**Homework 1 AERE355 Fall 2019 Due 9/6(F) SOLUTION**

**NOTE:** If your solution does not adhere to the format described in the syllabus, it will be grade as zero.

**Problem 1(25pts)** In the altitude region $0 < h < 10km$, we have the following atmospheric equations:

$$T = T_0 + \lambda h \quad (1.9^*) \quad ; \quad \frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{-g_o}{R\lambda}} \quad (1.55) \quad ; \quad \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{-(1+\frac{g_o}{R\lambda})} \quad \text{or } \rho = \frac{P}{RT} \quad (1.57).$$

**(a)(10pts)** Write a Matlab code that will generate a table with columns $[h \quad T \quad P \quad \rho]$ for the array of altitude values $h = 0:1000:10000$, give the table, and comment on how it compares to Table A.1 on p.395 in APPENDIX 1.

*Solution*: [See code @ 1(a).]

**Comment:** This table is almost identical to Table A.1

| $H_G$ | $h$ | $T$ | $P$ | $\rho$ |
|---|---|---|---|---|
| 0 | 0 | 288.16 | 101330 | 1.225 |
| 1000 | 999.84 | 281.66 | 89874 | 1.1116 |
| 2000 | 1999.4 | 275.16 | 79498 | 1.0065 |
| 3000 | 2998.6 | 268.67 | 70116 | 0.90919 |
| 4000 | 3997.5 | 262.18 | 61655 | 0.81927 |
| 5000 | 4996.1 | 255.69 | 54042 | 0.73634 |
| 6000 | 5994.4 | 249.2 | 47211 | 0.66001 |
| 7000 | 6992.3 | 242.71 | 41098 | 0.58992 |
| 8000 | 7990 | 236.23 | 35645 | 0.52568 |
| 9000 | 8987.3 | 229.74 | 30794 | 0.46696 |
| 10000 | 9984.3 | 223.26 | 26493 | 0.4134 |

**Table 1(a) Standard Atmosphere Table**

**(b)(10pts)** Run your code for $T_0 = 288.16^o K$ [i.e. the temperature at $h = 0\ km$ for your table in (a)], and for $T_0 = 316^o K$. Then obtain a plot of the percent change in air density vs. altitude associated with the increased air surface temperature.

*Solution*: [See code @ 1(b).] NOTE: To compute % change I used $\frac{\rho_{288} - \rho_{316}}{\rho_{288}} \times 100\%$. The problem ambiguity allows for other forms.

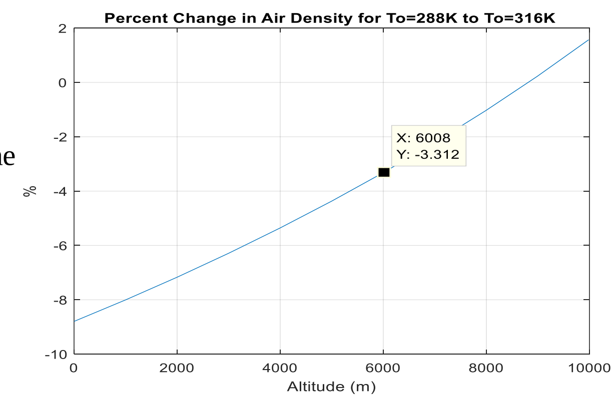


**Figure 1(b)** Percent change in Air density.

From https://aviation.stackexchange.com/questions/3011/how-is-pressure-related-to-air-density we have the following:

On a hot day what tends to happen is that the surface, which is being warmed by the sun, heats the lowest level of the atmosphere, reducing its density (it is at the same pressure as its surroundings and its T rises). This will eventually drive convection and mix this warmer air vertically. Given enough time, this will reduce the mass in the column of air and therefore reduce the pressure at the surface. These are called "heat lows" and you can see them forming in the desert areas and they play roles in sea breeze formation and the monsoons.

However, this question does not address the context of the increased temperature. We will here assume that the surface temperature has increased for a sufficiently short period, so that the ground pressure remains unchanged. Then, from the ideal gas law, for $T_0 = 316^o K$ , the resulting surface density is $\rho_0 = 1.1172 kg/m^3$

**(c)(5pts)** Based on your plot in (b) and the definition of the *lift coefficient*, explain, quantitatively, explain how the increased temperature influence the *lift* of a plane flying at an altitude of 6km. Include your reference in relation to the definition of the lift coefficient.

*Solution*: From https://en.wikipedia.org/wiki/Lift_coefficient , $C_L = L/(0.5\rho V^2 S)$ . Hence, for a plane having a given lift coefficient, its lift is directly proportional to $\rho$ . At $h = 6000\ km$ the lift will decrease by ~3.3%.

**PROBLEM 2(25pts) [Related to book Problem 1.2]** An airplane's altimeter reading is 5,000m.

**(a)(10pts)** If the outside ambient temperature is -20ºC and the plane's true airspeed is 300m/s, find the plane's indicated airspeed.
*Solution*: Recall that the altimeter uses pressure measurements to arrive at the altimeter reading. From Table A.1 on p.395, for $h = 5{,}000m$ we have $P = 54{,}012.5 N/m^2$. [Obtained from a linear interpolation of Table A.1 we obtain $5.4048 + 4\left(\frac{4.7217 - 5.4048}{5994 - 4996}\right) = 5.402e4$. Obtained directly from $P = P_o\left(1 + \frac{\lambda h}{T_o}\right)^{(-g_o/R\lambda)} = 54{,}012.5$. I will use the latter.]

Hence, for $T = -20^oC = 253^oK$, the air density is $\rho = P/RT = 54{,}012.5/[287(253)] = 0.744 kg/m^3$.

Hence, $\sigma = \rho/\rho_o = .744/1.225 = 0.607$. From (1.79) on p.25, we obtain $V_{EAS} = V_{TAS}\sqrt{\sigma} = 300\sqrt{.607} = 233.7 m/s$. To obtain $V_{IAS}$, I will forego the equations in the book, and simply use the Matlab command:
`correctairspeed(300,320.5,5.4e4,'TAS','CAS')`. [See also:
https://www.mathworks.com/help/aeroblks/idealairspeedcorrection.html . This gives: $V_{IAS} = 243 m/s$.

**(b)(5pts)** Suppose that the altimeter reading fluctuates between roughly 4,990m and 5,010m, and that the mean reading is $\mu_h = 5{,}000m$. Assume that the fluctuation $\Delta h$ has a normal distribution with $\sigma = 3m$. Use the Matlab command 'normpdf' to arrive at a plot of the *probability density function* (*pdf*) for the altimeter reading, *h*.
*Solution*: [See code @ 2(b).]

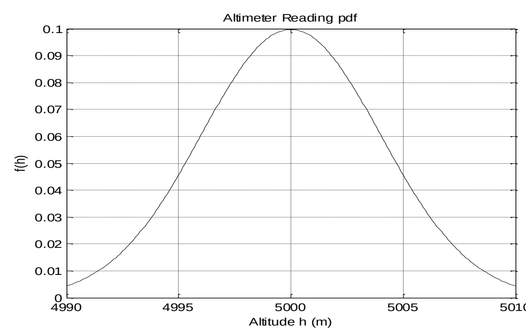


**Figure 2(b)** Altimeter reading *pdf*.

**(c)(5pts)** On p.17 we have equation (1.54): $\ln(P/P_1) = \frac{-g_o}{R\lambda}\ln\left(\frac{T_1 + \lambda(h - h_1)}{T_1}\right)$. Beginning with this equation, carry out detailed steps to show that $P = P_o\left(\frac{T_o + \lambda h}{T_o}\right)^{-g_o/R\lambda}$.

*Solution*: In our situation, $T_1 = T_o = 288.150^oK$, $P_1 = P_o = 101{,}325 N/m^2$ and $h_1 = 0m$.

Step 1: $\ln(P/P_o) = \ln\left(\frac{T_o + \lambda h}{T_o}\right)^{\frac{-g_o}{R\lambda}}$; Step 2: $P/P_o = \left(\frac{T_o + \lambda h}{T_o}\right)^{\frac{-g_o}{R\lambda}}$; Step 3: $P = P_o\left(\frac{T_o + \lambda h}{T_o}\right)^{-g_o/R\lambda}$

**(d)(5pts)** To arrive at the *pdf* of $P$ : (i) use the Matlab command 'normrnd' to generate *N*=10,000 simulations of *h*. (ii) Use these in the expression in (c) to obtain simulated measurements of *P*.(iii) Use the command 'histogram' to arrive at a plot of an estimate of the *pdf* of *P*. (iv) Use the commands 'mean' and 'std' to estimate the mean and standard deviation of *P*. NOTE: Recall that $R = 287 m^2/^oK - s^2$ and $\lambda = -.0065\ ^oK/m$.

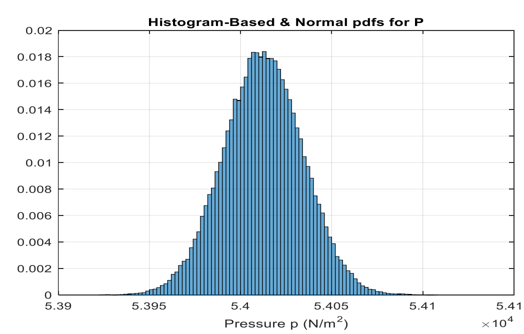


**Figure 2(d)** Simulation-based *pdf* for *P*.

*Solution*: [See code @ 2(d).]
$\hat{\mu}_P = 54{,}012\ N/m^2$ and $\hat{\sigma}_P = 21.6\ N/m^2$.

**Problem 3(25pts)** The plot below shows both lift and moment coefficients as functions of the angle of attack for the NACA 23012 airfoil. Also included are straight-line approximations of these relations. You are to use these approximations in this problem.

From:
http://www.google.com/search?q=naca+23012+airfoil+data&hl=en&prmd=imvns&tbm=isch&tbo=u&source=univ&sa=X&ei=WK87UIyzB8OZ2QW4pYCwDw&ved=0CDIQsAQ&biw=1280&bih=637

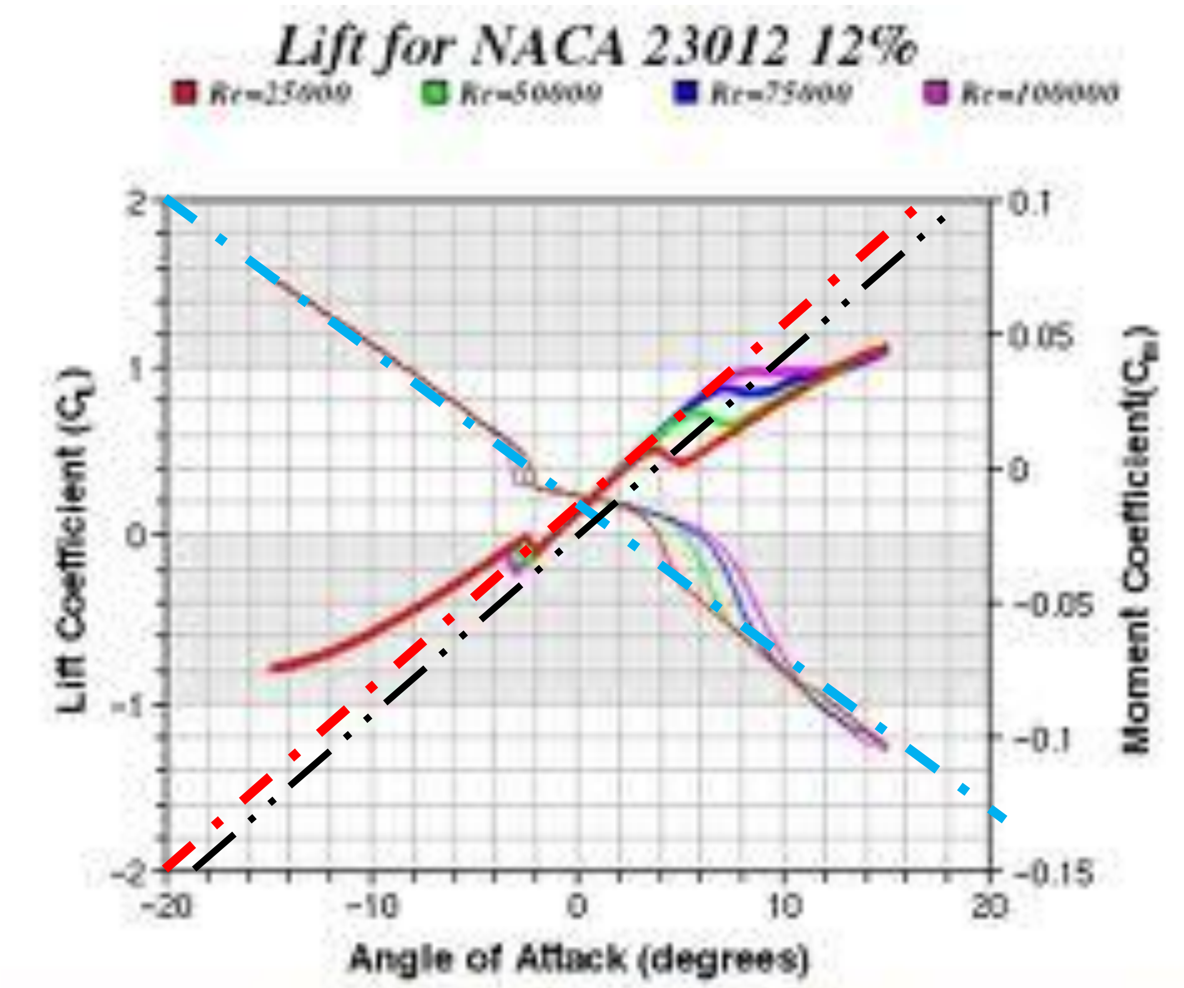


**Figure 1.** This plot also includes the $C_L(\alpha_{ZLL})$ line (in black).

**(a)(6pts)** Estimate the coefficients of the lift model: $C_L(\alpha) = C_{L_o} + C_{L_\alpha}\alpha$ using the RED line. Then plot your model for $-20^o < \alpha < 16^o$ to verify its correctness.
*Solution*: $C_{L_o} \cong 0.2^o$ & $C_{L_\alpha} \cong 4/36 = 0.11$.
Hence, my model is: $C_L(\alpha) = 0.2 + 0.11\alpha$

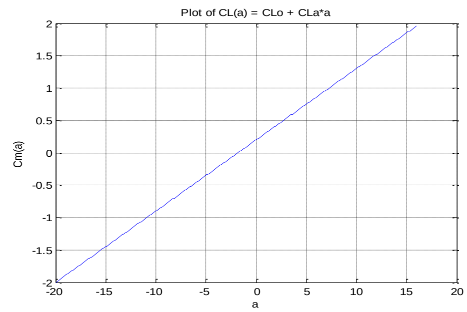


**Figure 1.1(a)** Plot of lift model.

**(b)(6pts)** Estimate the coefficients of the moment model: $C_m(\alpha) = C_{m_o} + C_{m_\alpha}\alpha$ using the BLUE line. Then plot your model for $-20^o < \alpha < 20^o$ to verify its correctness.
*Solution*: $C_{m_o} \cong -.01^o$ and $C_{m_\alpha} \cong -.23/40 = -.0057$.
Hence, my model is: $C_m(\alpha) = -0.01 - 0.0057\alpha$.

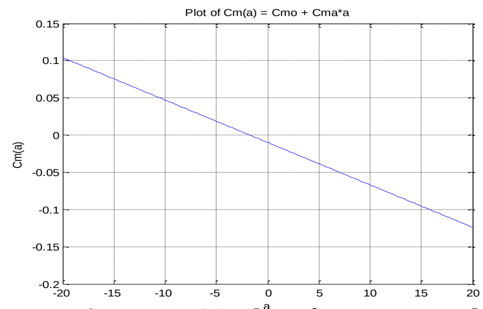


**Figure 1.1(a)** Plot of moment model.

**(c)(3pts)** Compute the numerical value of $\alpha_{trim}$ for your airfoil moment model in (b).

*Answer*: $C_m(\alpha_{trim}) = 0 \quad \Rightarrow \quad \alpha_{trim} \cong -1.75^o$.

**(d)(6pts)** Use your equations from parts (a) & (b) to estimate the parameters $C_{m_o}$ and $\Delta h = h - h_n$ of the moment model: $C_m = C_{m_o} + C_L \Delta h$ .

*Solution*: From (a) we have $\alpha = [C_L(\alpha) - 0.2]/0.11$. From (b) we have $C_m(\alpha) = -.01 - .0057\alpha$. Substituting the expression for $\alpha$ into this gives the model: $C_m = 0.0004 - 0.052 C_L$.

**(e)(4pts)** For your model in (d) determine whether the wing has negative pitch stiffness) or positive pitch stiffness).

*Explanation*: The wing *static margin* $K_n = -\Delta h = 0.052$ is greater than zero. So the wing has *positive pitch stiffness.*

For more on this problem, see pp.334-338 of Anderson *Fundamentals of Aerodynamics*.

**PROBLEM 4(25pts) [Related to book problem 2.1]** A given wing moment equation is: $C_{m_{cg_w}} = 0.08 - 0.15C_{L_w}$

**(a)(5pts)** Recall from the discussion below Figure 2.5 on p.43 that the *trim* condition is defined as that condition such that $C_{m_{cg}} = 0$. (i.e. the *trim* moment coefficient equals zero). Obtain the corresponding *trim* lift coefficient.

*Solution*: $C_{m_{cg_{trim}}} = 0 = 0.08 - 0.15C_{L_{w_{trim}}} \quad \Rightarrow \quad C_{L_{w_{trim}}} = .08/.15 = 0.53$

**(b)(5pts)** The (scaled) wing center of gravity of the wing is $h_{cg_w} \overset{\Delta}{=} \frac{x_{cg_w}}{\bar{c}} = 0.3$. Use (2.6) on p.45 to find the wing neutral point $h_{N_w} \overset{\Delta}{=} \frac{x_{N_w}}{\bar{c}}$. Assume that the neutral point equals the aerodynamic center point.

*Solution*: Using (2.6), we have: $C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w}(h_{gc_w} - h_{ac_w}) = 0.08 - 0.15C_{L_w}$.
Hence, $h_{gc_w} - h_{ac_w} = -0.15 \quad \Rightarrow \quad h_{ac_w} = h_{gc_w} + .15 = .3 + .15 = .45$

**(c)(10pts)** In (2.7) we see that $C_{L_w} = C_{L_{w0}} + C_{L_{\alpha_w}}\alpha_w$. Suppose that the wing lift equals zero when $\alpha_w = i_w = 2^o(\pi/180^o) = .0349rad$, and it is at *trim* when $\alpha_w = \alpha_{w_{trim}} = 5^o(\pi/180^o) = .0873rad$. Find $C_{L_{w0}}$ and $C_{L_{\alpha_w}}$. Then plot $C_{L_w}$ as a function of $\alpha_w$ over the range $0^o \le \alpha_w \le 10^o$.

*Solution*: From (a): $0.53 = C_{L_{w0}} + C_{L_{\alpha_w}}(.0873)$. Also, $0 = C_{L_{w0}} + C_{L_{\alpha_w}}(.0349)$.

These two equations can be written in matrix form: $\begin{bmatrix} 1 & .0873 \\ 1 & .0349 \end{bmatrix}\begin{bmatrix} C_{L_{w0}} \\ C_{L_{\alpha_w}} \end{bmatrix} = \begin{bmatrix} .53 \\ 0 \end{bmatrix}$.

Hence, (using Matlab): $\begin{bmatrix} C_{L_{w0}} \\ C_{L_{\alpha_w}} \end{bmatrix} = \begin{bmatrix} -.353 \\ 10.115 \end{bmatrix}$

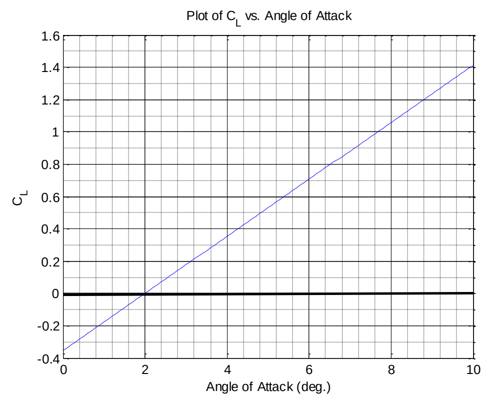


**Figure 2.1** Plot of $C_{L_w}$ versus $\alpha$.

**(d)(5pts)** For a wing-alone configuration as shown in Figure 2.7 (on p.45), explain why it is not statically stable.
*Explanation*: Because $x_{cg} > x_{ac}$.

## Appendix *Matlab* Code

```
%PROGRAM NAME: hw1.m
%PROBLEM 1
%PART (a):
hG = 0:1000:10000; hG = hG';  % Altitude array (km)
Ro = 6371e3; %Earth mean radius (m)
h = Ro*(Ro+hG).^-1 .* hG;
nh = length(h);
vec1 = ones(nh,1);
go = 9.807; %Gravity (m/s2) at sea level
R = 287; %Gas constant ((m2/K-s2)
To = 288.16; %Temp (K) at sea level
Po = 1.01325e5; %Pressure (N/m2) at sea level
ro = 1.225; %Density (kg/m3) at sea level
L = -.0065; %Temp gradiant (K/km)
c = go/(R*L);
T = To + L*h;
P = Po*((T/To).^(-c*vec1));
r = ro*(T/To).^-(1+c*vec1);
format shortg
Table = [hG h T P r]
%PART (b):
To=316; %~110F Temp (K) at sea level
Po = 1.01325e5; %Assume pressure (N/m2) at sea level remains constant
ro = Po/(R*To); %Density (kg/m3) at sea level
L = -.0065; %Temp gradiant (K/km)
c = go/(R*L);
T = To + L*h;
P = Po*((T/To).^(-c*vec1));
r1 = ro*(T/To).^-(1+c*vec1);
dr=100*(r-r1)./r;
figure(10)
plot(h,dr)
grid
xlabel('Altitude (m)')
ylabel('%')
title('Percent Change in Air Density for To=288K to To=316K')
%==============================================================
%PROBLEM 2
%Part(a):
correctairspeed(300,320.5,5.4e4,'TAS','CAS')
%PART (b):
hvals=4990:.1:5010;  % Altitude Range of Values
muh = 5000; % Altitude Mean
stdh = 3;  % Altitude Standard Deviation
f = normpdf(hvals,muh,stdh);
figure(20)
plot(hvals,f,'k')
xlabel('Altitude h (m)')
ylabel('f(h)')
title('Altimeter Reading pdf')
grid
% PART (d):
R=287; L=-.0065; To=288.15; Po=101325; go=9.807;
N = 10^5;
h = normrnd(muh,stdh,N,1);
hones = ones(N,1);
c1 = 1+(L/To)*h;
c2 = (-go/(R*L))*hones;
P = Po*c1.^c2;  % Pressure Simulated Data Array
%---------------------
% Compute Estimated pdf:
figure(21)
```

```
histogram(P,'Normalization','pdf');
grid
xlabel('Pressure p (N/m^2)')
title('Histogram-Based & Normal pdfs for P')
muPest = mean(P);
stdPest = std(P);
format shortg
[muPest , stdPest]
```