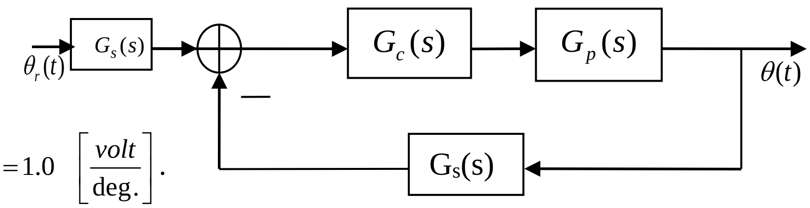
LECTURE 14 **Example of Lead/Lag Design** **(2/23/20)**

Example. The block diagram of an antenna angular position command control system is shown below.

motor: $G_m(s) = \frac{120}{s(s+1)}\left[\frac{\text{deg}}{volt}\right]$

antenna: $G_{ant}(s) = \frac{1000}{s^2+50s+1000}$ $\left[\frac{\text{deg.}}{\text{deg.}}\right]$

controller: $G_c(s)$ $\left[\frac{volts}{\text{deg}}\right]$. position sensor: $G_s(s)=1.0$ $\left[\frac{volt}{\text{deg.}}\right]$.



CLOSED LOOP SPECIFICATIONS:

(S1): unit ramp steady state error no greater than 0.025°.

(S2): closed $PM \cong 60^o$ at a gain crossover frequency $\omega_{gc} = 5r/s$.

(a)<u>Assuming</u> that the closed loop system is stable, then it is a *Type*-1 system. Because it is a *unity feedback* system, the error constant is: $c = \lim_{s\to 0} sG_c(s)G_m(s)G_{ant}(s) = \lim_{s\to 0} sG_c(s)\left[\frac{120}{s(s+1)}\right]\left[\frac{1000}{s^2+50s+1000}\right] = 120\,G_c(0) \overset{\Delta}{=} 120K$ ·

Hence, for a unit ramp, $e_{ss} = \frac{1}{120K} \le 0.025^o$, this requires $K \ge \frac{1}{120(.025)} = \frac{1}{3}$. So we must have $G_c(s=0) = K = 1/3$

It follows that our controller must have the form $G_c(s) = \frac{1}{3}G_c^*(s)$ where $G_c^*(s=0)=1$.

(b) Design a **unity static gain lead controller** to obtain open loop phase of 120° at $\omega = 5r/s$

The Bode plot for $G(s) = (1/3)G_m(s)G_{ant}(s)$ is at right.

At $\omega = 5r/s$, we currently have ~3.9dB and -183°.

We will center the lead controller here, with $\varphi_{\max} = 63^o$:

$$r_\omega \overset{\Delta}{=} \frac{\omega_2}{\omega_1} = \frac{1+\sin 63^o}{1-\sin 63^o} = \frac{1+.89}{1-.89} = 17.18 \quad \&$$

$$\frac{r_\omega\, dB}{2} = 10\log(17.18) = 12.35\, dB\,.\quad \omega_1 = \omega_{\max}/\sqrt{r_\omega} = 5/\sqrt{17.18} = 1.206$$

and $\omega_2 = r_\omega \omega_1 = 17.18(1.206) = 20.719\ rad/s$.

The augmenting lead controller is: $G_{lead}(s) = \frac{(s/1.206)+1}{(s/20.719)+1}$.

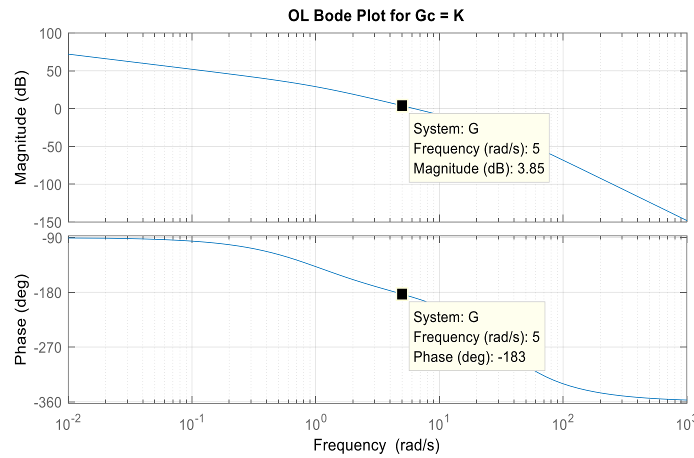


Figure 1. Bode plot for $G(s) = (1/3)G_m(s)G_{ant}(s)$.

(c) With the inclusion of the lead controller, at $\omega = 5r/s$ the OL phase is exactly -120°, and the magnitude is now 3.9+12.35 = 16.25 dB (= 6.494). Hence, we need a unity static gain lag controller must have high frequency attenuation $r_\omega \overset{\Delta}{=} \frac{\omega_2}{\omega_1} = \frac{1}{6.494}$. We must choose the upper frequency ω_1 to be sufficiently less than $\omega = 5r/s$, so that the lag controller phase is essentially back to zero at this frequency. I will set $20\omega_1 = 5$. So $\omega_1 = .25$. Then $\omega_2 = r_\omega \omega_1 = \left(\frac{1}{6.494}\right)(.25) = .0385$.

Hence, $G_{lag}(s) = \frac{(s/.25)+1}{(s/.0385)+1}$.

(d) The final controller is:

$$G_c(s) = \left(\frac{1}{3}\right) G_{lead}(s) G_{lag}(s) = \left(\frac{1}{3}\right)\left(\frac{(s/1.206)+1}{(s/20.719)+1}\right)\left(\frac{(s/0.25)+1}{(s/0.0385)+1}\right) \quad (1)$$

We see have CL $PM \cong 57.5^o$ at $\omega_{gc} \cong 5\, r/s$. The PM is slightly low due to the fact that the lag compensator phase at $\omega_{gc} \cong 5\, r/s$ is not exactly back to zero. In fact, it is -2.42°.

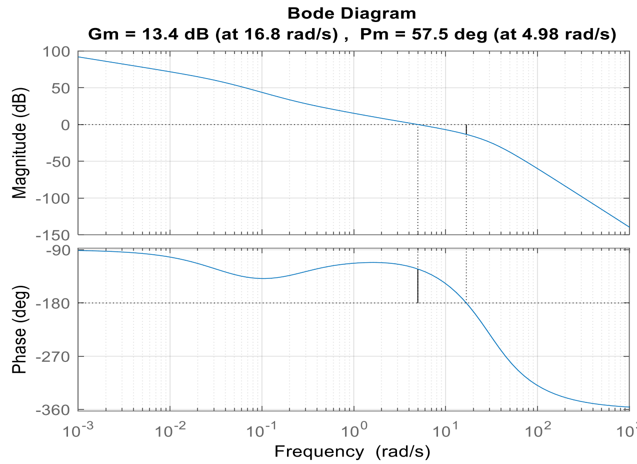
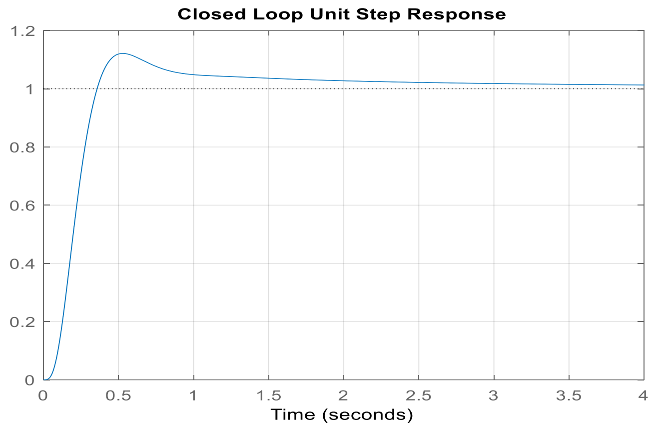
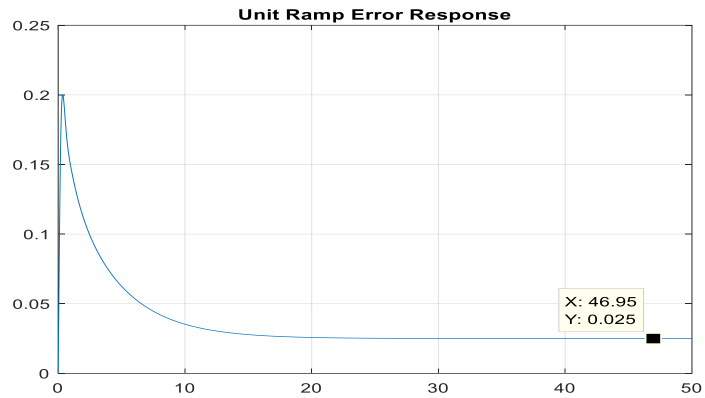


Figure 2. Final OL Bode plot that includes (1).

(e) The plots below validate the design. The left plot is the closed loop response to *a unit step* command. The right plot is the error response to a *unit* ramp command.





Matlab Code

```
% PROGRAM NAME: LEC14_leadlag.m
Gm = tf(120,[1 1 0]);
Gant=tf(1000,[1 50 1000]);
% PART (a):
K=1/3; %Satisfies (S1).
G=K*Gm*Gant;
figure(1)
bode(G)
grid
title('OL Bode Plot for Gc = K')
%(b):
Glead=tf([1/1.206 1],[1/20.719 1]);
%(c):
Glag=tf([1/.25 1],[1/.0385 1]);
%(d):
Gc=K*Glead*Glag;
G=Gc*Gm*Gant;
figure(2)
margin(G)
%(e):
W=feedback(G,1);
figure(3)
step(W)
title('Closed Loop Unit Step Response')
grid
E=1-W; %error transfer function
t=0:.01:50;
u=t;
e=lsim(E,u,t);
figure(4)
plot(t,e)
title('Unit Ramp Error Response')
grid
```