Properties of 1st order logic

- Recall, Frege introduced "higher-order" logic, but it was controversial (1879)
- Controversy settled by Russell by showing lack of soundness (1901)
- Sound "higher-order" logic by Russell & Whitehead (1910)
- 1st order logic fragment conceived by Hilbert & Ackermann (1928)
- Soundness: from soundness of higher-order logic
  Completeness: Gödel (1930)
- Undecidability (of satisfiability) 1930's

Example of undecidable problem (Post-correspondance problem)

Given finite pair of seqs. of binary nos $(s_1, t_1), \ldots, (s_n, t_n)$ does there exist indices $i_1, \ldots, i_k$ such that

$s_{i_1} s_{i_2} \cdots s_{i_k} = t_{i_1} t_{i_2} \cdots t_{i_k}$.

Say, $s_1 = 1, t_1 = 101$; $s_2 = 10, t_2 = 00$; $s_3 = 011, t_3 = 11$

Then $s_1 s_3 s_2 s_3 = 1\,011\,10\,011 = t_1 t_3 t_2 t_3$.

Checking satisfiability is like solving PCP ⇒ undecidable.

- 1st order logic sound & complete, but undecidable.

⇒ 1st order logic more expressive than 0-order logic (decidable)
  (Recall, satisfiability in 0-order logic is NP-complete)

- 1st order logic not powerful enough to express reachability.

$Path_0(x,y) \leftrightarrow x = y$

$Path_{n+1}(x,y) \leftrightarrow \exists z: Path_n(x,z) \wedge Edge(z,y)$

$Path(x,y) \leftrightarrow \exists n: Path_n(x,y)$

$\leftrightarrow Path_0(x,y) \vee Path_1(x,y) \ldots$

But this is a disjunction of infinitely many predicates, and so not a 1st order formula.