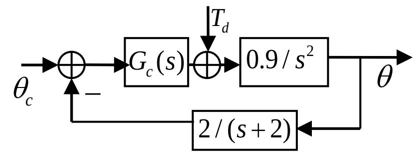
LECTURE 13 Examples of Bode-Based Control System Design

Example 1. [Example 6.20 on p.405]



In this example the authors address a PID controller for control the angular position of a spacecraft. They first appropriately address the torque disturbance ss error requirement. They correctly note that for zero ss error for a torque step disturbance, the controller must include a pole at the origin. However, they do not incorporate this needed pole in their OL Bode plot in Fig.6.68. I find this strange, to say the least. They then proceed with a long discussion of how they will arrive at the two controller zeros. Their final controller is:

$$G_c(s) = \frac{0.05(10s+1)(s+.005)}{s} \text{ . (p.408)}$$

They then give the command unit step response and the disturbance step response for magnitude $T_d = 0.0175$ in Fig.6.69. These responses are given below.

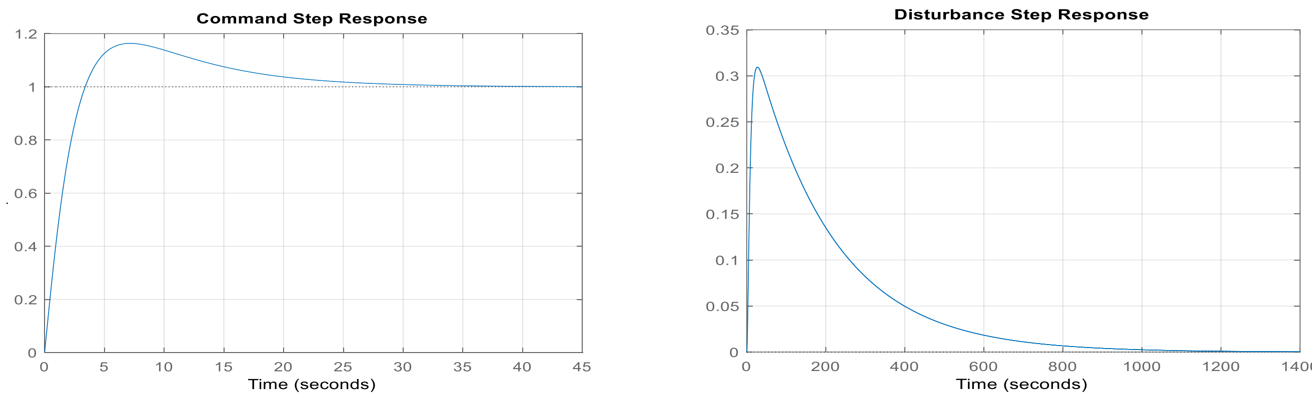


Figure 1. Command (LEFT) and disturbance step responses (per Fig.6.69).

While the LEFT response compares well to the authors', the right is smaller in magnitude by a factor of 6. In other words, Figure 6.69(b) on p.409 is incorrect. The maximum amplitude should be ~0.3.

It should be noted that while the disturbance system is a Type-1 system, the response to a step disturbance takes ~1000sec. to return to zero. To me, this is not impressive. The final system PM is shown at right. We see that, indeed, the system has the required 65° PM.

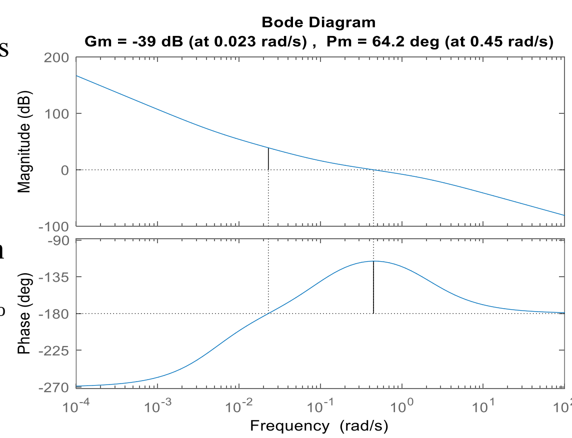


Figure 2. Final design PM.

Remark. The authors never explicitly show that for the disturbance system to be Type-1 the controller must have a pole at the origin. We will now show this. To begin, let the controller have the general form $G_c(s) = n(s)/d(s)$. Then the CL disturbance transfer function is:

$$W(s) = \frac{0.9/s^2}{1+(0.9/s^2)[2/(s+2)][n(s)/d(s)]} = \frac{0.9(s+2)d(s)}{s^2(s+2)d(s)+1.8n(s)}. \tag{1}$$

The error transfer function is $-W(s)$. For this to be Type-1 requires that $-W(0)=0$. This is achievable only of $d(0)=0$; that is, the controller denominator must include a factor s. Hence, the controller must have a pole at the origin.

We will now carry out our own design. To this end, we will first INCLUDE the controller pole at the origin in evaluating the PM of the resulting CL system. The OL TF is:

$$G(s)=\frac{2(0.9)}{s(s^2)(s+2)}. \tag{2}$$

The PM, as shown at right, is -115°. Ugh!!! It is those three poles at the origin!!! Even so, since we will not be using the frequency $\omega_{gc}=1$, it doesn't matter so much. The point is: At present the OL phase is very negative at all frequencies.

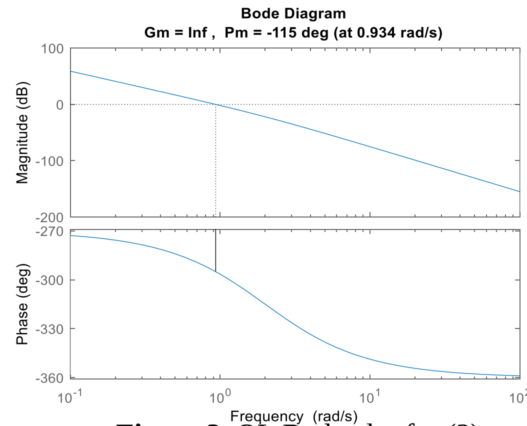


Figure 3. OL Bode plot for (2).

For this reason, we will begin our design by with a controller zero at $\omega_1=0.005$ to effectively cancel one of the three poles at the origin. Note that this is one of the zeros the authors arrived at. The OL TF is now:

$$G(s)=\frac{2(0.9)(s+0.005)}{s(s^2)(s+2)} \tag{3}$$

The associated PM is shown at right. We see that now the PM is only -24.7°. But who cares!!! If we place the second controller zero to contribute 71° at $\omega=0.1$, it will bump up the OL phase at $\omega=0.1$ to 115°. If needed, we can then incorporate a controller gain K to ensure that this is the gain crossover frequency. For the additional controller term $s+\omega_2$ the phase at $\omega=0.1$ must be 71°: : $i0.1+\omega_2$ gives $\tan(71^o)=0.1/\omega_2 \Rightarrow 0.0344$.

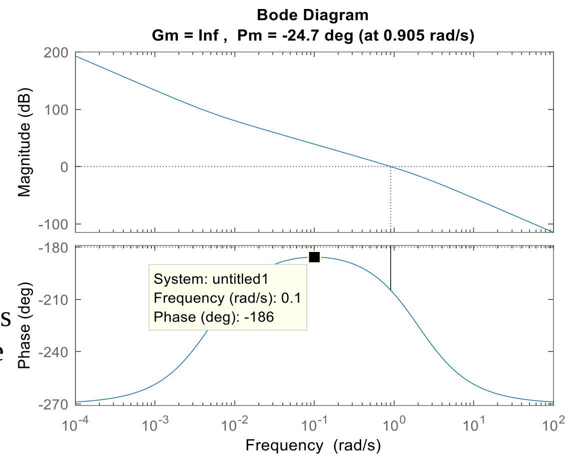
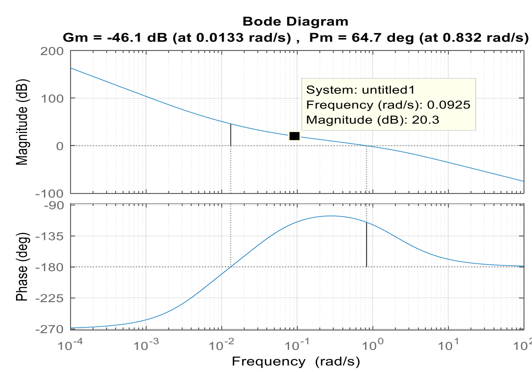


Figure 4. OL Bode plot for (3).

We now have: $G_c(s)=\frac{(s+0.005)(s+0.0344)}{s}$. The associated OL TF is:

$$G(s)+\frac{0.9}{s^2}\left(\frac{2}{s+2}\right)\left(\frac{(s+0.005)(s+0.0344)}{s}\right). \tag{4}$$

The Bode plot for (4) is shown at right. We see that the PM is close to 65°. It is not at $\omega_{gc}=0.1$. We could force it to be nearer to this frequency by incorporating a controller gain of -20dB. However, we currently have $\omega_{gc}=0.83$; which is better than $\omega_{gc}=0.1$. So, let's just see what the closed loop command response and torque disturbance response are, in comparison to what the authors got.



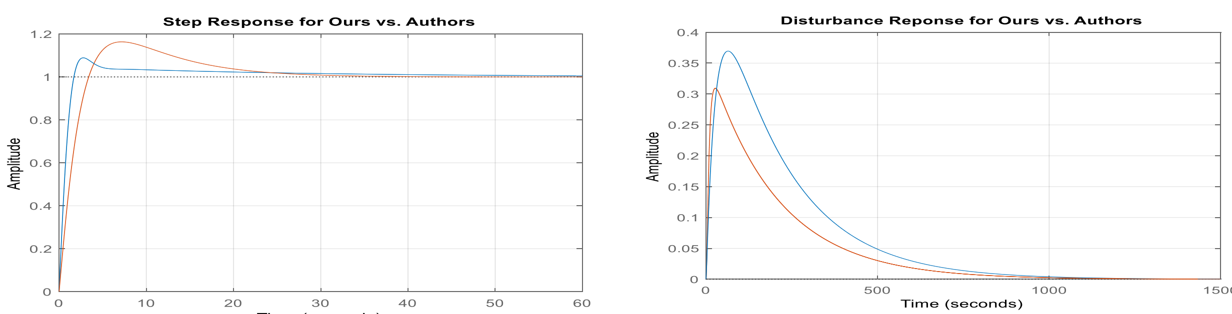


Figure 5. CL step responses for a unit step command (LEFT) and for a step disturbance $T_d=0.0175u(t)$ RIGHT).

I personally prefer our $G_c(s)=\dfrac{(s+0.005)(s+0.0344)}{s}$ to the authors' $D_c(s)=\dfrac{0.5(s+0.005)(s+0.1)}{s}$ on p.408. Our command response is faster (due to $\omega_{gc}=0.83$) and has less overshoot. Yes, our disturbance response is slightly larger and slightly longer. But I would accept that.

Summary: The authors designed a PID controller using a rather long argument for the selection of the controller zeros. Our argument was simple. We wanted to basically cancel out one of the open loop poles at the origin. We also wanted to bump up the phase to have a maximum of -115°. We were fortunate in that after doing this, the maximum phase was at a frequency close to that of where we wanted it to be. The authors never incorporated the controller pole at the origin in an OL Bode plot to see how it changed things. We did.

I believe that our design was more straightforward, and that it led to a better controller. This is not to say that I know more than the authors. I would never say that, in relation to feedback control systems. What it does illustrate is that it is possible to arrive at a better controller than one the experts designed- on occasion.

Finally, I should point out that our approach in arriving at the second term $s+.0344$ was based on it contributing 71° at $\omega=0.1$. We could now replace this single zero with a pole/zero pair that gives a net 71°. Why would we want to do this? The answer is that a PID controller is a high frequency noise amplifier.

Just for giggles ☺, let's carry out this alternative. The design equations for such a lead compensator (as opposed to $s+.0344$) are: $\omega_0=0.1=\sqrt{\omega_1\omega_2}$ and $\dfrac{\omega_2}{\omega_1}=\dfrac{1+\sin(71^o)}{1-\sin(71^o)}=35.71$. This gives: [W1 W2] =0.0167   0.5976. Our controller is now:

$$G_c(s)=\frac{(s+0.005)(s+0.0167)}{s(s+.5976)}. \qquad (5)$$

From the plot at right it is clear that we need a controller gain of -23.5dB, or $K=.067$.

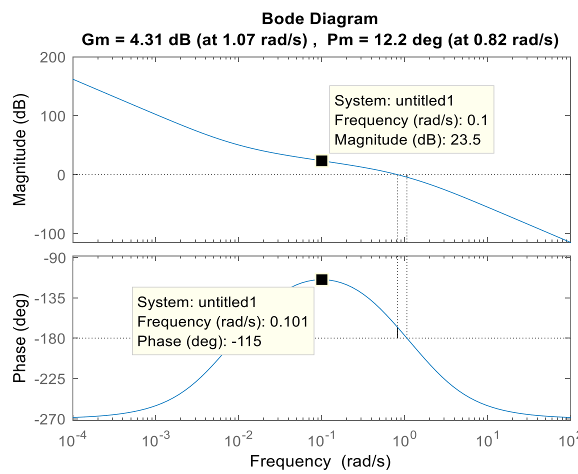


Figure 6. Bode plot of the OL TF using (5).

Our controller is now:

$$G_c(s)=0.067\frac{(s+0.005)(s+0.0167)}{s(s+.5976)}. \qquad (6)$$

From the OL Bode plot at right, we see that we exactly achieved the desired CL PM at the desired frequency. However, the plot below (LEFT) is not impressive. ☹ The response time is ~140 seconds. The reason is that we chose too low of a crossover frequency.

So, let's return back to the previous Figure 4 OL Bode plot (below RIGHT), prior to solving for the second controller zero.

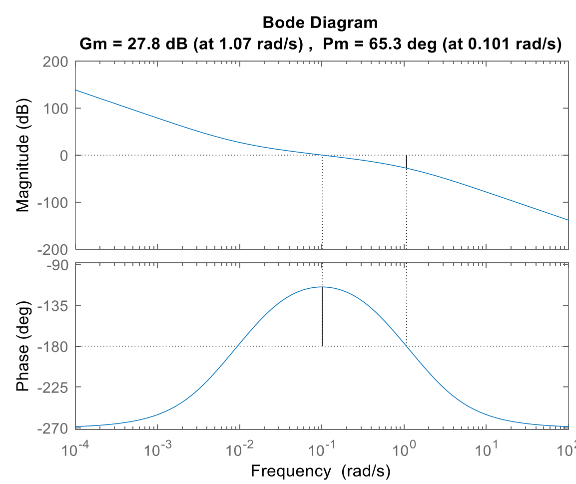


Figure 7. Bode plot of the OL TF using (6).

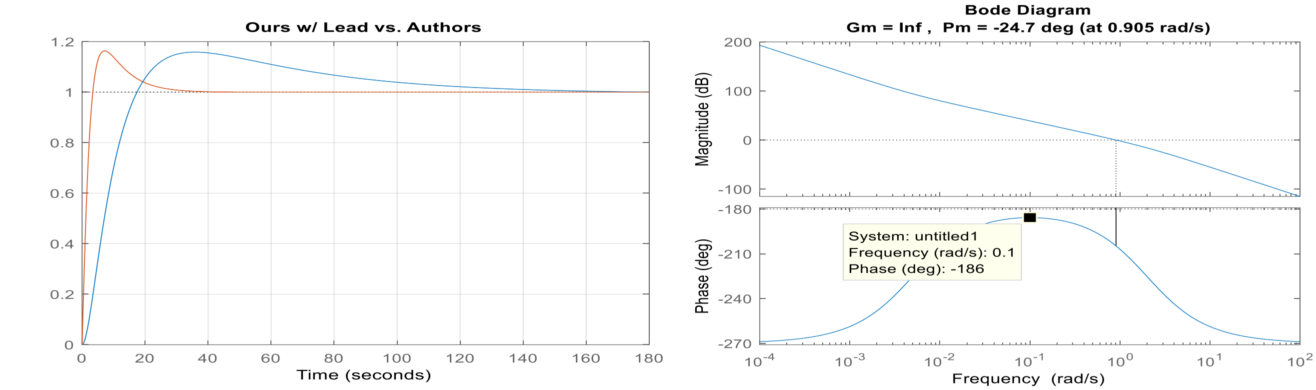


Figure 8. CL step response using (6) (LEFT). Repeat of Figure 4 (RIGHT).

It we place the lead compensator at $\omega_0 \cong 1$, we will need a net phase of $\phi_0 \cong 25^o + 65^o = 90^o$. Since this is unachievable with any proper lead compensator, let's choose $\phi_0 \cong 80^o$. Then $\omega_0 = 1 = \sqrt{\omega_1 \omega_2}$ and $\dfrac{\omega_2}{\omega_1} = \dfrac{1+\sin(80^o)}{1-\sin(80^o)} = 130.65$. This gives cc=sqrt(130.65) =11.4302; W1=1/cc; W2=cc; [W1 W2] =[0.0875 11.4302]. The resulting OL Bode plot is shown below (LEFT). The data cursor positions resulted from trying to achieve a maximum OL phase close to -115^o, while making the associated frequency as large as possible. I chose -118^o as the largest phase I could live with. Since the OL magnitude is -17.5dB at that frequency, we will need to include K=17.7db or K=7.5 in the controller. The OL Bode plot with this lead compensator is shown below (RIGHT).

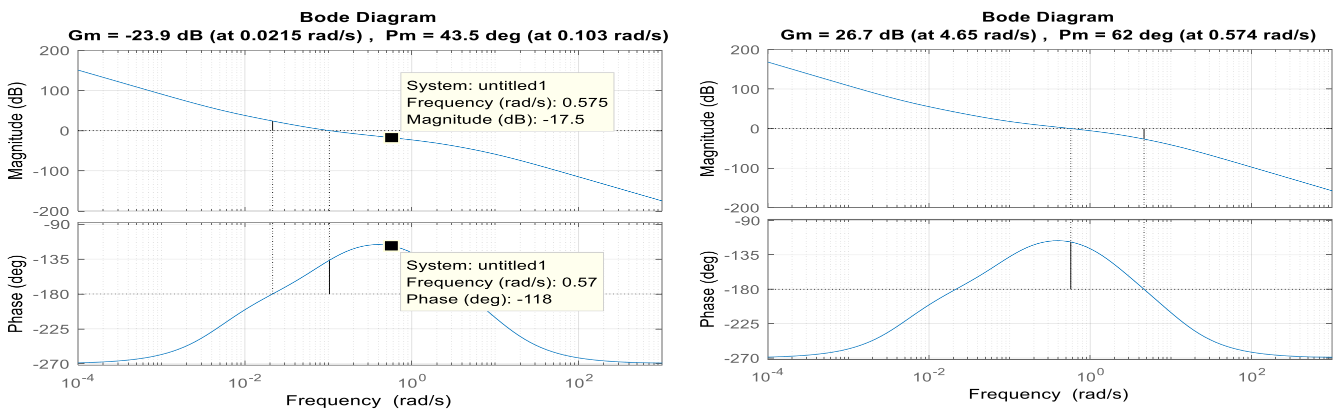


Figure 9. OL Bode plot to find needed K (LEFT), and including K=7.5 (RIGHT).

From it, we see that we now have a PM of 62^o (as expected) at a gain crossover frequency $\omega_{gc} = 0.575$. This is almost 6 times that of the previous design crossover frequency. The command and torque step responses are compared to the authors' below.

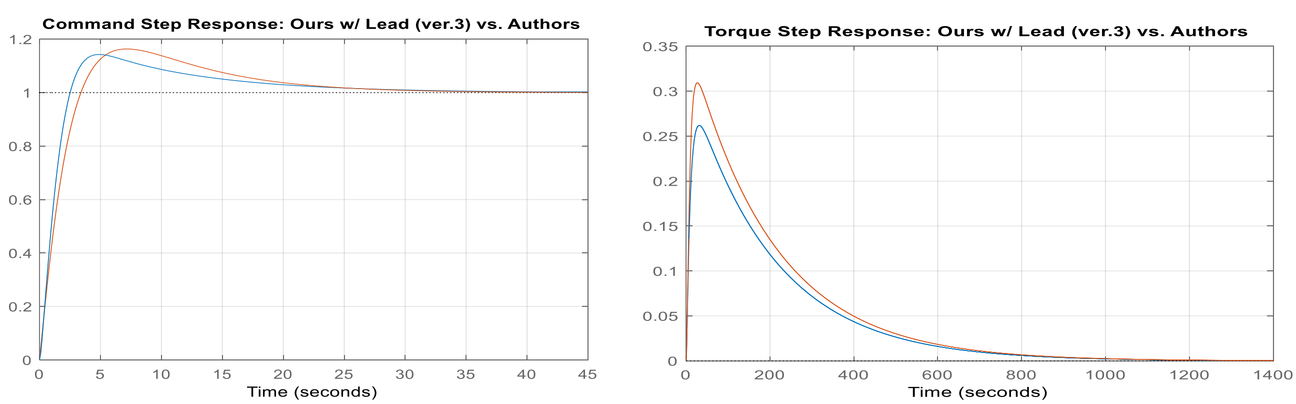
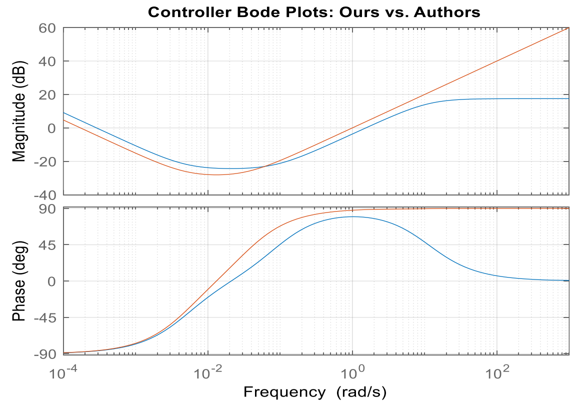


Figure 9. Closed loop command step response (LEFT) and torque disturbance step response (RIGHT).



The responses are similar enough. So then, why did we go through all the extra work? Ha! Ours and the authors' controller Bode plots are compared at right. They are very similar except at high frequencies, where the authors' gain keeps increasing and ours flattens out. The controller power is:

$$Pwr = \frac{1}{2\pi}\int_{-\infty}^{\infty} M(\omega)^2\, d\omega \cdot$$

Hence, it should be clear that mathematically, both controllers have infinite power. In reality, the high frequency magnitude of any controller must tend to zero (or -∞dB). If we replace the above limits of the integral by $(-200, 200)\, rad/\sec$, then we obtain the powers: [PWRGcV3 PWRGc] = 1.0e+05 * [0.0006   8.4883]. Our controller has ~$\mathbf{10^{-4}}$ the power of the authors'. □

THIS IS WHAT CAN HAPPEN WHEN YOUR OPPONENT HAS THE HIGHER TYPE NUMBER:

Matlab Code

```
%PROGRAM NAME: ex6_20.m
s=tf('s');
Gp=0.9/s^2;
H=2/(s+2);
GcPID=0.05*(10*s+1)*(s+.005)/s;
G=GcPID*Gp*H;
Wth=feedback(GcPID*Gp,H);
Wtq=feedback(Gp,GcPID*H);
figure(1)
step(Wth)
title('Command Step Response')
grid
figure(2)
Td=.0175; %Torque step disturbance magnitude
step(Td*Wtq)
title('Disturbance Step Response')
grid
%COMMENT 1: Fig. 6.69 on p.385 is wrong.
figure(3)
bode(G)
margin(G)
%COMMENT 2: We have proper PM but GM is negative.
%===============================================
%REVISIT THE DESIGN PROCESS:
G0=1/s; %Controller portion to ensure ess(tq)=0
figure(4)
margin(G0*Gp*H)
%The PM is -115deg. This will require TWO controller zeros
%Clearly, the 3 poles @ the origin are a challenge.
%Per the authors, try:
w1=0.005;
G1=s+w1;
figure(5)
margin(G1*G0*Gp*H)
%This gives PM=-25deg @ w=0.9 r/s
%Since we require PM=65deg the 2nd zero place here would
% require phimas = 85deg!!! Yuck.
%The OL phase @w=0.1 r/s is -186deg.
%If we place 2nd zero there we need phimax=71deg.
%We know that a single zero will give max 90%.
%So find w2 such that it gives 71deg:
w2=0.1/tand(71); %= .0344
G2=s+w2;
Gc=G0*G1*G2;
figure(6)
margin(Gc*Gp*H)
grid
%We get PM=64.7deg @ w=0.832 r/s
WWth=feedback(Gc*Gp,H);
figure(7)
step(WWth)
hold on
step(Wth)
title('Step Response for Ours vs. Authors')
grid
figure(8)
WWtq=feedback(Gp,Gc*H);
step(Td*WWtq)
hold on
step(Td*Wtq)
title('Disturbance Reponse for Ours vs. Authors')
grid
K=0.1;
GcK=K*Gc;
WWWth=feedback(GcK*Gp,H);
WWWtq=feedback(Gp,GcK*H);
```

```
figure(7)
step(WWWth)
figure(8)
step(Td*WWWtq)
%==============================
c=(1+sind(71))/(1-sind(71)); % w2/w1
w0=0.1; % sqrt(w1*w2)
W1=w0/sqrt(c);
W2=w0*sqrt(c);
GG2=(s+W1)/(s+W2);
GGc=G0*G1*GG2;
figure(9)
margin(GGc*Gp*H)
KK=.067
GGc=KK*GGc;
WFth=feedback(GGc*Gp,H);
figure(10)
step(WFth)
hold on
step(Wth)
title('Command Step Response: Ours w/ Lead vs. Authors')
grid
%===============================
W1=.0875; W2=11.43;
G2V3=(s+W1)/(s+W2);
GcV3=G0*G1*G2V3;
figure(13)
margin(GcV3*Gp*H)
grid
K=7.5;
GcV3=K*GcV3;
figure(15)
margin(GcV3*Gp*H)
WV3th=feedback(GcV3*Gp,H);
step(WV3th)
hold on
step(Wth)
title('Command Step Response: Ours w/ Lead (ver.3) vs. Authors')
grid
figure(16)
WV3tq=feedback(Gp,GcV3*H);
step(Td*WV3tq)
hold on
step(Td*Wtq)
title('Torque Step Response: Ours w/ Lead (ver.3) vs. Authors')
%--------------------------
figure(17)
bode(GcV3)
hold on
bode(Gc)
grid
title('Controller Bode Plots: Ours vs. Authors')
%-------------------------
%Controller powers:
MC2=@(w) abs((1i*w).^-1.*(w1+1i*w).*(w2+1i*w)).^2;
PWRGc=(1/pi)*integral(MC2,0,200);
MCV3=@(w) abs((1i*w).^-1.*(w1+1i*w).*(W1+1i*w)./(W2+1i*w)).^2;
PWRGcV3=(1/pi)*integral(MCV3,0,200);
[PWRGcV3 PWRGc]
```