Lecture 19 **Simulation of Turbulence and Phugoid Response to It (3/11/20)**

Simulation of atmospheric turbulence is essential for conducting realistic flight dynamics simulations. Matlab's Aerospace Blockset/Environment/Wind includes tools for simulating specified types of turbulence. One example is the Von Karman frontal turbulence tool. It is summarized below:

The spatial bandwidth (BW) of atmospheric turbulence is inversely related to its spatial correlation length, L_{u_g}. The temporal turbulence BW that a plane flying at a speed u_0 experiences is: $\omega_{BW} = u_0 /(1.34 L_{u_g})\ rad/s$. The Von Karman frontal turbulence is described by the *power spectral density* (PSD): $\Phi_{u_g}(\omega) \cong \left(\frac{\sigma_{u_g}^2 2L_{u_g}}{\pi}\right)\left[\frac{1}{(\omega/\omega_{BW})^2+1}\right]$. Define the 'transfer function' $H_{u_g}(s) = \frac{\sqrt{\sigma_{u_g}^2 2L_{u_g}/\pi}}{1+s/\omega_{BW}}$. It should be clear that $H_{u_g}(s)$ is simply a low pass filter (LPF). Turbulence is simulated by running fictitious white noise through this filter. Because white noise has equal power at every frequency, the filter will remove the high frequency power, while letting the low frequency power pass. Specifically, the turbulence power spectral density (PSD) [i.e. power, as a function of frequency] will have exactly the same shape as $|H_{u_g}(i\omega)|$. For this reason, $H_{u_g}(s)$ is referred to as a ***shaping filter***.

***Example* 1.** Let $H_{u_g}(s) = \frac{1}{s+1}$. The -3dB BW of this first order shaping filter is 1 rad/sec and its static gain is one. It is designed to simulate band-limited turbulence, as shown in Figure 1.

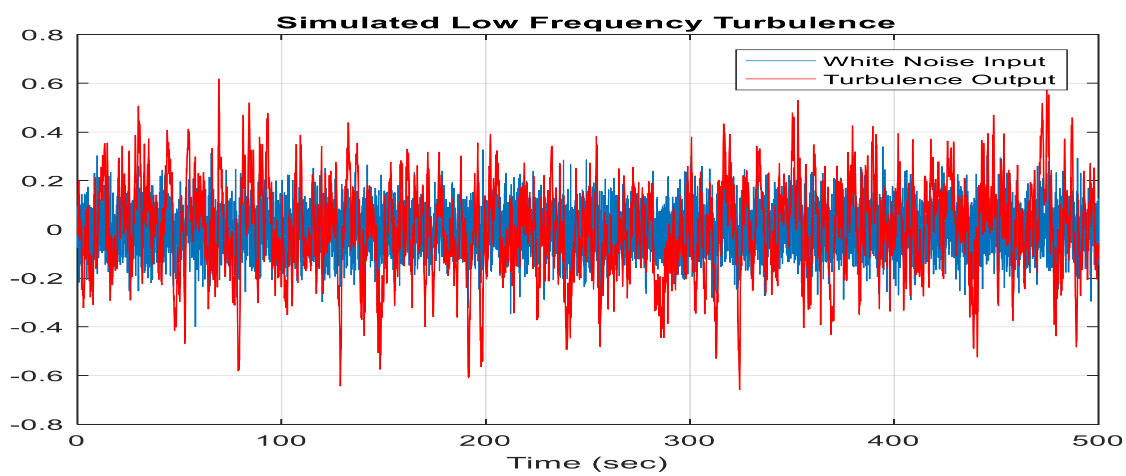
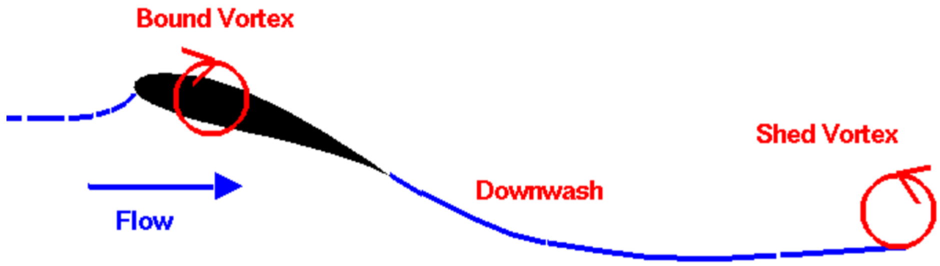


Figure 1. Simulation of low frequency turbulence.

We see that the turbulence is not nearly as erratic as the fictitious white noise that was used to simulate it. This is because that noise has been run through a low pass filter (LPF) having a cutoff frequency of $\omega_{co} = 1 rad/s$, or $f_{co} = 1/2\pi \cong 0.16 Hz$. The filter time constant is $\tau = 1/\omega_{co} = 1\sec$. Hence, whereas the white noise is unpredictable from one sample to the next, the turbulence will be relatively predictable over 1-2 seconds. Since $T = 0.078\sec$. this translates into 12-25 samples. □

What is Vortex Shedding in relation to a Wing?

[https://www.grc.nasa.gov/www/k-12/airplane/shed.html]:



Net vorticity in the flow domain is zero.

Lift is the force which holds an aircraft in the air. From a Newtonian perspective, lift is generated by turning a flow of air. The flow turning creates a downwash from the wing which can be observed in flight.

The flow turning that occurs in the creation of lift also creates **bound vorticity** within the airfoil. For a general shaped airfoil, there is some distribution of vorticity which we can think of as small vortices. For the simple Joukowski airfoil, shown in this figure, there is a single vortex present at the center of the generating cylinder. The flowfield from this generating cylinder has been conformally mapped into the airfoil, but the vorticity has been maintained.

The existence of the bound vortex (or vortices) within the airfoil created an important theoretical problem when it was first proposed. A static fluid has no vorticity within it; vorticity is zero in a static fluid. From the fluid conservation laws, if a fluid initially has no vorticity within it, then the net vorticity must remain zero within the domain. With a bound vortex within the object, there would then have to be another vortex of **opposite strength** present within the flow domain. Then the sum of the two vortices, one spinning clockwise, the other counter clockwise, would be zero as required by the conservation laws. Where is the other vortex?

It took some very careful experimental work by Ludwig Prandtl to actually "catch" the other vortex on film. In his experiment he placed an airfoil in a tunnel with no flow. He turned the tunnel on and as the flow began he photographed the flowfield at the trailing edge of the airfoil. What he saw was a vortex shed from the trailing edge, spinning opposite to the predicted bound vortex in the airfoil. The shed vortex was convected downstream and eventually mixed out due to viscous effects in the air stream. Without viscosity, the shed vortex would remain with constant strength and would be carried downstream away from the airfoil. This is depicted by the vortex at the right of the figure.

***Example* 2.** In this example, we will simulate vortex shedding turbulence. Vortex shedding is clearly related to the wing speed and geometry. When the wing velocity, u_0 'coincides' with its characteristic dimension, vortices having a distinct period of shedding will be generated. This regime of shedding is often referred to as the *Strouhal Number*. [https://en.wikipedia.org/wiki/Strouhal_number]. At such a speed, the *power spectral density* of the turbulence will have a sharp peak. In this example, we will model this type of *psd*.

To this end, let $H_{u_g}(s) = \frac{1}{s^2 + 0.2s + 1}$. A resulting simulation is shown in Figure 2.

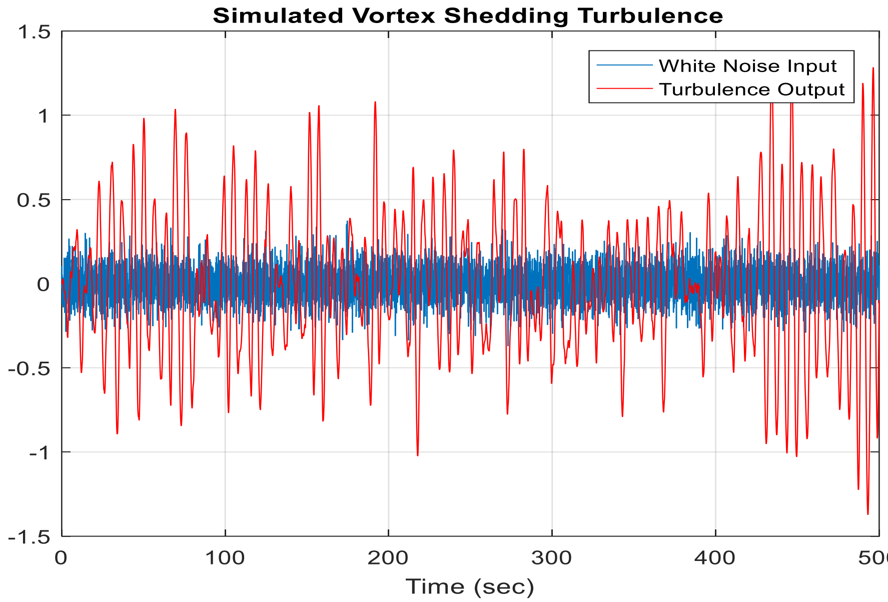


Figure 2(a). Simulation of vortex shedding turbulence.

Notice that the structure of this turbulence is much different than that shown in Figure 1. This is because the shaping filter is that of a low-damped second order system. The Bode plot at right shows that the white noise will be highly amplified at frequencies near the shedding frequency. The oscillations in Figure 2(a) correspond to an average frequency of 1 rad/sec. □

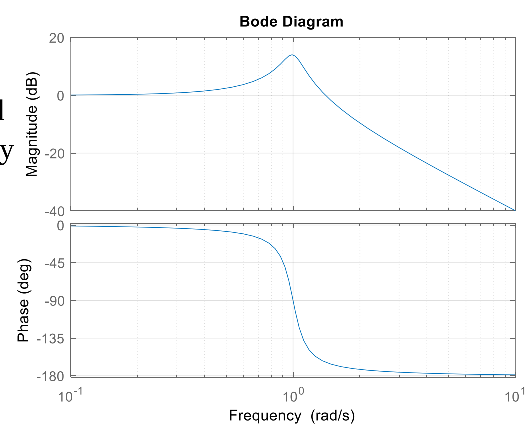


Figure 2(b). Vortex shaping filter Bode plot.

***Example* 3.** Typically, textbooks model a wind gust as a simple step of a given duration. The figure below shows a different type of gust; one that is modeled as a brief amplification of vortex shedding turbulence.

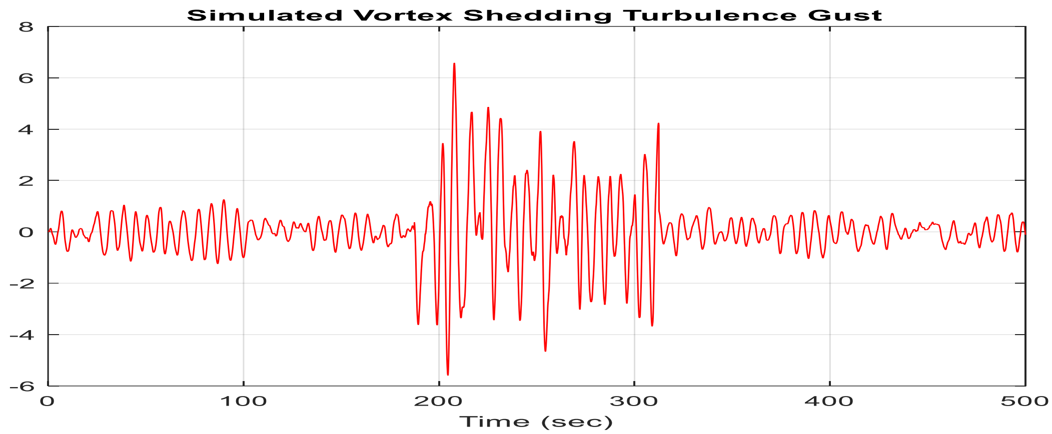
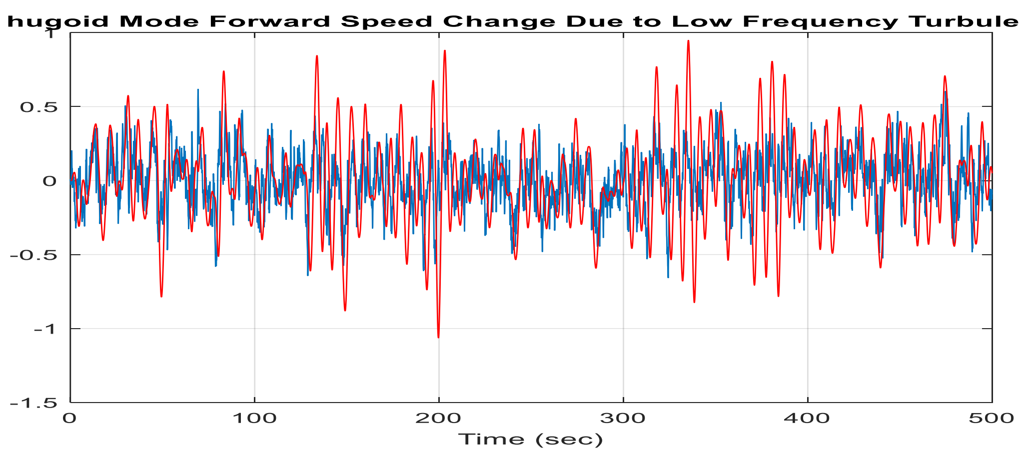


Figure 3. Simulate vortex shedding wind gust.

Recall that in simple terms, a stationary random process has the property that its temporal behavior is generally the same everywhere. This is an example of a nonstationary random process. The pulse was simulated by simply amplifying the magnitude of the by a factor of five in the region shown. Hence, while the spectral shape is unchanged, the power of the process (which is its statistical variance) is time-varying.

We could go to the next level of realism by generating a sequence of gusts that are randomly separated in time and have varying degrees of power. However, for now we will forego this fun. ☹ □

***Example* 4** Suppose that for a given forward speed, the phugoid forward speed mode transfer function is $G_p(s) = \frac{1}{s + 0.2s + 1}$. The response to low frequency and vortex shedding turbulence is shown below.



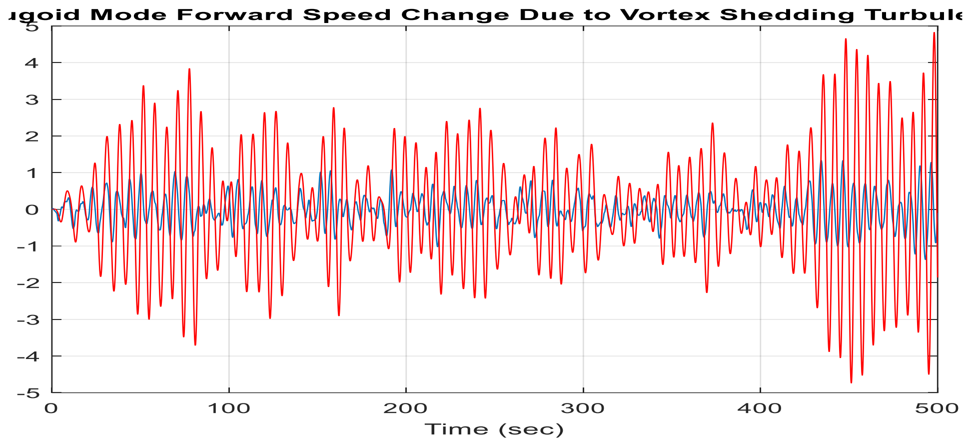


Figure 4. Forward speed response to low frequency (TOP) and vortex shedding frequency (BOTTOM) turbulence. The BLUE processes are the turbulence, and the RED ones are the resulting dynamics of the forward speed.

Notice the increase in the forward speed variance when the plane's forward speed is such that vortex shedding occurs, and when that shedding coincides with the phugoid resonant frequency. □

Summary

We have demonstrated how to simulate turbulence having a given spectral shape, by running fictitious white noise through a shaping filter whose magnitude has the same shape as the desired shape of the turbulence PSD. Shaping filters that simulated low frequency turbulence and vortex shedding turbulence were then demonstrated. Finally, we showed how such differing types of turbulence connect to the phugoid dynamics of a plane.

The concepts of a transfer function, a Bode plot, and filtering are concepts that relate centrally to this course. The only new concept is that of a random process. One can create all types of random processes by running fictitious white noise through shaping filters. Since filters have played a big role in the course material, the student should hopefully see how they can be applied to areas not covered in the course. Such areas are not limited to aircraft dynamics. I have applied them to areas ranging from early detection of an impending epileptic seizure, to characterizing blow-down pressure in high speed turbomachinery, to energy spot pricing in the financial market, to even quantum random walk dynamics. Suffice to say, the value of the course material can extend far beyond aerodynamic applications.

Matlab Code

```
%PROGRAM NAME: lec27.m
%Turbulence Examples
%*****************************************
%Example 1: Low Frequency Turbulence
wBW=1;
Hlp=tf(1,[1/wBW ,1]); %Shaping Filter
%---------------------------------
%Generate fictitious white noise
wN=40*wBW; %Nyquist Frequency
T=pi/wN; %Sampling Period
tmax=500;
t=0:T:tmax; t=t'; nt=length(t);
q=normrnd(0,1,nt,1);
xlp=lsim(Hlp,q,t);
figure(1)
plot(t,0.1*q) %Scale for plotting purposes
hold on
plot(t,xlp,'r')
title('Simulated Low Frequency Turbulence')
legend('White Noise Input','Turbulence Output')
xlabel('Time (sec)')
grid
%********************************************
%Example 2: Vortex Shedding Turbulence
wn=1; zeta=0.1;
Hvs=tf(wn^2,[1 , 2*zeta*wn , wn^2]);
wBW=10*wn;
T=pi/wN; %Sampling Period
t=0:T:tmax; t=t'; nt=length(t);
q=normrnd(0,1,nt,1);
xvs=lsim(Hvs,q,t);
figure(2)
plot(t,0.1*q) %Scale for plotting purposes
hold on
plot(t,xvs,'r')
title('Simulated Vortex Shedding Turbulence')
legend('White Noise Input','Turbulence Output')
xlabel('Time (sec)')
grid
%==============================================
%Simulation of a gust
ng=nt/4; %Gust Duration
Wg=ones(nt,1);
n1=fix(nt/2 - ng/2); n2=fix(nt/2 + ng/2);
Wg(n1:n2)=5*Wg(n1:n2);
xg=Wg.*x;
figure(3)
plot(t,xg,'r')
title('Simulated Vortex Shedding Turbulence Gust')
xlabel('Time (sec)')
grid
%***************************************************
%Phugoid Mode Excitation
Gp=Hvs; %Choose mode TF to be equal to VS shaping filter
%------------------
%Response to Low Frequency Turbulence:
ylp=lsim(Gp,xlp,t);
figure(4)
plot(t,xlp)
hold on
plot(t,ylp,'r')
title('Phugoid Mode Forward Speed Change Due to Low Frequency Turbulence')
xlabel('Time (sec)')
grid
%------------------
%Response to Vortex Shedding Turbulence:
yvs=lsim(Gp,xvs,t);
figure(5)
plot(t,xvs)
hold on
plot(t,yvs,'r')
title('Phugoid Mode Forward Speed Change Due to Vortex Shedding Turbulence')
```

```
xlabel('Time (sec)')
grid

figure(5)
figure(4)
figure(3)
figure(2)
figure(1)
```