Inference rule for while-loop

- While loop is of form,

  while B do S $\equiv$ if B then S; if B then S; ... ad-infinitum
- while loop is a source for non-termination (eg, while TRUE do S)

partial correctness assumes termination of while-loop, whereas total correctness also proves termination of while-loop.

- **Loop-invariant:** Predicate that is pre & post-cond. of "if B then S"

$$\alpha(x) \{\text{if } B \text{ then } S\} \alpha(x)$$

$$\equiv [\alpha(x) \wedge B(x) \ \{S\} \ \alpha(x)] \ \wedge \ \underbrace{[\alpha(x) \wedge \neg B(x) \rightarrow \alpha(x)]}_{\text{TRUE}}$$

$$\equiv \alpha(x) \wedge B(x) \ \{S\} \ \alpha(x)$$

Note: $\alpha(x)$ remains invariant under assignment of "if B then S"

$\Rightarrow$ $\alpha(x)$ remains invariant under assignment of "while B do S".

(since "while B do S" is repeated assignment of "if B then S")

Examples of loop-invariants: (i) TRUE: TRUE $\wedge B(x)$ $\{S\}$ TRUE

(ii) FALSE: FALSE $\wedge B(x)$ $\{S\}$ FALSE

(iii) $\neg B(x)$: $\neg B(x) \wedge B(x)$ $\{S\}$ $\neg B(x)$

(Several loop-invariants exist; we need to find an appropriate one)

- **Inference rule for while-loop:**

$$\text{(vi)} \quad \frac{\alpha(x) \wedge B(x) \ \{S\} \ \alpha(x) \qquad \text{loop-terminates}}{\alpha(x) \ \{\text{while } B \text{ do } S\} \ \alpha(x) \wedge \neg B(x)}$$

1^{st} premise says $\alpha(x)$ an inv. for "while B do S"

Termination can occur after finite no. of iterations if post-cond. becomes $\alpha(x) \wedge \neg B(x)$ (inv. and negation of gaurd)