Example involving while-loop (ctnd.)

- loop-invariant for white/black ball puzzle:

  $Odd(w) \Leftrightarrow Odd(W)$

  For each assignment it holds that

  $Odd(w) \Leftrightarrow Odd(W) \quad \{S_i\} \quad Odd(w) \Leftrightarrow Odd(W)$

  S1) $w, b \Leftarrow w, b-1$

  S2) $w, b \Leftarrow w-2, b+1$

  S3) $w, b \Leftarrow w, b-1$

- This means no. of white balls remains odd iff initially it was odd. So last ball is white iff W is odd.
- Note another loop-invariant: $(w+b \geq 1)$
- We can now claim:

precond. $[w, b = W, B]$

while-loop
while $w+b \geq 2$ do
S1) $w, b \Leftarrow w, b-1$
S2) $w, b \Leftarrow w-2, b+1$
S3) $w, b \Leftarrow w, b-1$

postcond. $[w+b=1] \wedge [(w=1) \Leftrightarrow Odd(W)]$

- Since $[Odd(w) \Leftrightarrow Odd(W)] \wedge [w+b \geq 1]$ is loop-invariant

$$[w+b \geq 1] \wedge [Odd(w) \Leftrightarrow Odd(W)] \{\text{while } \underbrace{w+b \geq 2}_{\text{gaurd}} \text{ do } \ldots\} [Odd(w) \Leftrightarrow Odd(W)] \wedge [w+b \geq 1] \wedge \neg[w+b \geq 2]$$

$$\begin{aligned}
\text{post. cond.} &\equiv [Odd(w) \Leftrightarrow Odd(W)] \wedge [w+b \geq 1] \wedge [w+b<2] \\
&\equiv [Odd(w) \Leftrightarrow Odd(W)] \wedge [w+b=1] \\
&\equiv [w+b=1] \wedge [(w=1) \Leftrightarrow Odd(W)].
\end{aligned}$$