A Logic System for Numbers (Dedekind-Peano)

- Notation: N set of numbers; $\forall n \in N$, $n' \in N$ successor of n
  variables: n, m; functions: successor, addition, multiplication, exp.
- Axiomatic definition of N: $N = 1 \mid n \mid n'$

  (A1) $\exists 1 \in N, \forall n \in N, n' \neq 1$ (1 not a successor of any no.)

  (A2) $n' = m' \Rightarrow n = m$ (successor is unique)

  (A3) $(X \subseteq N) \wedge (X' \subseteq X) \wedge (1 \in X) \Rightarrow (X = N)$

- Axiomatic definition of addition:

  (A4) $n + 1 = n'$

  (A5) $m + n' = (m+n)'$

- Axiomatic definition of multiplication:

  (A6) $n \cdot 1 = n$

  (A7) $m \cdot n' = m \cdot n + m$

- Axiomatic definition of exponentiation:

  (A8) $m^1 = m$

  (A9) $m^{n'} = m^n \cdot m$

- Above example of logic system illustrates:
  1) Power of logic (any reasoning mechanism can be represented)
  2) Precison of logic (reasoning mechanism formally & precisely defined)
  3) Compactness of logic (reasoning mechanism succintly defined)

- Godel proved that number system is sound iff it is incomplete.
  Sound $\equiv$ (provable $\rightarrow$ True); Complete $\equiv$ (True $\rightarrow$ provable).
  Godel showed how to construct formula f in lang. of number system such that $\boxed{f \Leftrightarrow f \text{ not provable}}$.