# Syntax / Semantics of Propositional CTL*

- Propositional logic + temporal operators = prop. LTL

  Prop. LTL + path operators = Prop. CTL*

  (LTL: Linear temporal logic / CTL*: Computational-tree temporal logic)

- CTL* formula either state-formula ($\phi$) or path formula ($\alpha$):

$$\phi = T \mid F \mid p \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \neg \phi \mid A\alpha \mid E\alpha$$

$$\alpha = \phi \mid \alpha_1 \wedge \alpha_2 \mid \alpha_1 \vee \alpha_2 \mid \neg \alpha \mid X\alpha \mid F\alpha \mid G\alpha \mid \alpha_1 U \alpha_2$$

- Semantics of CTL* wrt a model $M = (S, \rightarrow, L)$.

  Given model $M$, state $s \in S$, path $\pi = s_0(\pi)\, s_1(\pi) \ldots \in S^\omega$ recursively define the "satisfies" relation "$\models$" as follows:

1) $M, s \models p \iff p \in L(s)$
2) $M, s \models \phi_1 \wedge \phi_2 \iff (M, s \models \phi_1) \wedge (M, s \models \phi_2)$
3) $M, s \models \neg \phi \iff \neg (M, s \models \phi)$
4) $M, s \models A\alpha \iff \forall \pi$ in $M$ with $s_0(\pi) = s$: $M, \pi \models \alpha$
5) $M, \pi \models \phi \iff M, s_0(\pi) \models \phi$
6) $M, \pi \models \neg \alpha \iff \neg (M, \pi \models \alpha)$
7) $M, \pi \models \alpha_1 \wedge \alpha_2 \iff (M, \pi \models \alpha_1) \wedge (M, \pi \models \alpha_2)$
8) $M, \pi \models X\alpha \iff M, \pi' \models \alpha$ $\quad (\pi' = s_1(\pi)\, s_2(\pi) \ldots)$
9) $M, \pi \models F\alpha \iff \exists k: M, \pi^k \models \alpha$ $\quad (\pi^k = s_k(\pi)\, s_{k+1}(\pi) \ldots)$
10) $M, \pi \models \alpha_1 U \alpha_2 \iff \exists k: (M, \pi^k \models \alpha_2) \wedge (\forall j < k: M, \pi^j \models \alpha_1)$
11) $M, \pi \models G\alpha \iff \forall k: (M, \pi^k \models \alpha)$.