AERE355 Fall 2019 Take-Home EXAM 1 Due 9/27(F) SOLUTION
[Note: You MUST use the homework format, or risk zero credit. Begin each PROBLEM on a new page. Unless otherwise stated, Matlab code should be placed in the APPENDIX. However, the work needed to arrive at the code should be included at the problem part.]

PROBLEM 1 (15pts) Recall that $Q \overset{\Delta}{=} 0.5\rho V^2$ is needed to scale lift, drag and moment information. Typically, the value for ρ at a given height *h* is taken to be in relation to standard sea-level conditions: $T_0 = 287^o K$, $P_0 = 1.01325e5\ N/m^2$, and hence $\rho_0 = 1.225\,kg/m^2$. The assumed temperature $T_0 = 287^o K$ is equivalent to $57.22^o F$. In this problem you will begin by investigating how ignoring the fact that the ground temperature is $T_0 = 310.93^o K$ (i.e. 100°F) can contribute to an error in relation to the *assumed* $\rho_{a12} = 0.31194\,kg/m^3$ at a height $h_{12} = 12\,km$. [See APPENDIX A on p.394 of Nelson.].

(a)(10pts) Use equations (1.57) and (1.60) of Nelson (as well as other equations needed in order to use these) to arrive at the percent error $e = \frac{\rho_{12} - \rho_{a12}}{\rho_{a12}} \times 100\%$. Show all basic computations of key variables HERE. [HINT: You should find that this error is sufficiently small that using the standard sea-level conditions, no matter what the actual conditions, is reasonable.]

<u>*Solution*</u>: [See code @ 1(a).]
Since $h_{12} = 12\,km$ is in the isothermal region, in order to use (1.60) we need to compute ρ_{11} at the 11 km boundary. From (1.57) we have $\rho_{11} = \rho_{gnd}(T_{11}/T_{gnd})^{-(1+g_0/(R\lambda))}$. To use this, we must first find: (i) $T_{11} = 310.93 - 11000(-.0065) = \mathbf{239.43^oK}$, and (ii) $\rho_{gnd} = P_0 / RT_{gnd} = 1.012(10^5)/(287 \times 310.93) = \mathbf{1.1341\,kg/m^3}$. For $R = 287\,m^2/^o\,K - s^2$ and $g_0 = 9.81\,m/s^2$, equation (1.57) gives: $\rho_{11} = \mathbf{0.3727\,kg/m^3}$. Equation (1.60) for the isothermal region is: $\rho_{12} = \rho_{11} e^{-g_0(h_{12}-h_{11})/(RT_{11})} = \mathbf{0.3144\,kg/m^2}$. Hence, the percent error is $e = \mathbf{0.78\%}$

(b)(5pts) Consider the B747 transport flying at $h \cong 40{,}000\ ft.$ and M=0.9. Information on this craft is given in Table B.27 and in Figure B.27. (i): Verify that $C_{L_0} \cong 0.5$ [See column 1 in Table B.27. The book denotes it as C_L] Show all work HERE (i.e. nothing in the APPENDIX).
<u>*Solution*</u>:
(i) From Table A.2, at 40,000 ft. & M=0.9 we have $\rho \cong 5.87(10^{-4})$ and $V = 0.9(968) = 871.2$.
Hence, $Q = 0.5\rho V^2 = 0.5(5.87 \times 10^{-4})(871.2^2) = \mathbf{222.77}$. From Figure B.27 we have $S = 5500$.
Hence, $QS = 1{,}225{,}235$. From Figure B.27 we also have $W = 636{,}600\,lb$. Hence, $C_{L_{trim}} = C_{L_0} = W/QS = \mathbf{0.52} \cong 0.5$.

PROBLEM 2 (40pts) Row 3 of Table B.27 (pp.416) is repeated here for convenience.

Table 1 (from TABLE B.27 on p.416 of the book) Longitudinal, M=0.90.

$C_L = .5$	$C_D = .042$	$C_{L_\alpha} = 5.5$	$C_{D_\alpha} = .47$	$C_{m_\alpha} = -1.6$	$C_{L_{\dot\alpha}} = .006$	$C_{m_{\dot\alpha}} = -9$
$C_{L_q} = 6.58$	$C_{m_q} = -25$	$C_{L_M} = .2$	$C_{D_M} = .025$	$C_{m_M} = -.1$	$C_{L_{\delta_e}} = .3$	$C_{m_{\delta_e}} = -1.2$

(a)(6pts) Use only Table 1 to find the value for $\alpha = \alpha_{trim}$ (in *degrees*) relative to the ZLL.

<u>Solution</u>: $C_{L_0} = C_{L_\alpha}\alpha_{trim} \quad \Rightarrow \quad \alpha_{trim} = \dfrac{C_{L_0}}{C_{L_\alpha}} = \dfrac{0.5}{5.5} = 0.091\,rad = \mathbf{5.48^o}$

(b)(6pts) Recall that $C_m = C_{m_0} + C_{m_\alpha}\alpha$. Use your result in (a) and Table 1 to find the value for C_{m_0}.

<u>Solution</u>: $0 = C_{m_0} + C_{m_\alpha}\alpha_{trim} \quad \Rightarrow \quad C_{m_0} = -C_{m_\alpha}\alpha_{trim} = -(-1.6)(.091) = \mathbf{0.15}$

(c)(6pts) Use (26) of the Ch.2 Notes to compute the value of the *static margin*, $h_n - h \overset{\Delta}{=} K_n$

<u>Solution</u>: Equation (26) is: $h_n = h - (C_{m_\alpha}/C_{L_\alpha})$. So: $K_n = -C_{m_\alpha}/C_{L_\alpha} = -(-1.6)/5.5 = \mathbf{0.29}$

(d)(6pts) Use your answer in (c) to arrive at the largest value for the *cg*, call it $x_{\max}$ for which the plane will be longitudinally stable.

<u>Solution</u>: From Figure B.27 we have "CG at 25% MAC". In other words, $h = 0.25$.

So, from (c): $h - h_n = 0.25 - h_n = -0.29 \quad \Rightarrow h_n = 0.504 = x_{\max}/\bar{c} \quad \Rightarrow x_{\max} = 0.504(27.31) = \mathbf{13.76ft}$ aft of the leading edge of the wing chord.

(e)(6pts) Use lift and moment arguments to determine if signs associated with $C_{L_{\delta_e}}$ and $C_{m_{\delta_e}}$ are correct.

<u>Determination</u>: A value of $\delta_e > 0$ corresponds to downward rotation of the elevator. This will result in positive tail lift, which is reflected in the <u>positive</u> nature of $C_{L_{\delta_e}}$. The resulting moment will be stabilizing (i.e. negative). Since this moment is $C_{m_{\delta_e}}\delta_e$ and $\delta_e > 0$, the sign of $C_{m_{\delta_e}}$ must be <u>negative</u>. Hence, both signs are correct.

(f)(10pts) As the plane approaches landing at sea level, its speed is M=0.25. If the desired angle of attack is 8°, find the necessary value for δ_e. Assume standard sea level conditions. [See † at the end of this exam.]

<u>Solution</u>:

$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e = 0$ gives $\delta_e = \dfrac{-(C_{m_0} + C_{m_\alpha}\alpha)}{C_{m_{\delta_e}}}$. From Table B.27 we have $C_{m_\alpha} = -1.26$ and $C_{m_{\delta_e}} = -1.34$. To find C_{m_0} we need to first find α_{trim} for $\delta_e = 0$: $\alpha_{trim} = \dfrac{C_{L_0}}{C_{L_\alpha}} = \dfrac{1.25}{5.7} = 0.2193\,rad = 12.565^o$.

Set $C_{m_0} = -C_{m_\alpha}\alpha_{trim} = -(-1.26)(0.2193) = \mathbf{0.2763}$. The desired $\alpha_{trim_d} = 8^o = 0.1396\,rad$.

Hence, we arrive at: $\delta_e = -\dfrac{0.2763 - 1.26(0.1396)}{-1.34} = 0.0749\,rad = \mathbf{4.29^o}$

PROBLEM 3 (45pts) For the NAVION plane, Table B.1 gives:

$C_{y_\beta} = -0.564 \quad C_{l_\beta} = -0.074 \quad C_{n_\beta} = \mathbf{0.071} \quad C_{l_{\delta_a}} = -0.134$

$C_{n_{\delta_a}} = -0.0035 \quad C_{y_{\delta_r}} = 0.157 \quad C_{l_{\delta_r}} = -0.107 \quad C_{n_{\delta_r}} = -0.072$

The entry $C_{n_\beta} = \mathbf{0.071}$ is in bold because in class we determined that the sign of the entry $C_{n_\beta} = -0.071$ is incorrect.

A determination of whether the sign of C_{n_β} is correct proceeds as:

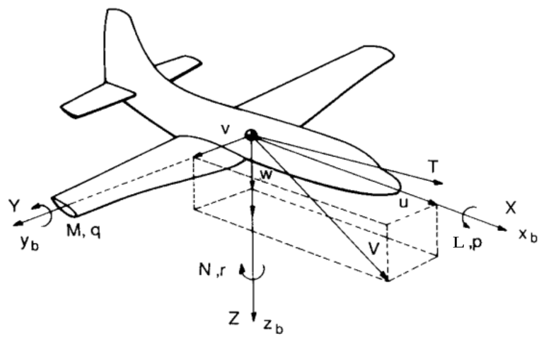

Figure 3 Reprint of book Fig.1.10.

Determination: For **V** as shown in the figure, $\beta > 0$. Application of a ccw moment $C_{n_\beta}\beta > 0$ will tend to bring β back to zero. [See the sign of the moment N in Figure 3.] Hence, we must have $C_{n_\beta} > 0$. Hence, the sign is incorrect.

(a)(5pts) Use a similar determination in relation to the sign of $C_{y_{\delta_r}}$. In doing so, <u>refer to the appropriate figure in the book, as well as to the above figure.</u>

Determination: Rotation of the rudder to the *right* gives $\delta_r < 0$. [See Fig.2.32 on p.77.] This will result in a negative side force $C_y = C_{y_{\delta_r}}\delta_r < 0$. Since $\delta_r < 0$, we must have $C_{y_{\delta_r}} > 0$. Since $C_{y_{\delta_r}} = 0.157$, the sign is correct.

(b)(5pts) Figure 2.33 on p.79 implies that for roll angle disturbance $\phi > 0$, the plane will begin to sideslip to the right (i.e. $\beta > 0$). Prior to the perturbation angle $\phi > 0$ the velocity vector V is as shown in Figure 3(b).hown at right.

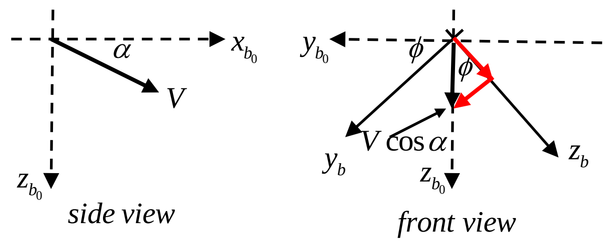

Figure 3(b). Views for $\phi = 0$.

Draw the rotated ($\phi > 0$) front view body axes. Then use it show that in this plane-body coordinate system we have $\mathbf{V} = [u \quad v \quad w] = V[\cos\alpha \quad \sin\alpha\sin\phi \quad \sin\alpha\cos\phi]$

Solution: For $\phi = 0$ we have $\mathbf{V}_0 = [u_0 \quad v_0 \quad w_0] = V[\cos\alpha \quad 0 \quad \sin\alpha]$. In the rotated body coordinate system the components of $w_0 = V\sin\alpha$ are shown in RED. Clearly, the z_b component of $V\sin\alpha$ is $w = (V\sin\alpha)\cos\phi$, and the y_b component of $V\sin\alpha$ is $w = (V\sin\alpha)\sin\phi$.

(c)(8pts) From (b) it follows that a roll angle perturbation $\phi > 0$ will be accompanied by a side slip angle $\beta = \sin^{-1}(v/V) = \sin^{-1}(\sin\alpha\sin\phi)$. For sufficiently small θ, we have the small angle approximation $\sin\theta \cong \theta$. Taking the arcsine of both sides of this equation gives $\theta \cong \sin^{-1}\theta$. Hence, for small α and ϕ, we have $\beta \cong \alpha\phi$. (i) For $\alpha = 10^o$ overlay plots of $\beta = \sin^{-1}(\sin\alpha\sin\phi)$ and $\hat{\beta} = \alpha\phi$ over the range $0^o \le \phi \le 30^o$. Then (ii) find the value $\phi_{\max}$ such that the error in this approximation is 5% by plotting the percent error vs. ϕ and using the data cursor.

Solution: [See code @ 3(c).]

From the data cursor we see that $\phi_{\max} \cong 30^o$

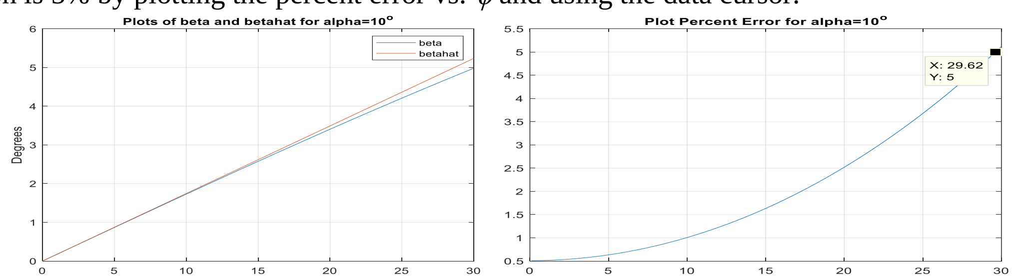

Figure 3(c). Plots of β and $\hat{\beta}$ (LEFT) and %Error (RIGHT).

(d)(5pts) In (c) you should have found that the approximation $\beta \cong \alpha\phi$ is accurate for quite a large range of values of ϕ; even for $\alpha = 10^o$. Since throughout this course we will be using small angle approximations, we can then assume that $\beta = \alpha\phi$. Determine if the sign of C_{l_β} is correct.

Determination: $C_l = C_{l_\beta}\beta = C_{l_\beta}(\alpha\phi)$. From Figures 1.10 and 2.33, we must have $C_l < 0$. Since $\beta = \alpha\phi > 0$, this requires that $C_{l_\beta} < 0$. The given value is $C_{l_\beta} = -0.074$. Hence, the sign is correct.

(e)(5pts) Differential application of the ailerons with right aileron **up** will tend to result in $\phi > 0$. If this is defined as $\delta_a > 0$, determine if the sign of $C_{l_{\delta_a}}$ is correct.

Solution: Right aileron up will impart a cw moment $C_l = C_{l_{\delta_a}}\delta_a > 0$. Since $\delta_a > 0$, we must have $C_{l_{\delta_a}} > 0$. Since we are given $C_{l_{\delta_a}} = -0.134$, the sign is incorrect. [See also all of the other tables.]

(f)(7pts) Suppose that for a certain crosswind situation a value for β is specified. (i) In the absence of aileron activation, show that the rudder angle must be $\delta_r = -(C_{n_\beta} / C_{n_{\delta_r}})\beta$. Then (ii): Recalling that the parameters involved in this expression are *derivatives*, convert this expression to one that is more intuitively obvious.

Solution:

(i) $C_n = 0 = C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r \quad \Rightarrow \delta_r = -(C_{n_\beta} / C_{n_{\delta_r}})\beta$.

(ii) $C_{n_\beta} / C_{n_{\delta_r}} = [dC_n / d\beta]/[dC_n / d\delta_r] = d\delta_r / d\beta$. Hence, we arrive at: $\delta_r = -(d\delta_r / d\beta)\beta$.

(g)(10pts) It should be clear that: $C_n = C_{n_\beta}\beta + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r$ and $C_l = C_{l_\beta}\beta + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r$.

(i) Reformulate these equations into a single matrix equation involving the vector $[\delta_r \quad \delta_a]^{tr}$ for the case of steady level flight. Then (ii) implement them in a Matlab code to find the values for δ_r and δ_a needed to maintain $\beta = 5^o$. Give your answers in degrees.

Solution: [See code @ 4(e).]

(i) For steady level flight, we have $C_n = C_l = 0$. In this case, the above equations become:

$$\begin{bmatrix} C_{n_{\delta_r}} & C_{n_{\delta_a}} \\ C_{l_{\delta_r}} & C_{l_{\delta_a}} \end{bmatrix}\begin{bmatrix} \delta_r \\ \delta_a \end{bmatrix} = -\begin{bmatrix} C_{n_\beta} \\ C_{l_\beta} \end{bmatrix}\beta \quad \Rightarrow \begin{bmatrix} \delta_r \\ \delta_a \end{bmatrix} = \begin{bmatrix} C_{n_{\delta_r}} & C_{n_{\delta_a}} \\ C_{l_{\delta_r}} & C_{l_{\delta_a}} \end{bmatrix}^{-1}\begin{bmatrix} C_{n_\beta} \\ C_{l_\beta} \end{bmatrix}(-\beta)$$

(ii) The code @ 4(g) gives: [dr da] = [**5.27^o -6.97^o**]

[†[] [See PROBLEM 2(f).] Correction of $C_L = 1.11$ for M=0.25 in Table B.27: The speed of sound is $a = 1116.45\ ft/s$. It follows that the plane speed is $V = Ma = 0.25(1116.45) = 279.11\ ft^2/s$. The air density is $\rho_0 = 2.3769(10^{-3})\ slug/ft^3$. Hence, $Q = 0.5\rho V^2 = 0.5(0.0023769)(279.11^2) = 92.583$ From Figure B.27, the plane weight is $W = 636,600\ lb$ and the wing area is $S = 5,500\ ft^2$. Hence, the lift coefficient is $C_L = W/(QS) = 636600/(92.583\times 5500) = 1.25$.

Appendix Matlab Code

```
%PROGRAM NAME: exam1.m  (9/19)
%PROBLEM 1:
%Standard Ground Conditions:
rho0=1.225; %kg/m^2
T0=287.16; %deg.K = 57.22 deg.F
%Ground Conditions:
Pg= 1.012e5; % N/m^2
Tg=310.93; % deg.K = 100 deg.F
R=287; %Ideal gas constant m^2/(deg.K - sec^2)
g0=9.81; %Gravity constant (N/s^2)
lambda=-0.0065; % deg.K/m
%(a):
h11=11000; % m
rhog=Pg/(R*Tg); %Air density @ ground
T11=Tg+lambda*h11;
%Nelson (1.57) on p.17:
rho11=rhog*(T11/Tg)^-(1+g0/(R*lambda)); %density @ 11km
%Nelson (1.60) on p.18:
h12=12192; %m
rho12=rho11*exp(-g0*(h12-h11)/(R*T11));
%Precent Error:
rhoa12=0.31194;
err=100*(rho12-rhoa12)/rhoa12;
%=================================
%PROBLEM 3
%(c):
a_degrees=10;
a=a_degrees*pi/180;
phi_degrees=0:.1: 30;
phi=phi_degrees*pi/180;
beta=asin(sin(a)*sin(phi));
betahat=a*phi;
beta_degrees=beta*180/pi;
betahat_degrees=betahat*180/pi;
figure(31)
plot(phi_degrees,[beta_degrees;betahat_degrees])
title('Plots of beta and betahat for alpha=10^o')
xlabel('phi (Degrees)')
ylabel('Degrees')
grid
legend('beta','betahat')
figure(32)
err=100*(betahat-beta)./beta;
plot(phi_degrees,err)
title('Plot Percent Error for alpha=10^o')
xlabel('phi (Degrees)')
grid
%(g):
Cnb=0.071; Clb=-0.074;
Cndr=-0.072; Cnda=-0.0035; Cldr=-0.107; Clda=-0.134;
b=5; %Specified beta (degrees)
A=[Cndr Cnda;Cldr Clda]; B=[Cnb; Clb]*(-b);
D=A^-1*B;
dr=D(1); da=D(2);
[dr da] %These are in units of degrees, since b was.
```