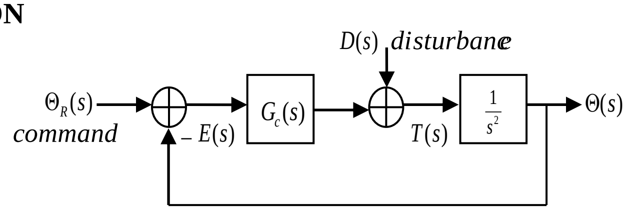
Homework 3 AERE331 Spring 2020 Due 2/28(F) SOLUTION



PROBLEM 1(35pts). In *Example* 6.14 on pp.382-385 of the book, the authors design a spacecraft attitude control system. For the unity feedback control system shown at the right, the authors propose the PD controller $G_c(s) = 0.2(s+.05)$ to satisfy the closed loop (CL) command system specifications (S1) good damping, and (S2) a bandwidth (BW) of approximately 0.2 rad/sec.

Figure 1. Closed loop block diagram.

(a)(5pts) Use a Bode plot of the command system to (i) assess how well (S2) is satisfied, per the authors' definition of BW. Specifically, compute the percent error relative to (S2). Then discuss whether that error is a good error or a bad error.
<u>*Solution*</u>: [See code @ 1(a).]

The -3dB BW is 0.247rad/sec. which is $\left(\frac{.247-.2}{.2}\right)\times 100\% = 23.5\%$ greater than (S2). Because it is greater than (S2), it's a good error. [Higher BW is usually a good thing.]

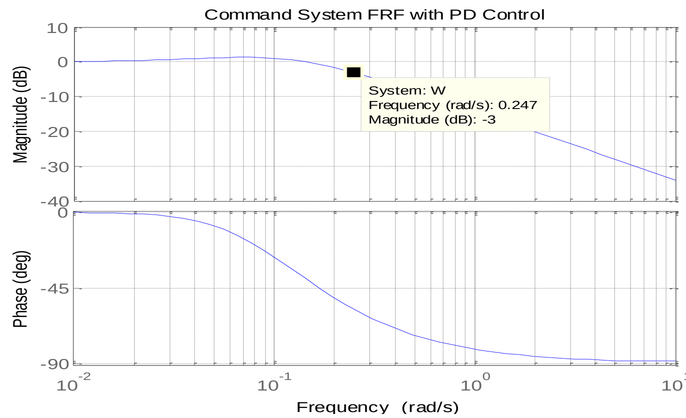


Figure 1(a) CL Bode plot with BW data.

(b)(6pts) Use the open loop FRF to obtain the command system CL *gain margin* (GM) and *phase margin* (PM).
<u>*Solution*</u>: [See code @ 1(b).]
Because the open loop phase never crosses -180°, the closed loop $GM = +\infty$. From the data cursor,
$PM = 180^o - 104^o = 76^o$.

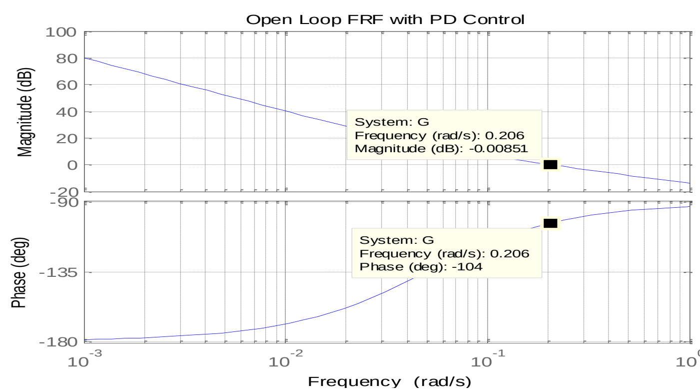


Figure 1(b) OL Bode plot with CL GM/PM data.

(c)(6pts) Plot the disturbance system FRF, then use the data cursor to estimate the system (i) static again, (ii) -3dB BW, and (iii) resonant frequency, and amplification (re: static gain) at resonance.
<u>*Solution*</u>: [See code @ 1(c).]

(i) $g_s^{dB} \cong 40dB$ [i.e. $g_s \cong 100$.] (ii) $BW \cong 0.04 r/s$

(iii)There is no resonance. [Both CL roots are at -1+i0.]

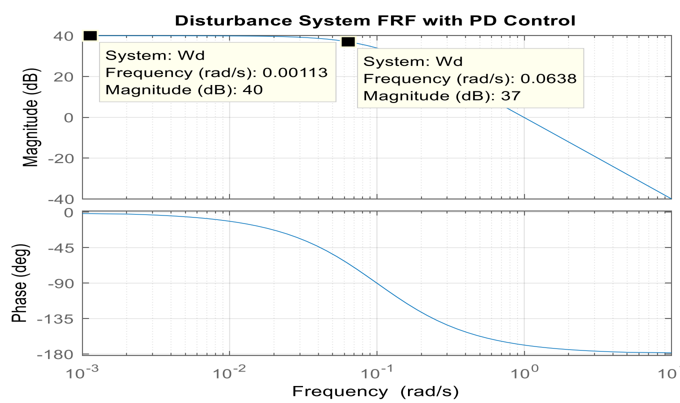


Figure 1(c) Disturbance FRF.

(d)(10pts) Consider a unit step command input, $\theta_R(t) = 1(t)$ plus a disturbance input $d(t) = .001\sin(.01t)$. Let the observation time be the interval $[0,6000)$ seconds, with a sampling interval $\Delta = 0.3142$ sec. (i) Plot the command response. Then (ii) use the 'lsim' command to compute the disturbance response, and then add this to the command response and overlay it on your *noiseless* command response.
Solution: [See code @ 1(d).]

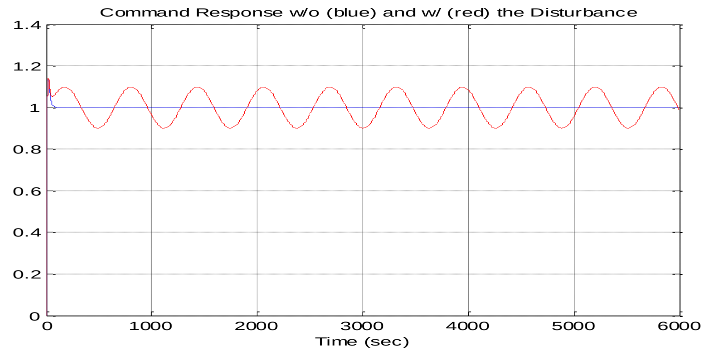


Figure 1(d) CL step response without and with noise.

(e)(8pts) The disturbance input in (d) had an amplitude 1/1000th of the command input. (i) Use your FRF in (c) to explain why the disturbance response is as large as it is. Then (ii) Use your answer in (c) to point out a fundamental problem with this feedback control system.

Solution:

(i) At $\omega_n = .01 rad/\sec$ the disturbance amplification is 40 dB; which is equivalent to a factor of $10^{40/20} = 100$. Hence, the disturbance input amplitude .001 will yield an output amplitude 0.1 (as is observed in the plot).

(ii) The disturbance system has a low frequency gain of 40dB, or a gain of 100. This is huge! The system is extremely sensitive to small low frequency disturbance inputs.

PROBLEM 2(25pts) (Book Problem 6.2 on p.389) Consider a system with transfer function $G(s) = \dfrac{1}{s+10}$.

(a)(8pts) Develop the expressions for the magnitude (in dB) and phase (in degrees) of $G(i\omega)$.

<u>*Solution*</u>: The magnitude is: $|G(i\omega)| = \dfrac{1}{|i\omega + 10|} = (100 + \omega^2)^{-0.5} \Rightarrow M(\omega)_{dB} = 20\log(|G(i\omega)| = -10\log(100 + \omega^2)$.

The phase is: $\theta(\omega) = 0^o - (180/\pi)\tan^{-1}(\omega/10)$.

(b)(7pts) Use your expressions in (a) to write a simple *Matlab* code to compute the numerical values of these expressions at the 7 radial frequencies $\{1,2,5,10,20,50,100\}$. [Give magnitude in dB and phase in degrees.]

<u>*Solution*</u>: [Include your code HERE.]

```
w = [1 2 5 10 20 50 100]; M = -10*log10(100 + w.^2); th = -(180/pi)*atan(w/10);
[M ; th].
```
Running this code gives:

Magnitude (dB):	-20.0432	-20.1703	-20.9691	-23.0103	-26.9897	-34.1497	-40.0432
Phase (degrees):	-5.7106	-11.3099	-26.5651	-45.0000	-63.4349	-78.6901	-84.2894

(c)(10pts) Construct *straight-line* approximations of the Bode plot over the frequency range $[0.1, 1000]r/s$ and overlay your numbers from (b) on it.

<u>*Solution*</u>: [See code @ 2(c).]
The magnitude break frequency is 10 r/s. The phase break frequencies are: {0.1 , 1.0 , 100 , 1000}. The low frequency system gain is 0.1 = -20dB. The plot was constructed with the simple *Matlab* code included in the Appendix.

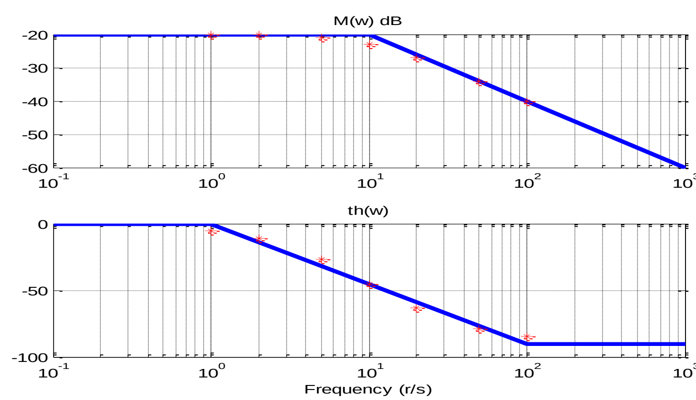


Figure 2(c) Straight line Bode plot and data from (b).

PROBLEM 3(40pts) The Root Locus-based *pole placement* method was used to design a unity-feedback control system for the plant $G_p(s) = \dfrac{10}{s(s+2)}$. The result was a *lead controller* $G_c(s) = \dfrac{4.13(s+2.75)}{s+9.55}$. The resulting OL and CL transfer functions are: $G(s) = G_c(s)G_p(s) = \dfrac{41.3(s+2.75)}{s(s+2)(s+9.55)}$ and $W(s) = \dfrac{41.3s+113.57}{s^3+11.55s^2+60.4s+113.57}$.

In this problem, you will analyze that design in the frequency domain.

(a)(5pts) Give the plant Bode plot, and use it to estimate GM and PM of the corresponding closed loop transfer function obtained by incorporating unity feedback alone (i.e. with $G_c(s) = 1$).

Solution: [See code @ 3(a).]

The open loop *gain crossover* frequency is ~2.8r/s, and so the closed loop PM is ~40°. The GM is infinite, since the open loop phase never crosses -180°.

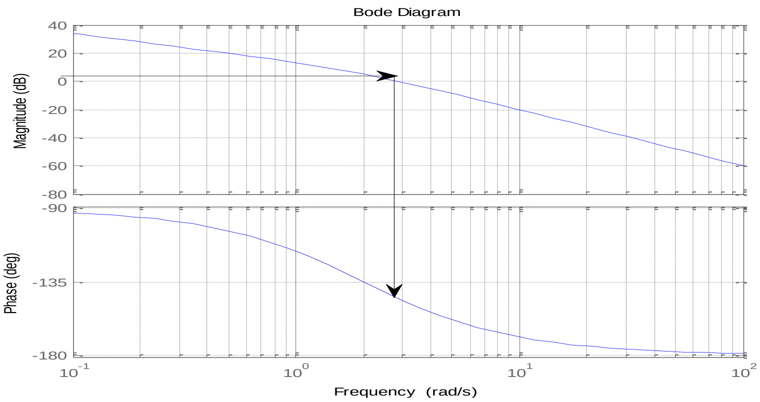


Figure 3(a) OL Bode plot and arrows to show CL GM and PM.

(b)(10pts) Give the controller Bode plot, and use it to estimate (i) the frequency, $\omega_{\max}$ at which the controller phase is maximum, (ii) the value of this maximum phase, $\theta_{\max}$, and (iii) the relative gain of the controller from low to high frequencies, α_{dB}.

Solution: [See code @ 3(b).]

From the plot at the right, we have:

$\omega_{\max} \cong 4.65 r/s$; $\theta_{\max} \cong 35^o$; $\alpha_{dB} \cong 11.8 dB$

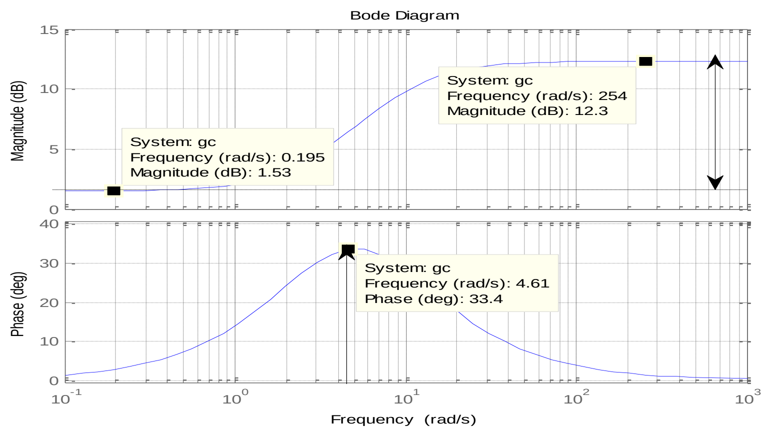


Figure 3(b) Controller Bode plot with data cursor information.

(c)(5pts) Give the open loop (i.e. the plant/controller combination) Bode plot, and use it to determine (i) the lead-controller-based closed loop GM and PM, and (ii) how much phase the controller contributed to the PM.

Solution: [See code @ 3(c).]

$GM = +\infty$; $PM \cong 60^o$ @ $\omega_{co} \cong 4.35 r/s$.

Hence, the controller contributed ~20°, which is 80% of its maximum phase.

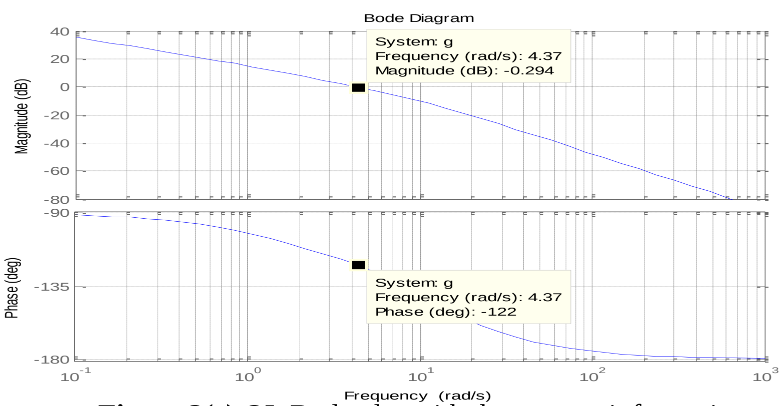


Figure 3(c) OL Bode plot with data cursor information.

(d)(10pts) (i) Overlay the CL Bode plots associated with (a) and (c). (ii) Use the data cursor to estimate the $\pm 3dB$ bandwidth of each system. (iii) Use the data cursor to estimate the Q factor of each system.

<u>*Solution*</u>: The CL BW re: (a) is **3.0 r/s** and that for (c) is **6.74 r/s**. The Q factor for (a) is **4.31dB** or 1.66. For (b) you could also say it has **no** Q factor. However, were one to go a little deeper in relation to $\frac{1}{2\zeta} \overset{\Delta}{=} Q$ [See Lecture 10]:

$$W_{(b)}(s) = \frac{41.3s + 113.6}{s^3 + 11.55s^2 + 60.4s + 113.6}$$ has poles:

roots([1 11.55 60.4 113.6]) = -4.0000 +/- 4.0000i; -3.55.

It is critically damped. So, in fact, $Q = 0.5$.

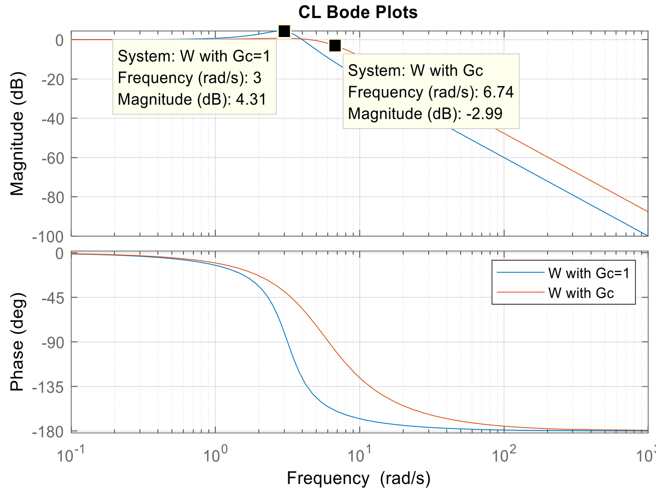


Figure 3(d) CL Bode plots re: (a) and (c) with data cursor informati

(e)(10pts) You should have found that the CL PM re: (a) was ~40º, while that re: (c) was ~60º. It is often claimed that a CL PM of ~40º is a good design goal. From overlaid CL step responses, comment on this claim, based on numbers.

<u>*Solution*</u>:

The settling time re: (a) is 4 times that of (c).

The response re: (a) has over twice the overshoot as (b) has.

The response re(a) has notable oscillations.

Hence, I would conclude that the claim is not well-founded.

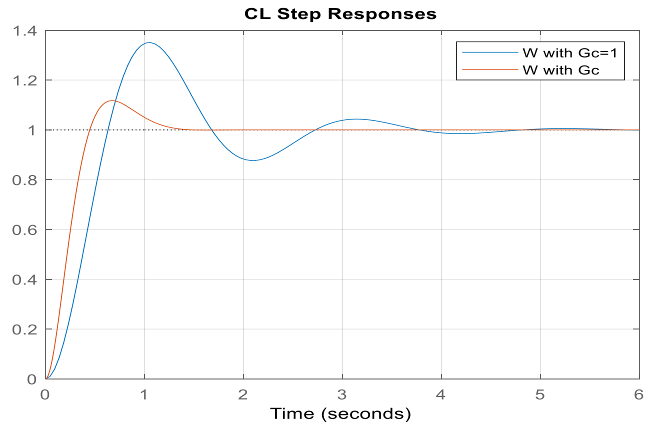


Figure 3(e) CL Bode plots re: (a) and (c).

Appendix Matlab code

```
% PROGRAM NAME: hw3.m   2/10/20
% PROBLEM 1:
% 1(a):
Gp = tf(1,[1 0 0]);
Gc = 0.2*tf([1 .05],1);
G = Gc*Gp;
H = tf(1,1);
W = feedback(G,H);
figure(10)
bode(W)
grid
title('Command System FRF with PD Control')
% 1(b):
figure(11)
bode(G)
grid
title('Open Loop FRF with PD Control')
% 1(c):
Wd = feedback(Gp,Gc);
figure(12)
bode(Wd)
grid
title('Disturbance System FRF with PD Control')
% 1(d)
T = 6000; % Observation time
dt = pi/10; % sampling interval
n = fix(T/dt);
tvec = 0:dt:(n-1)*dt;
thc = step(W,tvec);
%d = normrnd(0,.01,n,1);
d = .001*sin(.01*tvec);
thd = lsim(Wd,d,tvec);
th = thc + thd;
figure(13)
plot(tvec,thc,tvec,th,'r')
grid
xlabel('Time (sec)')
title('Command Response w/o (blue) and w/ (red) the Disturbance')
%==============================================
%PROBLEM 2
%(b):
w=[1 2 5 10 20 50 100];
M=-10*log10(100+w.^2);
PH=-atand(w/10);
[M;PH]
%(c);
%MAGNITUDE:
W1=10^-1; W2=10^3; %Plot frequency range
wb=10; %break frequency
wM=[W1 wb W2];
MSL=[-20 -20 -60];
figure(20)
subplot(2,1,1), semilogx(wM,MSL,'b','LineWidth',2)
title('M(w)')
ylabel('dB')
grid
hold on
semilogx(w,M,'r*','LineWidth',2)
%PHASE:
wPH=[W1 0.1*wb 10*wb W2];
PHSL=[0 0 -90 -90];
subplot(2,1,2), semilogx(wPH,PHSL,'b','LineWidth',2)
title('TH(w)')
ylabel('degrees')
xlabel('Frequency (r/s)')
grid
hold on
semilogx(w,PH,'r*','LineWidth',2)
%===========================================
% PROBLEM 3:
Gp = tf(10,[1 2 0]);
```

```
Gc = 4.13*tf([1 2.75],[1 9.55]);
G = Gc*Gp;
%(a):
figure(30)
bode(Gp)
grid
%(b):
figure(31)
bode(Gc)
grid
%(c):
figure(32)
bode(G)
grid
%(d):
H = tf(1,1);
W1 = feedback(Gp,H);
W2 = feedback(G,H);
figure(33)
bode(W1)
hold on
bode(W2)
title('CL Bode Plots')
legend('W with Gc=1','W with Gc')
grid
%(e):
figure(34)
step(W1)
hold on
step(W2)
title('CL Step Responses')
legend('W with Gc=1','W with Gc')
grid
```