

## Inference involving Assumptions

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- Assumption: Axiom that tentatively introduced.
  - Will be denoted by placing  $\neg$  bracket around it.
  - Also all deductions using it will be indented.

- Reduction to absurdity:

$$\begin{array}{c} [\neg p] \\ \hline \text{FALSE} \end{array}$$

Assuming  $p$  to be TRUE, later established  $\neg p$ , then  $\neg p$  holds.

- $\rightarrow$  introduction:

$$\begin{array}{c} [\neg p] \\ \hline q \\ \hline p \rightarrow q \end{array}$$

Assuming  $p$  to be TRUE, later establishes  $q$ , then  $p \rightarrow q$  holds.

- Example: Prove  $((p \vee q) \wedge \neg p) \rightarrow q$

1.

$$[(p \vee q) \wedge \neg p]$$

assumption

2.

$$p \vee q$$

 $\wedge$  elimination in 1

3.

$$\neg p$$

 $\wedge$  elimination in 1

4.

$$p \rightarrow q$$

vacuous proof from 3.

5.

$$q \rightarrow q$$

tautology

6.

$$q$$

case analysis from 2,4,5

$$7. ((p \vee q) \wedge \neg p) \rightarrow q$$

 $\rightarrow$  introduction from 1,6