Homework 2 AerE355 Fall 2019 Due 9/20(F) SOLUTION

Problem 1(30pts) Consider a small aircraft of the flying wing type, without elevator. Assume that the following parameters apply to the aircraft and flight condition: Weight (empty + pilot) = W = 3000 lb, Wing area = S = 150 ft^2, c.g. location = $h = 0.14$ $h_n = 0.26$, $\bar{c} = 5\ ft.$, ρ = 0.002378 slug/ft^2, $C_{m_{ac}} = 0.015$, $C_{L_\alpha} = 0.08$ per degree.

(a)(10pts) Recall that $C_{m_\alpha} = C_{L_\alpha}(h - h_n)$. (i)Compute this value. Then (ii) use it to determine α_{trim} (re: ZLL). Finally, based on (i) and (ii) explain why the plane is longitudinally stable.

Solution:

(i) $C_{m_\alpha} = C_{L_\alpha}(h - h_n) = .08(.14 - .26) = -.0096/\deg$

(ii) $C_m = 0 = C_{m_{ac}} + C_{m_\alpha}\alpha_{trim} \Rightarrow \alpha_{trim} = -C_{m_{ac}} / C_{m_\alpha} = -.015/(-.0096) = 1.56^o$

(iii)Because $C_{m_\alpha} < 0$ and $\alpha_{trim} > 0$, the plane is longitudinally stable.

(b)(5pts) Use (ii) in (a) to arrive at the trim velocity, V , for level, steady flight at the given density.

Solution: For level flight W=L, $C_L = L/(1/2\rho SV^2) = C_{L_\alpha}\alpha$, so $V = \sqrt{\dfrac{2W}{\rho S C_{L_\alpha}\alpha}}$ =366.8 ft/s.

(c)(10pts) Add a 500 lb payload to the airframe-pilot combination of part (a). Let the position of the load be a distance Δ behind the original x_{cg}^{old}. Begin with summation of moments to show that $x_{cg}^{new} = x_{cg}^{old} + \Delta/7$.

Solution: $\sum M_O = 3000x_{cg}^{old} + 500(x_{cg}^{old} + \Delta) = 3500x_{cg}^{old} + 500\Delta \equiv 3500x_{cg}^{new}$, Hence, $x_{cg}^{new} = x_{cg}^{old} + \Delta/7$.

(d)(5pts) Use the expression in (c) to arrive at the value for Δ such that the plane will be neutrally stable. [Note that h_n is not influenced by the added weight.]

Solution: $\dfrac{x_{cg}^{new}}{\bar{c}} - h_n = \dfrac{x_{cg}^{old}}{\bar{c}} + \dfrac{\Delta}{7\bar{c}} - h_n = 0$. Hence, $\Delta = 7\bar{c}\left(h_n - \dfrac{x_{cg}^{old}}{\bar{c}}\right) = 7(5)(.26 - .14) = 4.2\ ft.$

PROBLEM 2(30pts) In this problem we will investigate use of the elevator in relation to the NAVION general plane operating at sea level and M=0.158. Information on this plane is also given in Table B.1 on p.400, on p.401, and in EXAMPLE PROBLEM 2.2 on p.57.

(a)(6pts) The discussion in section 2.4.2 on p.65 includes the following:

$$C_{L_{trim}} = C_{L_\alpha}\alpha_{trim} + C_{L_{\delta_e}}\delta_{e_{trim}} \,. \tag{2.48}$$

For $\delta_{e_{trim}} = 0$, use <u>only</u> the information in Table B.1 to find α_{trim}. Then comment on how this relates to the value associated with the information on the <u>Wing airfoil characteristics</u> in Figure 2.16. [Hint: See also the 4th eqn. on p.58.]

<u>Solution</u>: For the entire plane, Table B.1 gives: $\alpha_{trim} = C_{L_{trim}} / C_{L_\alpha} = C_{L_0} / C_{L_\alpha} = 0.41/4.44 = .0923 = 5.29^o$.

<u>Comment</u>: In Figure 2.16 we are given $\alpha_{O_{L_w}} = -5^o$. This is the angle of attack (AOA) (relative to horizontal) for zero lift line (ZLL) of the wing alone. The 4th equation on p.58 uses the <u>negative</u> of this angle to compute $C_{L_{0_w}}$. When the wing is mounted on a horizontal fuselage so that it is parallel to it, the wing will have a 5° AOA. For the entire plane we have $\alpha_{trim} = 5.29^o$ relative to the plane ZLL. Hence, at the steady *level* flight condition (i.e. the fuselage is horizontal and the plane velocity direction is also horizontal), the vast majority (5/5.29 =0.945) of the lift is contributed by the wing.

(b)(6pts) Find the value of $C_{L_{trim}}$ using <u>only</u> the information on p.401. Then comment on how it compares to the value given in Table B.1.

<u>Solution</u>: $C_{L_{trim}} = \dfrac{W}{QS}$ where $W = 2750$, $S = 184$, and $Q = 0.5\rho(M \bullet a)^2 = 0.5(.002377)(.158 \times 1116.45)^2 = 36.98$. So $C_{L_{trim}} = .404$.

<u>Comment</u>: The value given in Table B.1 is 0.41, which is slightly greater than the value found here.

(c)(5pts) Suppose that the maximum elevator deflection is $\delta_{e_{\max}} = 15^o$. Find the minimum achievable $\alpha_{trim_{\min}}$ (degrees). [Hint: See book Section 2.4.2.]

<u>Solution</u>: From (2.50): $\alpha_{trim_{\min}} = \dfrac{C_{L_{trim}} - C_{L_{\delta_e}}\delta_{e_{\max}}}{C_{L_\alpha}}$. Rather than converting $\delta_{e_{\max}} = 15^o$ to radians, computing $\alpha_{trim_{\min}}$, and converting it back into degrees, we can simply multiply $C_{L_{trim}}$ by $180/\pi$ and leave $\delta_{e_{\max}} = 15^o$. Then

$$\alpha_{\min} = \frac{(180/\pi)C_{L_{trim}} - C_{L_{\delta_e}}\delta_{e_{\max}}}{C_{L_\alpha}} = \frac{(180/\pi)(0.41) - (0.355)(15)}{4.44} = 4.09^o \,.$$

(d)(5pts) Use equations (11) (for V_H), (31) and (32) in the Ch.2 notes to show that $C_{m_{\delta_e}} = -\left(\dfrac{l_t}{\bar{c}}\right)C_{L_{\delta_e}}$.

<u>Solution</u>: These equations are: $V_H = \dfrac{l_t S_t}{\bar{c}S}$ (11) ; $C_{L_{\delta_e}} = \left[\eta\tau\left(\dfrac{S_t}{S}\right)\right]C_{L_{\alpha_t}}$ (31) ; $C_{m_{\delta_e}} = -(\eta\tau V_H)C_{L_{\alpha_t}}$ (32).

Substituting (11) into (32) gives: $C_{m_{\delta_e}} = -\left[\eta\tau\left(\dfrac{S_t}{S}\right)\right]C_{L_{\alpha_t}}\left(\dfrac{l_t}{\bar{c}}\right)$. Substituting (31) into this gives the desired result.

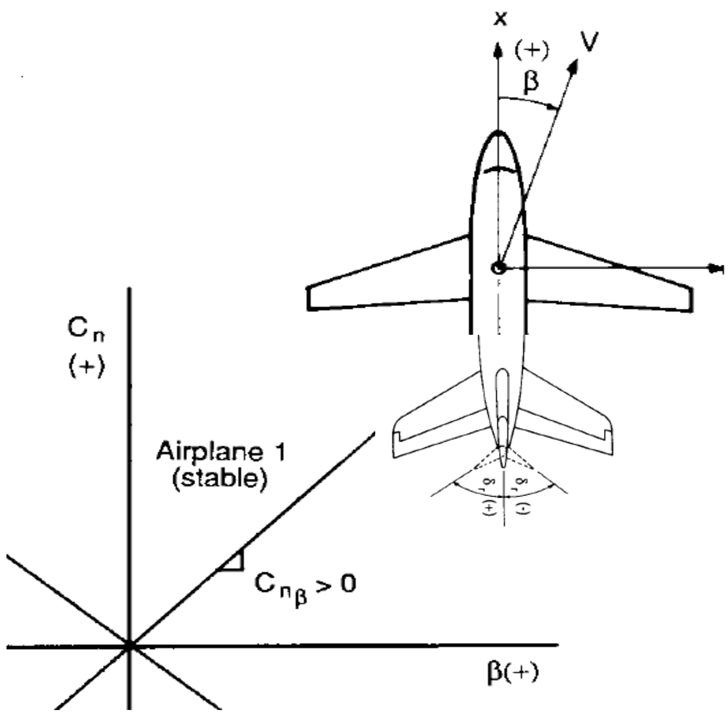
(e)(5pts) From the equation in (d), (i) find the numerical value for l_t, and then (ii) comment on how this value compares to that value given in EXAMPLE PROBLEM 2.2.

<u>Solution</u>: $l_t = -\bar{c}C_{m_{\delta_e}} / C_{L_{\delta_e}} = -(5.7)(-.923)/.355 \;=\; 14.82\ ft$

<u>Comment</u>: The value given in the example problem is $l_t = 16\ ft$, which is ~8% larger.

(f)(3pts) Identify where in the book the *elevator control power* is defined, and give its numerical value.
Answers: It is defined at the top of p.64 & again on p.66. From Table B.1, it is $C_{m_{\delta_e}} = -0.923$.

PROBLEM 3(15pts) Consider the combined Figures 2.28 & 2.32 on pp. 73 & 77, respectively. Sideslip is commonly performed intentionally, in order to accommodate crosswinds during landing. See, for example, segment 1:00 – 2:00 of [https://www.youtube.com/watch?v=QhV3BqPA1II]



(a)(5pts) Assume that the plane at right is at lateral equilibrium with a designated sideslip angle β_o. At a time $t = 0$ the sideslip angle is increased an amount $\Delta\beta$. Given that the fuselage contributes to *destabilization* lateral stability (c.f. p.74), describe how the sideslip angle would evolve, in the absence of a vertical tail.
Explanation: The sideslip angle would increase further.

(b)(5pts) Suppose that, even with a vertical tail, the plane became laterally unstable. Which rudder direction (+ or -) would be appropriate to try to stabilize the plane? Explain. [Hint: Consider the camber of the vertical tail/rudder.]
Explanation: A lateral instability would continue to *increase* $\Delta\beta$. This would occur if, for example, the tail vertical area was insufficient to generate large enough negative side force to stop the increase in $\Delta\beta$. Activating right rudder (i.e. $\delta_r < 0$) will result in an increase in vertical camber, thereby generating a larger tail *negative* vertical lift. This, in turn, would result in a larger cw moment. This moment would act to reduce $\Delta\beta$.

(c)(5pts) In addition to the rudder, what other control surface would need to be activated in order to level the plane? Justify your answer. [Hint: Watch the above video. Then, think about it.]
Answer: The short answer is: The ailerons. Here's my personal elaboration. The video @ 1:24 states that the wind is blowing 30° from the left at 17mph. This will cause the plane to slip to the right, as shown in the video and in the above figure. At 1:40 the video states that, in order to maintain a desired sideslip $\beta_o > 0$ a right rudder/left aileron is appropriate. I disagree. The vertical tail will tend to align the nose in the direction of the velocity vector. A right rudder would increase this tendency. A left rudder would oppose it. In maintaining β_o by using the left rudder, the right side of the wing will have greater lift than the left side, causing the plane to roll left. Use of the left aileron will increase the lift on the left side of the wing to counter this roll. Hence, I would claim that in the video setting, what is needed to maintain β_o is left rudder/left aileron. But then, what do I know. ☹

Problem 4(25pts) In this problem we consider the NAVION general plane operating as described in PROBLEM 2.

(a)(8pts) For a sideslip angle $\beta = 10^o$, find the magnitude and sign of the rudder step input needed to bring the NAVION to $\beta_o = 5^o$. [Hint: Notice that $C_{n_\beta} = dN/d\beta$, $C_{n_{\delta_r}} = dN/d\delta_r$, and $\delta_r \cong (d\delta_r/d\beta)\beta$. Also, note that in Table B.1 the entry $C_{n_\beta} = -0.071$ is wrong. It should be $C_{n_\beta} = +0.071$.]

Solution: $\dfrac{C_{n_{\delta_r}}}{C_{n_\beta}} = \dfrac{-.072}{.071} = -1.014 = \dfrac{dN/d\delta_r}{dN/d\beta} = \dfrac{d\beta}{d\delta_r}$. Hence, $\dfrac{d\delta_r}{d\beta} = -0.986$

Hence, a rudder angle $\delta_r = \left(\dfrac{d\delta_r}{d\beta}\right)\Delta\beta = -.986(-5^o) = 4.93^o$ is needed.

(b)(5pts) Identify where in the book the rudder *control effectiveness* is defined, and give its numerical value.
<u>*Answer*</u>: It is defined directly above (2.84) on p.78. From Table B.1: $C_{n_{\delta_r}} = -.072$.

(c)(5pts) Use the appropriate entries in Figure 1.10 to define C_{l_β}. Then explain why the line with slope $C_{l_\beta} < 0$ in Figure 2.33 on p.79 corresponds to a plane that possesses roll stability.
<u>*Definition*</u>: In Figure 1.10 we have the *roll moment* $L = C_l QSb$. The rate of change of the scaled moment, C_l, with respect to the *side slip* angle β is C_{l_β}.

<u>*Explanation*</u>: For the positive β, it is noted in Figure 2.33 that a positive roll moment is created. In order to correct for this, we must have $C_{l_\beta} < 0$. [See also Figure 1.10.]

(e)(7pts) In the Ch.2 Notes we have

$$C_{l_{\delta_a}} \cong \frac{2C_{L_{\alpha_a}} \tau\, c_r \Delta_a \mu_a}{Sb}\left[1 - \left(\frac{1-\lambda}{b/2}\right)\mu_a\right]. \qquad (69)$$

The moment derivative (68) [and its approximation (69) is called the aileron roll *control power*. In EXAMPLE PROBLEM 2.4 on p.83, the roll control power is ultimately found to be $C_{l_{\delta_a}} = 0.155$. Use the approximation (69) to estimate it.
<u>*Solution*</u>: $\Delta_a = y_2 - y_1 = 4.9$; $\mu_a = (y_1 + y_2)/2 = 13.55$. We will retain all the other terms in the equation in the middle of p.84:

$$C_{l_{\delta_a}} \cong \frac{2C_{L_{\alpha_a}} \tau\, c_r}{Sb}\left\{\Delta_a \mu_a \left[1 - \left(\frac{1-\lambda}{b/2}\right)\mu_a\right]\right\} = \frac{2(4.3)(.36)(7.2)}{184(33.4)}\left\{4.9(13.55)\left[1 - \left(\frac{1-.54}{16.7}\right)13.55\right]\right\} = 0.151.$$

The approximation error is (0.151-0.155)/0.155 = -2.6%.