EXAM 3 AERE355 Fall 2019 (Take-Home) Due 11/1(F) SOLUTION

PROBLEM 1 (40pts) This problem concerns the longitudinal linearized dynamics of the NAVION plane. The state equation is $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$. For the state $\mathbf{x} = [u \quad w \quad q \quad \theta]^{tr}$, Nelson develops the numerical values in **A** in EXAMPLE PROBLEM 4.3 on pp.155-158. This matrix is given in the Matlab code in Appendix of this exam. The command eig(A) results in the following eigenvalues:

eig(A)=[-2.4894 + 2.5977i -2.4894 - 2.5977i -0.0171 + 0.2146i -0.0171 - 0.2146i]

(a)(10pts) For each of the two modes, (i) give its name, and (ii) compute values for the parameters $(\tau, \omega_d, \omega_n, \zeta)$

Solution: Recall that stable complex-conjugate roots have the form: $\lambda_{1,2} = -\zeta\omega_n \pm i\omega_d = -1/\tau \pm i\omega_d$.

short period $\lambda_{1,2} = -2.4894 \pm i\,2.5977$: $\tau = 1/2.4897 = 0.4017\,\text{sec}$, $\omega_d = 2.5977\,\text{r/s}$, $\omega_n = |\lambda_1| = 3.598\,\text{r/s}$, $\zeta = 0.692$

phugoid: $\lambda_{3,4} = -0.0171 \pm i\,0.2146$: $\tau = 1/.0171 = 58.64\,\text{sec}$, $\omega_d = 0.2146\,\text{r/s}$, $\omega_n = |\lambda_3| = 0.2153\,\text{r/s}$, $\zeta = 0.079$

(b)(10pts) We will consider the elevator angle input $\mathbf{u}(t) = \delta_e(t)$. It follows from (4.51) on p.149 that $\mathbf{B} = \begin{bmatrix} X_{\delta_e} & Z_{\delta_e} & M_{\delta_e} & 0 \end{bmatrix}^{tr}$. This presumes that $M_{\dot{w}} = 0$. Because no value is given in relation to X_{δ_e} in Table B.1 (or anywhere else), it is reasonable to assume that $X_{\delta_e} = 0$. Use information given in Table 3.3, Table 3.5, Table B.1 and EXAMPLE PROBLEM 4.3 to show that $\mathbf{B} = [0 \quad -28.146 \quad -11.874 \quad 0]^{tr}$. Show all work HERE.

Solution: From Table 3.5: $Z_{\delta_e} = C_{Z_{\delta_e}}(QS/m)$ and $M_{\delta_e} = C_{m_{\delta_e}}(QS\bar{c}/I_y)$. From Table 3.3: $C_{Z_{\delta_e}} = -C_{L_{\delta_e}}$.

From Table B.1: $C_{L_{\delta_e}} = 0.355$ and $C_{m_{\delta_e}} = -0.923$. From EP 4.3: $QS/m = 6771/85.4 = 79.286$ and $QS\bar{c}/I_y = 6771(5.7)/3000 = 12.865$. Hence: $Z_{\delta_e} = -0.355(79.286) = \mathbf{-28.146}$ and $M_{\delta_e} = -0.923(12.865) = \mathbf{-11.874}$.

(c)(10pts) As it stands, the state model $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$ is undesirable in two respects. First, in view of the recent Boeing MCAS issues, it is desirable that the state be $\mathbf{x} = [u \quad \alpha \quad q \quad \theta]^{tr}$. Second, it is desirable that all state angles and the input angle be in degrees, as opposed to radians. Hence, we need to convert $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$ to $\dot{\mathbf{x}}' = \mathbf{A}'\mathbf{x}' + \mathbf{B}'\mathbf{u}'$. To this end, first note the following FACT: For any invertible matrix **V**, let $\mathbf{x}' = \mathbf{Vx}$. Then clearly $\dot{\mathbf{x}}' = \mathbf{V}\dot{\mathbf{x}}$. And so $\mathbf{V}\dot{\mathbf{x}} = \mathbf{VAV}^{-1}\mathbf{Vx} + \mathbf{VBu}$, which is $\dot{\mathbf{x}}' = (\mathbf{VAV}^{-1})\mathbf{x}' + (\mathbf{VB})\mathbf{u} \overset{\Delta}{=} \mathbf{A}'\mathbf{x}' + \mathbf{VBu}$. To convert *w* to α, requires the matrix $\mathbf{Q} = diag\{1 \quad 1/u_0 \quad 1 \quad 1\}$, and to convert from radians to degrees requires the matrix $\mathbf{U} = diag\{1 \quad 180/\pi \quad 180/\pi \quad 180/\pi\}$. Hence, setting $\mathbf{V} \overset{\Delta}{=} \mathbf{UQ}$ will result in $\dot{\mathbf{x}}' = \mathbf{A}'\mathbf{x}' + \mathbf{VBu}$. Finally, to convert **u** (radians) to $\mathbf{u}'$ (degrees) requires that $\mathbf{u} = (\pi/180)(180/\pi)\mathbf{u} = (\pi/180)\mathbf{u}'$. From this, it follows that setting $\mathbf{B}' \overset{\Delta}{=} (\pi/180)\mathbf{VB}$. Hence, the desired state model becomes $\dot{\mathbf{x}}' = \mathbf{A}'\mathbf{x}' + \mathbf{B}'\mathbf{u}'$. Use Matlab to arrive at the values for $\mathbf{A}'$ and $\mathbf{B}'$.

Solution: [See code @ 1(c).]

$$\mathbf{A}' = \begin{bmatrix} -0.0450 & 0.1106 & 0 & -0.5620 \\ -0.1201 & -2.0200 & 1 & 0 \\ 0.1140 & -6.9696 & -2.9480 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad \mathbf{B}' = \begin{bmatrix} 0 \\ -0.1599 \\ -11.8747 \\ 0 \end{bmatrix}$$

(d)(10pt) Regardless of your answers in (c), use $\mathbf{A}' = \begin{bmatrix} -0.045 & 0.111 & 0 & -0.562 \\ -0.120 & -2.020 & 1 & 0 \\ 0.114 & -6.970 & -2.948 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $\mathbf{B}' = \begin{bmatrix} 0 \\ -0.160 \\ -11.875 \\ 0 \end{bmatrix}$.

For the input $\mathbf{u}(t) = \delta_e \delta(t) = -5^o\,\delta(t)$ use the 'impulse' command to obtain plots of the state response. [Note: The 'impulse' command does not include an argument for scaling it. So, you will need to scale the $\mathbf{B}'$ by -5.] Run the code twice: first with no maximum time value, and second, with a maximum time value of 2 seconds.

Solution: [See code @ 1(d).]

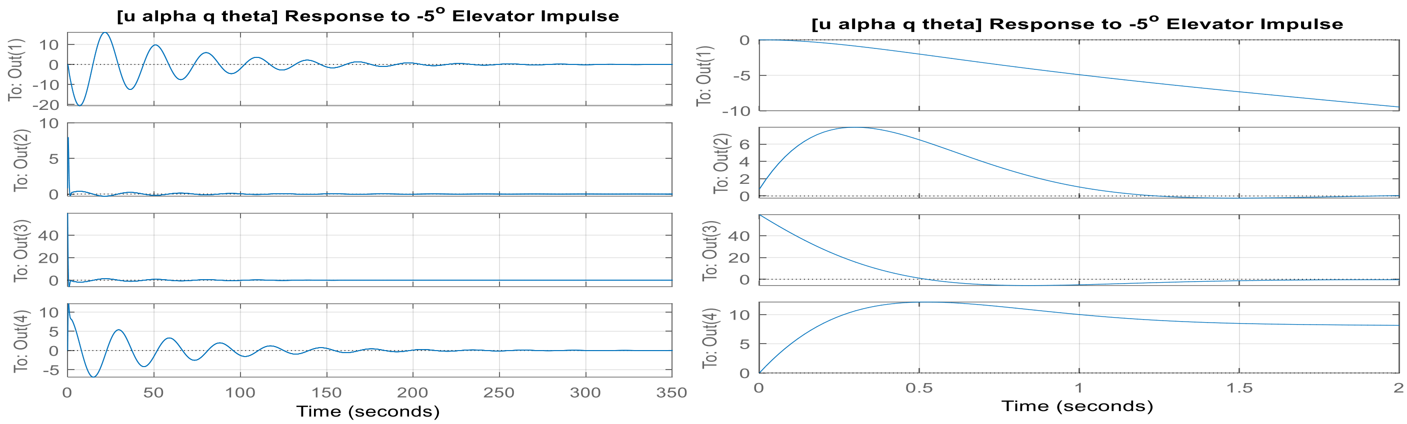

Figure 1(d) State response to $\mathbf{u}(t) = -5^o\,\delta(t)$: Complete response (LEFT), and initial response (RIGHT).

PROBLEM 2(35pts) The four transfer functions associated with PROBLEM 1, obtained using the 'ss2tf' command, are:

$$G_u(s)=\frac{u(s)}{\delta_e(s)}=\frac{-0.0177s^2+5.3082s+12.8541}{s^4+5.0130s^3+13.1614s^2+0.6722s+0.6}\ ;\ G_\alpha(s)=\frac{\alpha(s)}{\delta_e(s)}=\frac{-0.1599s^3-12.3534s^2-0.5556s-0.8119}{s^4+5.0130s^3+13.1614s^2+0.6722s+0.6}$$

$$G_q(s)=\frac{q(s)}{\delta_e(s)}=\frac{-11.8747s^3-23.3534s^2-1.1890s}{s^4+5.0130s^3+13.1614s^2+0.6722s+0.6}\ ;\ G_\theta(s)=\frac{\theta(s)}{\delta_e(s)}=\frac{-11.8747s^2-23.3534s-1.1890}{s^4+5.0130s^3+13.1614s^2+0.6722s+0.6}$$

(a)(5pts) Each transfer function has the denominator polynomial $p(s)=s^4+5.0130s^3+13.1614s^2+0.6722s+0.6$. Verify that this is the system characteristic polynomial by computing its roots and commenting.

<u>*Solution*</u>:

roots(ps) = -2.4894 +/- 2.5977i   -0.0171 +/- 0.2146i. These are the eigenvalues of **A** (or equally, $\mathbf{A}'$). The system characteristic polynomial is $p(s)=|s\mathbf{I}-\mathbf{A}|=0$ The eigenvalues of **A** are the roots of $p(s)$.

(b)(10pts) What is unique to flight dynamics is that the various transfer functions have a significant number of zeros in practically every transfer function. These zeros can, and usually do have a significant impact on the dynamical responses. To investigate the influence of the zeros we consider here $G_\alpha(s)$. In this part, use the command 'residue' to show that

$$G_\alpha(s)=G_{\alpha1}(s)+G_{\alpha2}(s),\ \text{where}\ \ G_{\alpha1}(s)\approx\frac{-0.16s-12}{s^2+5s+13}\ \ \text{and}\ \ G_{\alpha2}(s)\approx\frac{-0.003s-0.02}{s^2+0.03s+0.05}.$$

HINT: $$G(s)\overset{\Delta}{=}\frac{z(s)}{p(s)}\overset{\Delta}{=}\frac{z(s)}{(s-p)(s-\bar{p})}=\frac{r}{(s-p)}+\frac{\bar{r}}{(s-\bar{p})}=\frac{r(s-\bar{p})}{(s-p)(s-\bar{p})}+\frac{\bar{r}(s-p)}{(s-\bar{p})(s-p)}=\frac{(r+\bar{r})s-(r\bar{p}+\bar{r}p)}{(s-\bar{p})(s-p)}. \qquad \text{(b1)}$$

<u>*Solution*</u>: [See code @ 2(b).]

residue(zs,ps) = [-0.0786 +/ 2.2952i   -0.0014 +/- 0.0433i] and p = [-2.4894 +/- 2.5977i   -0.0171 +/- 0.2146i] allows us to apply (b1) to each set of residues and corresponding poles, gives:

$$G_{\alpha1}(s)=\frac{-0.1572s-12.32}{s^2+4.979s+12.95}\ \ \text{and}\ \ G_{\alpha2}(s)=\frac{-0.002745s-0.01863}{s^2+0.0341s+0.04635}.$$ These are close to those given above.

(c)(5pts) Overlay the impulse responses $g_{\alpha1}(t)$, $g_{\alpha2}(t)$, and $g_\alpha(t)=g_{\alpha1}(t)+g_{\alpha2}(t)$. [Note: Run your code twice; once using $t_{max}=150$, then again, using $t_{max}=5$.]

<u>*Solution*</u>: [See code @ 2(d).]

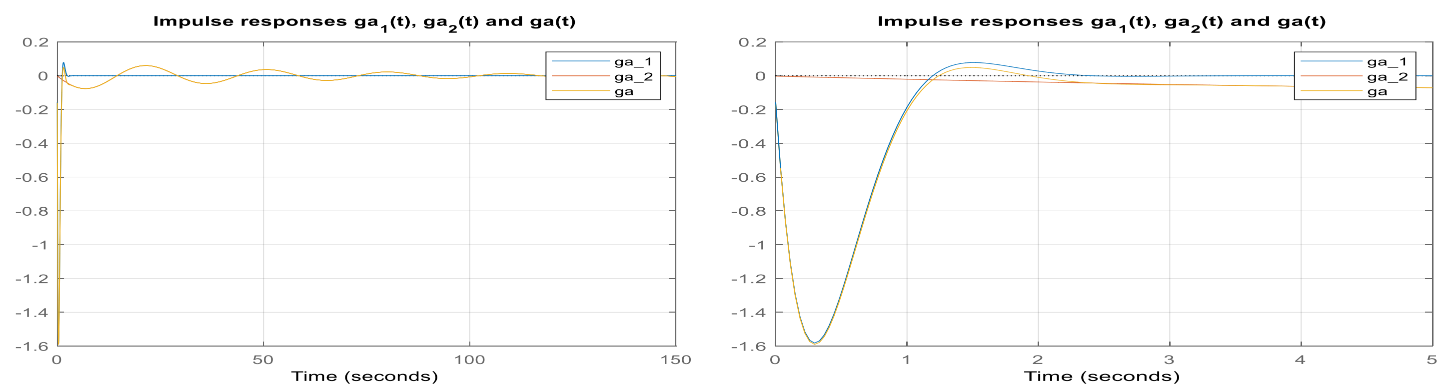

Figure 2(c) Impulse responses $g_{\alpha1}(t)$, $g_{\alpha2}(t)$ and $g_\alpha(t)$ for $t_{max}=150$ (LEFT) and $t_{max}=5$ (RIGHT).

(d)(5pts) From (b) you should have found that the short period mode *exact* transfer function is $G_{\alpha 1}(s)=\dfrac{-0.1572s-12.32}{s^2+4.979s+12.95}$. Arrive at the 2D *approximation* transfer function, call it $G_{\alpha_{2D}}(s)$, of this mode by using $\mathbf{A}'_{2D}\overset{\Delta}{=}\mathbf{A}'(2:3,2:3)$ and $\mathbf{B}'_{2D}\overset{\Delta}{=}\mathbf{B}'(2:3)$. Then overlay plots of the impulse responses $g_{\alpha 1}(t)$ and $g_{\alpha_{2D}}(t)$ for $t_{\max}=5$ and comment.

<u>*Solution*</u>: [See code @ 2(d).] $G_{\alpha_{2D}}(s)=\dfrac{-0.1599s-12.35}{s^2+4.968s+12.92}$

As expected, they are visually identical.

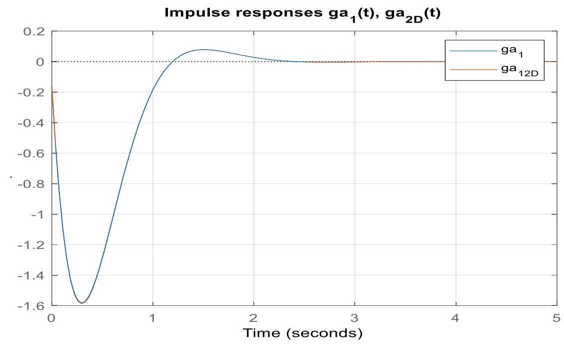

Figure 2(d) Impulse responses $g_{\alpha 1}(t)$ and $g_{\alpha_{2D}}(t)$.

(e)(5pts) In (d) you should have found that the $G_{\alpha_{2D}}(s)$ and $G_{\alpha 1}(s)$ are nearly identical; thereby suggesting that, at least in this case, we need not have gone through the residue approach in (b) to arrive at a nearly exact approximation of the short period mode transfer function. In this part we will investigate the influence of the zero in $G_{\alpha 1}(s)$ on the corresponding impulse response $g_{\alpha 1}(t)$. To this end, let $G_{\alpha 10}(s)=\dfrac{-12.32}{s^2+4.979s+12.95}$. Overlay impulse responses $g_{\alpha 1}(t)$ and $g_{\alpha 10}(t)$. Then comment.

<u>*Solution*</u>: [See code @ 2(e).] The zero has essentially no effect.

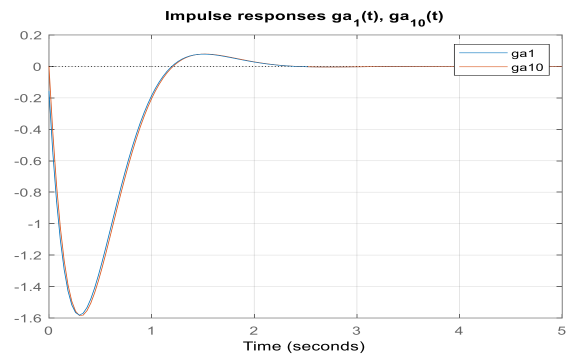

Figure 2(e) Impulse responses $g_{\alpha 1}(t)$ and $g_{\alpha 10}(t)$.

(f)(5pts) In (e) you should have found that the influence of the zero is visually negligible. In this part we quantify this influence. Let $G_{\alpha 11}(s)\overset{\Delta}{=}\dfrac{-0.1572s}{s^2+4.979s+12.95}$. Then $G_{\alpha_1}(s)=G_{\alpha 11}(s)+G_{\alpha 10}(s)$. Overlay the impulse response $g_{\alpha 11}(t)$ and $g_{\alpha 10}(t)$. Then comment.

<u>*Solution*</u>: [See code @ 2(f).]

The peak value of $g_{\alpha 11}(t)$ is ~0.04, and it occurs when $g_{\alpha 10}(t)\cong -1.2$. This sum of -1.16 is visually negligible relative to -1.20. The largest contribution of $g_{\alpha 10}(0)\cong -0.15$ while significant, decays in relation to that of $g_{\alpha 11}(t)$ after no more than ~0.05 sec.

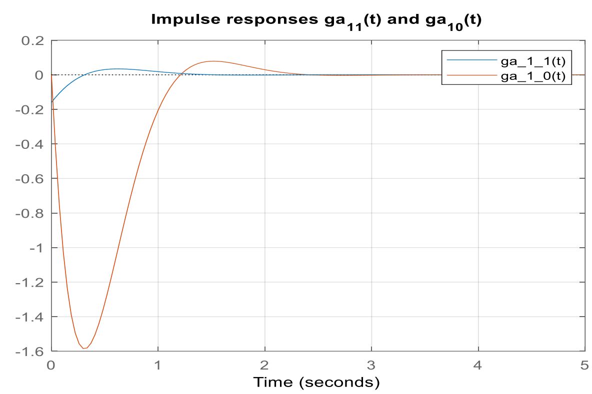

Figure 2(f) Impulse responses $g_{\alpha 11}(t)$ and $g_{\alpha 10}(t)$.

Summary and Conclusions

This problem addressed the short period mode for the angle of attack, $\alpha(t)$, in detail. (i) First, it was shown that it dies out well within 5 second; which is a small fraction of the decay time of ~200 seconds for the phugoid mode. (ii) We then used the concept of *residues* (also known as partial fraction expansions) to obtain the exact short period mode impulse response, and compared it to the total impulse response. It was found that the short period impulse response dominates the total impulse response for the first 5 seconds. (iii) We then showed that the exact short period mode and the 2D approximate modes were nearly identical; thereby obviating the need for the residue approach. (iv) Finally, we showed that the zero in the transfer function for the short period mode had a negligible influence on the impulse response. The knowledge of transfer functions was essential in carrying out such a detailed investigation. This is why I am a proponent of bringing that concept into play, in parallel with the state space concept.

PROBLEM 3(25pts) As noted in HW4 PROBLEM 3, consider the approximate *short period* transfer function:

$$\frac{\alpha(s)}{\delta_e(s)} \overset{\Delta}{=} G_{\alpha 2D}(s) = \frac{\left(Z_{\delta_e}/u_0\right)s + \left(M_{\delta_e} - M_q Z_{\delta_e}/u_0\right)}{s^2 - \left(M_q + M_{\dot\alpha} + Z_\alpha/u_0\right)s - \left(M_\alpha - M_q Z_\alpha/u_0\right)} \overset{\Delta}{=} \frac{z(s)}{p(s)}.$$

This system will be *dynamically stable* if and only if both of its poles lie in the LHP. It can be shown (e.g. using the Routh array) that both poles will lie in the LHP when the coefficients $c_1 = M_q + M_{\dot\alpha} + Z_\alpha/u_0$ and $c_0 = M_\alpha - M_q Z_\alpha/u_0$ are both less than zero.

(10pts) Use Tables 3.3 and 3.5 to explain that it is not possible for $c_1 = M_q + M_{\dot\alpha} + Z_\alpha/u_0$ to ever be greater than zero.

<u>Solution</u>:

(i) $c_1 = M_q + M_{\dot\alpha} + Z_\alpha/u_0$: Since these variables are positive constants times the variables $C_{m_q}; C_{m_{\dot\alpha}}; C_{Z_\alpha}$ it suffices to check the signs of the latter.

$C_{m_q} = -2\eta C_{L_{\alpha_t}} V_H l_t/\bar{c}$: All the parameters are greater than zero, and so this is always less than zero. So $M_q < 0$.

$C_{m_{\dot\alpha}} = -2\eta C_{L_{\alpha_t}} V_H (l_t/\bar{c})(\partial\varepsilon/\partial\alpha) = C_{m_q}(\partial\varepsilon/\partial\alpha)$. This is less than zero, since $\partial\varepsilon/\partial\alpha = 2C_{L_{\alpha_w}}/\pi AR_w > 0$. So $M_{\dot\alpha} < 0$.

$Z_\alpha = -(C_{L_\alpha} + C_{D_0})QS/m$. Since $C_{L_\alpha}, C_{D_0} > 0$, we have $Z_\alpha < 0$. Conclusion: $c_1 < 0$, always!

(b)(5pts) We now address $c_0 = M_\alpha - M_q Z_\alpha/u_0$. From (2.36) on p.56 we see that the stick-fixed neutral point (i.e. setting $M_\alpha = 0$) satisfies:

$$h_{NP} - h_{ac} = \eta V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) - \frac{C_{m_{\alpha_f}}}{C_{L_{\alpha_w}}}. \qquad \text{(c1)}$$

The condition (c1) is the condition for *static neutral stability*. The plane will be *dynamically neutrally stable* when $M_\alpha = M_q Z_\alpha/u_0$. Using Tables 3.3 and 3.5, it can be shown that this condition is equivalent to:

$$h'_{NP} - h_{ac} = \eta V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left[(C_{L_\alpha} + C_{D_0})\left(\frac{QSl_t}{mu_0^2}\right) + \left(1 - \frac{d\varepsilon}{d\alpha}\right)\right] - \frac{C_{m_{\alpha_f}}}{C_{L_{\alpha_w}}} \qquad \text{(c2)}$$

where h'_{NP} is what we call the stick-fixed neutral point for *dynamic stability*.

Compute the difference between (c1) and (c2), and then comment on whether or not a plane can be dynamically stable, yet statically unstable.

<u>Solution</u>: $h'_{NP} - h_{NP} = \eta V_H \frac{C_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left[(C_{L_\alpha} + C_{D_0})\left(\frac{QS}{mu_0^2}\right)\right]$. Since this difference is positive it would appear that the neutral point for dynamic stability is further aft than for static stability. Hence, the plane can by statically unstable, but dynamically stable in relation to perturbation dynamics.

Remark. I did not intend to cause the considerable consternation that many students had in relation to this part. ☹ though, upon it being pointed out by students that o n p.24 of Nelson the authors state that "for a vehicle to be dynamically stable it must be statically stable", this consternation is understandable. Moreover, it stems from the fact that static and dynamic stability are two distinctly different types of stability. As defined in the boo, an object has static stability if its cg returns to equilibrium when subjected to a perturbation. This say nothing about the dynamics associated with the object. Dynamic stability relates to the dynamics of the object, itself. This is especially true in the case of small perturbations of the object's dynamics. If a plane has $C_{m_\alpha} \overset{\Delta}{=} dC_m/d\alpha\,|_{\alpha=0}$, then it does not possess static equilibrium near $\alpha = 0$. A small static perturbation, say α_0 will result in $\alpha(t)$ moving away from $\alpha = 0$. As the plane is doing so, suppose that the pilot gives the elevator a small perturbation $\delta_e(t) = \varepsilon\delta(t)$. Then the plane short period dynamics will be felt. Suppose that $\alpha(t) \cong \alpha_1$ over a given 20 second interval. Then the short period perturbation dynamics are in relation to α_1 . If the plane is dynamically stable, then $\Delta\alpha(t) \to \alpha_1$. Hence, in relation to perturbation dynamics: YES a plan can be statically unstable and yet in relation to small perturbations it can be dynamically stable.

(c)(10pts) The fact it, it takes a fair bit of work to arrive at (c2). Hence, in this part, use the above tables to arrive at expressions for M_α , M_q and Z_α , from which (c2) could be arrived at through simple algebra.

<u>*Solution*</u>:

(i): $M_\alpha = C_{m_\alpha}\left(\frac{QS\bar{c}}{I_y}\right) = \left[C_{L_{\alpha_w}}(h - h_n) + C_{m_{\alpha_f}} - \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]\left(\frac{QS\bar{c}}{I_y}\right)$

(ii): $M_q = C_{m_q}\left(\frac{\bar{c}}{2u_0}\right)\left(\frac{QS\bar{c}}{I_y}\right) = \left[-2\eta V_H C_{L_{\alpha_t}}\left(\frac{l_t}{\bar{c}}\right)\right]\left(\frac{\bar{c}}{2u_0}\right)\left(\frac{QS\bar{c}}{I_y}\right)$

(iii): $Z_\alpha = -(C_{L_\alpha} + C_{D_0})QS / m$

Appendix Matlab code

```
%PROGRAM NAME: exam2.m  10/25/19
%PROLEM 1
%(a):
%NAVION A-matrix for Longitudinal dynamics (p.158)
A=[-.045 .036 0 -32.2;-.369 -2.02 176 0;.00199 -.0396 -2.948 0;0 0 1 0];
eigsA=eig(A);
speig=eigsA(1);
sptau=-1/real(speig);
spwd=imag(speig);
spwn=abs(speig);
spzeta=1/(sptau*spwn);
disp('sptau spwd spwn spzeta')
[sptau spwd spwn spzeta]
pheig=eigsA(3);
phtau=-1/real(pheig);
phwd=imag(pheig);
phwn=abs(pheig);
phzeta=1/(phtau*phwn);
disp('phtau phwd phwn phzeta')
[phtau phwd phwn phzeta]
%(b):
% B= [Xde Zde Mde 0]' where Zde=C_Zde*(QS/m) & Mde=C_mde*(QScbar/Iy)
C_Zde=-0.355; %From Table 3.3: C_Zde = -C_Lde & Table B.1
C_mde=-0.923; %From Table B.1
QS=6771; m=85.4; QScbar=38596; Iy=3000; u0=176; %p.157
Zde=C_Zde*(QS/m);
Mde=C_mde*(QScbar/Iy);
B=[0 Zde Mde 0]';
%(c):
U=diag([1 180/pi 180/pi 180/pi]); %Convert radians to degrees
Q=diag([1 1/u0 1 1]); %Convert w to alpha
UQ=U*Q;
AA=UQ*A*UQ^-1;
BB=(pi/180)*UQ*B; %The pi/180 converts de to degrees.
%(d):
de_d=-5; %elevator angle in degrees
BBde_d=de_d*BB; %This allows use of the unit impulse command.
C=eye(4); D=zeros(4,1);
sysc=ss(AA,BBde_d,C,D);
figure(10)
impulse(sysc)
title('[u alpha q theta] Response to -5^o Elevator Impulse')
grid
figure(11)
impulse(sysc,2)
title('[u alpha q theta] Response to -5^o Elevator Impulse')
grid
%=====================================================
%PROBLEM 2
[zs,ps]=ss2tf(AA,BB,C,D);
%(a):
roots(ps)
%(b):
Ga=tf(zs(2,2:5),ps);
GGa=tf(zs(2,5),ps);
[r,p,k]=residue(zs(2,2:5),ps);
s=tf('s');
P1s=s^2-(p(1)+p(2))*s+p(1)*p(2);
A11=r(1)+r(2); A10=-(r(1)*p(2)+r(2)*p(1));
Z1s=A11*s+A10;
Ga1=Z1s/P1s;
P2s=s^2-(p(3)+p(4))*s+p(3)*p(4);
A21=r(3)+r(4); A20=-(r(3)*p(4)+r(4)*p(3));
Z2s=A21*s+A20;
Ga2=Z2s/P2s;
%(c)
figure(20)
impulse(Ga1,Ga2,Ga,150)
title('Impulse responses ga_1(t), ga_2(t) and ga(t)')
legend('ga_1','ga_2','ga')
grid
figure(21)
```

```
impulse(Ga1,Ga2,Ga,5)
title('Impulse responses ga_1(t), ga_2(t) and ga(t)')
legend('ga_1','ga_2','ga')
grid
%(d): The 2-D Approximation for x=[alpha q]'
AA2=AA(2:3,2:3);
BB2=BB(2:3);
CC2=eye(2); DD2=zeros(2,1);
[ZZ2s PP2s]=ss2tf(AA2,BB2,CC2,DD2);
Ga2D=tf(ZZ2s(1,2:3),PP2s);
figure(22)
impulse(Ga1,Ga2D,5)
title('Impulse responses ga_1(t), ga_2_D(t)')
legend('ga_1','ga_1_2_D')
grid
%(e):
Ga10=tf(ZZ2s(1,3),PP2s);
figure(23)
impulse(Ga1,Ga10,5)
title('Impulse responses ga_1(t), ga_1_0(t)')
legend('ga1','ga10')
grid
%(f):
figure(24)
impulse(Ga11,Ga10,5)
title('Impulse responses ga_1_1(t) and ga_1_0(t)')
legend('ga_1_1(t)','ga_1_0(t)')
grid
```