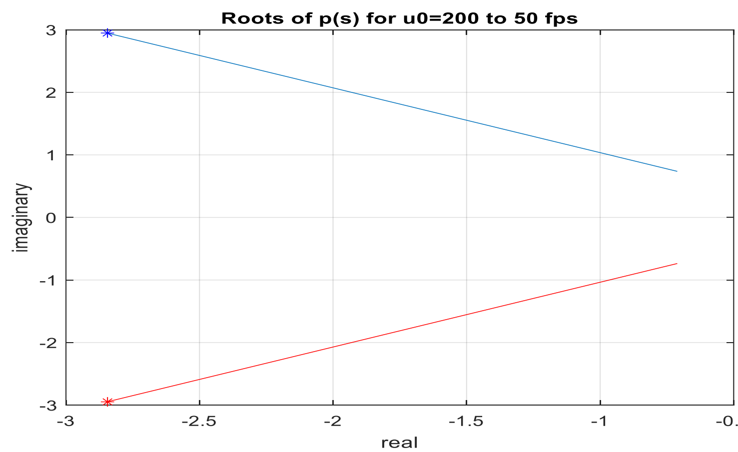

Figure 3(b) Roots of $p(s)$ for the speed range u_0 = 200: -1 : 50.

(c)(5pts) In view of the fact that the damping ratio is independent of the plane speed, explain how the time constant and undamped natural frequency vary with plane speed.

<u>*Explanation*</u>: $\zeta\,\omega_n = 1/\tau \quad \Rightarrow \zeta = 1/(\omega_n\tau) = \text{constant}$. From the plot it is clear that as speed decreases τ increases and ω_n decreases such that their product is constant.

(d)(5pts) Define $\lambda \overset{\Delta}{=} u_0\tau$. The units of τ are seconds, and the units of λ are feet. Since τ is called the system *time constant*, we will call λ the system *length constant*. Similarly, define $\Lambda_n \overset{\Delta}{=} \omega_n/u_0$. The units of ω_n are radians per second, and the units of λ are radians per foot. Since ω_n is called the system *temporal natural frequency*, we will call Λ_n the system *spatial* *natural frequency*. In view of (c), both λ and Λ_n are independent of the plane speed u_0. Use your results in (a) to compute their numerical values for the NAVION plane. Then, from Λ_n compute the spatial natural period.

<u>*Solution*</u>:

$\lambda = u_0\tau = 176(0.3995) = \mathbf{70.3}\text{ft}$ and $\Lambda_n = 3.6039/176 = \mathbf{0.0205}\text{rad/ft}$. $P_n = 2\pi/\Lambda_n = 3.6039/176 = \mathbf{306.5}\text{ft}$

Appendix Matlab Code

```
%PROGRAM NAME: hw4.m    10/7/17
%PROBLEM 2
%(c):
figure(20)
G=tf([24 250],[1 12 45 250]);
Ghat=tf(25,[1 2 25]);
figure(20)
step(G)
hold on
step(Ghat)
legend('g(t)','ghat(t)')
grid
%==============================================
%PROBLEM 3
rho=0.002377; %air density @ sea level
%NAVION Parameters (pp.400-401)
S=184; cbar=5.7; Iy=3000;  W=2750; g=32.18; m=W/g;
CD0=0.05; CLa=4.44; Cma=-0.683; Cmadot=-4.36;
Cmq=-9.96; Cmde=-0.923; CLde=0.355;
Czde=-CLde;   %Table 3.3 p.116
%----------------
%u0vec=176; %CODE for 3(a)
u0vec=200:-1:50;  %CODE for 3(b)
nu0=length(u0vec);
R1=zeros(nu0,1); R2=R1;
for k=1:nu0
u0=u0vec(k);
Q=0.5*rho*u0^2;
Mq=0.5*Cmq*Q*S*cbar^2/(Iy*u0);
Ma=Cma*Q*S*cbar/Iy;
Madot=0.5*Cmadot*Q*S*cbar^2/(Iy*u0);
Zw=-(CLa+CD0)*Q*S/(m*u0);
B=-(Mq+Madot+Zw);
C=Mq*Zw-Ma;
ABC=[1 B C];
R=roots(ABC);
R1(k)=R(1); R2(k)=R(2);
end
figure(30)
plot(real(R1),imag(R1))
hold on
plot(real(R1(1)),imag(R1(1)),'b*')
plot(real(R2),imag(R2),'r')
plot(real(R2(1)),imag(R2(1)),'r*')
title('Roots of p(s) for u0=200 to 50 fps')
xlabel('real')
ylabel('imaginary')
grid
```