Lecture 16 Lateral Dynamics of the NAVION Airplane

Consider the state space model (5.35) on p.195 of Nelson for the lateral dynamics $[\beta \quad p \quad r \quad \phi]^{tr}$ [sideslip, roll rate, yaw rate, roll angle] of a plane in response to input $[\delta_a(t) \quad \delta_r(t)]^{tr}$. This represents a 2-input/4-output system if we set $\mathbf{C} = \mathbf{I}_{4\times4}$ and $\mathbf{D} = \mathbf{0}_{4\times2}$. The values for **A** and **B** are given in the code in the Appendix of these notes.

(a) Arrive at plots of the four responses to each of an impulse rudder input and an impulse aileron input.
Solution: [See code @ 3(a).]

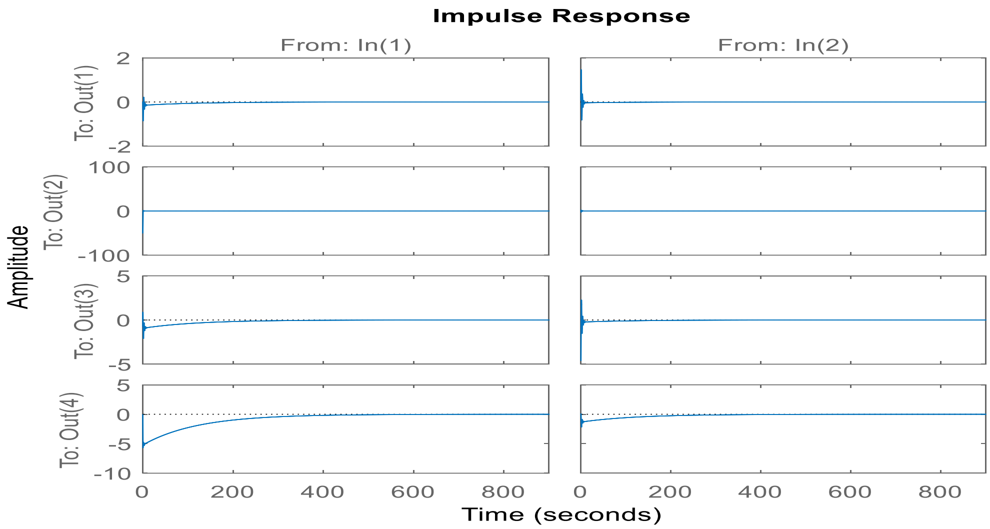


Figure 3(a) State impulse responses.

(b) Repeat (a) but for Tfinal=10.
Solution: [See code @ 3(b).]

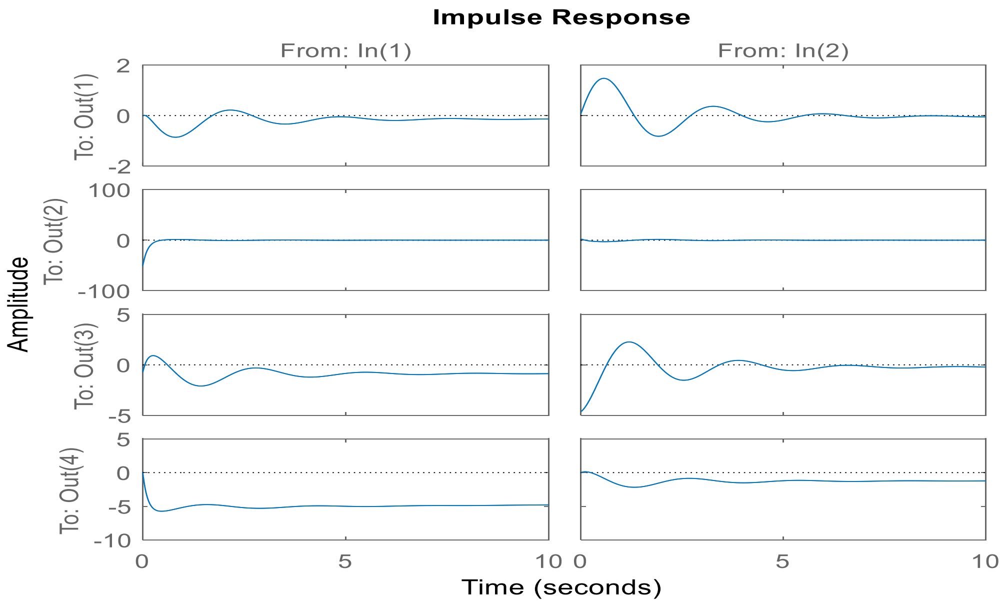


Figure 3(a) State impulse responses for Tfinal = 10 sec.

(c) Compute the eigenvalues of A.

Solution:

Aeigs = eigs(A) = [-8.449 -0.488 +/- 2.339i -0.008]

(d) Compute the transfer functions re: the aileron input.

Solution:

[Na,Da]=ss2tf(A,B,C,D,1)

```
Na =  0        0    0.7505  -20.5240   -7.3558          Da = 1.0000    9.4339   14.0396   48.3644    0.3985
      0  -50.7425  -53.2431 -251.6703         0
      0   -0.7590   11.2019    2.9004  -44.5274
      0         0  -50.7425  -53.2431 -251.6703
```

(d) Compute the transfer function re: the rudder input.

Solution:

[Nr,Dr]=ss2tf(A,B,C,D,2)

```
Nr =  0    0.1173    5.6609   40.8030   -1.4957          Dr = 1.0000    9.4339   14.0396   48.3644    0.3985
      0    2.6022   -9.4205  -64.1362         0
      0   -4.6353  -40.5725   -4.9888  -11.3977
      0         0    2.6022   -9.4205  -64.1362
```

(e) Explain the relation between the second and fourth transfer function numerators.

Explanation: Well?

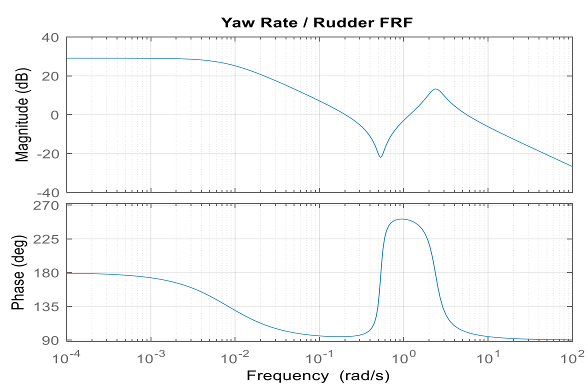
(f) Verify that the eigenvalues of A are the roots of the characteristic polynomial.

Verification:

proots=roots(Dr)

proots = [-8.4490 -0.4883 +/- 2.3393i -0.0083] ; Aeigs = eigs(A) = [-8.449 -0.488 +/- 2.339i -0.008]

(g) Give the FRF of the yaw rate/rudder transfer function, and plot it.



Gyr=tf(Nr(3,:),Dr) Gyr =

```
  -4.635 s^3 - 40.57 s^2 - 4.989 s - 11.4
 ---------------------------------------------
 s^4 + 9.434 s^3 + 14.04 s^2 + 48.36 s + 0.3985
```

(h) Investigate the behavior of the FRF in (g).

Investigation: Let the games begin! ☺

Matlab Code

```
%PROGRAM NAME: lec16.m
%4-D Lateral Dynamics [b,
%NAVION Values:
W=2750; Iz=3530; S=184; b=33.4; cbar=5.7;
g=32.174; m=W/g;
M=.158; a=1116.4; u0=M*a;
rho=.002377; Q=0.5*rho*u0^2;
Cnb=.071; Cnr=-.125; Cndr=-.072;
c1=Q*S*b/Iz;
Cyb=-.564;
Cydr=.157;
Cyr=0.26;
c2=Q*S/m;
Yb=c2*Cyb; Yr=(c2*.5*b/u0)*Cyr;
Ydr=c2*Cydr;
Ix=1048;
Cyp=0;
Clb=-.074; Clp=-.41; Cnp=-.0575; Clr=.107;
Lb=Clb*(Q*S*b/Ix);
Yp=(c2*0.5*b/u0)*Cyp;
Np=(c1*0.5*b/u0)*Cnp;
Lp=Clp*(Q*S*b^2)/(2*Ix*u0);
Lr=Clr*(Q*S*b^2)/(2*Ix*u0);
%----------------------------
% A MATRIX:
A1=[Yb/u0,Yp/u0,-(1-Yr/u0),g/u0];
A2=[Lb,Lp,Lr,0];
A3=[Nb,Np,Nr,0];
A4=[0,1,0,0];
A=[A1;A2;A3;A4];
%----------------------------
%B MATRIX:
Cydr=.26; Cndr=-.072; Cldr=.012;
Ydr=Cydr*c2; Ndr=Cndr*c1; Ldr=Cldr*(Q*S*b/Ix);
Clda=-.234; Cnda=-.0035;
Lda=(Q*b*S/Ix)*Clda; Nda=(Q*b*S/Ix)*Cnda;
B=[0,Ydr/u0; Lda,Ldr; Nda,Ndr;0,0];
%---------------------------
%C & D  MATRICES:
C=eye(4); D=zeros(4,2);
%-----------------------------
figure(1)
impulse(A,B,C,D)
%---------------------------------
%(b):
tfinal=10;
figure(2)
sys=ss(A,B,C,D);
impulse(sys,tfinal)
%-------------------------------
%(c):
Aeigs=eigs(A)
%(d):
[Na,Da]=ss2tf(A,B,C,D,1)
%-------------------------------
%(e):
[Nr,Dr]=ss2tf(A,B,C,D,2)
%------------------------------
%(f):
proots=roots(Dr)
%-----------------------------
%(g):
Gyr=tf(Nr(3,:),Dr)
figure(3)
bode(Gyr)
title('Yaw Rate / Rudder FRF')
grid
```