Understanding LMPs

6.0 Locational marginal price

The following paper provides good additional insight.
T. Organogianni and G. Gross, "A General Formulation for LMP Evaluation," IEEE Trans. On Power Systems, Vol 22, No 3, Aug 2007.

Armed with the envelope theorem, we may now identify the meaning to (17), which is repeated here for convenience:

$$k \in load: \qquad \frac{\partial L}{\partial P_{dk}} = \lambda \left(1 + \frac{\partial P_{loss}}{\partial P_{dk}}\right) + \sum_{j=1}^{M} \mu_j t_{jk} \qquad (17)$$

Equation (17) gives the change in the optimal value of the objective function due to a small change in the parameter P_{dk}.

In other words, if we solve the optimization problem with $P_{dk}=P_{dk0}$, obtaining $G^*(P_{dk0})$, and then resolve the optimization problem with $P_{dk}=P_{dk0}+1$, obtaining $G^*(P_{dk0}+1)$, then

$$G^*(P_{dk0}+1) - G^*(P_{dk0}) = \frac{\partial L}{\partial P_{dk}} \qquad (23)$$

We call $\frac{\partial L}{\partial P_{dk}}$ the LMP for bus k, that is,

$$k \in load: \qquad LMP_k = \lambda \left(1 + \frac{\partial P_{loss}}{\partial P_{dk}}\right) + \sum_{j=1}^{M} \mu_j t_{jk} \qquad (24)$$

Written slightly different, it is

$$k \in load: \qquad LMP_k = \lambda + \lambda \frac{\partial P_{loss}}{\partial P_{dk}} + \sum_{j=1}^{M} \mu_j t_{jk} \qquad (25)$$

And (25) show us a very useful way to think about LMPs. They consist of three components:

$$
\begin{aligned}
k \in load: \quad LMP_k &= \lambda && \text{Energy component} \\
&+ \lambda \frac{\partial P_{loss}}{\partial P_{dk}} && \text{Loss component} \\
&+ \sum_{j=1}^{M} \mu_j t_{jk} && \text{Congestion component}
\end{aligned} \tag{26}
$$

We discuss each one of these terms in what follows.

7.0 Energy component

We are considering the components of the LMP at a particular bus *k*. The first component is the energy component, represented by λ.

To gain better understanding of exactly what this is, we will neglect losses in our original formulation (14), resulting in

$$
\begin{aligned}
&\min \quad G(\underline{P}) = \sum_{k=1}^{N} s_k P_{gk} \\
&s.t. \\
&\sum_{k=1}^{N} P_{gk} - P_{dk} = 0 \\
&\sum_{k=1}^{N} t_{jk} (P_{gk} - P_{dk}) \leq F_{j\max}, \ j = 1, \ldots, M
\end{aligned} \tag{27}
$$

Rewriting the equality constraint in (27) so that the function of decision variables is on the left-hand-side and constants on the right-hand-side, we have

$$\min \quad G(\underline{P}) = \sum_{k=1}^{N} s_k P_{gk}$$

$$s.t.$$

$$\sum_{k=1}^{N} P_{gk} = \sum_{k=1}^{N} P_{dk} = P_{D,tot}$$

$$\sum_{k=1}^{N} t_{jk}(P_{gk} - P_{dk}) \le F_{j\max}, \; j = 1,\ldots,M \tag{28}$$

Now write the Lagrangian function:

$$L(\underline{P}_g, \lambda, \underline{\mu}) = \sum_{k=1}^{N} s_k P_{gk} - \lambda\left[\sum_{k=1}^{N} P_{gk} - \sum_{k=1}^{N} P_{dk}\right] - \sum_{j=1}^{M} \mu_j \left[\sum_{k=1}^{N} t_{jk}(P_{gk} - P_{dk}) - F_{j\max}\right] \tag{29}$$

or

$$L(\underline{P}_g, \lambda, \underline{\mu}) = \sum_{k=1}^{N} s_k P_{gk} - \lambda\left[\sum_{k=1}^{N} P_{gk} - P_{D,tot}\right] - \sum_{j=1}^{M} \mu_j \left[\sum_{k=1}^{N} t_{jk}(P_{gk} - P_{dk}) - F_{j\max}\right] \tag{30}$$

Notice that λ is the Lagrange multiplier (or dual variable) on the power balance equality constraint. This immediately gives us an interpretation of λ.

> ➔The energy component λ of the LMP is the increase in the objective function (in this case, cost per hour) if demand $P_{D,tot}$ increases by 1 unit.

Without losses, the LMP expression becomes (from (29)):

$$k \in load: \qquad LMP_k = \frac{\partial L}{\partial P_{dk}} = \lambda + \sum_{j=1}^{M} \mu_j t_{jk} \tag{31}$$

The summation is the congestion component. If there is no congestion, then

$$k \in load: \qquad LMP_k = \lambda \tag{32}$$

Equation (32) makes the interesting point that, if we ignore losses, and if there is no congestion, then the LMP will equal to λ, *and this will be true for every load bus in the network*.

One last comment here. It is worthwhile to identify what determines λ. We may gain insight to this via the first order condition (16) which, without losses, becomes

$$k \in gen: \qquad \frac{\partial L}{\partial P_{gk}} = s_k - \lambda - \sum_{j=1}^{M} \mu_j t_{jk} = 0 \tag{33}$$

Solving for λ, we obtain:

$$k \in gen: \quad \lambda = s_k - \sum_{j=1}^{M} \mu_j t_{jk} \tag{34}$$

Under the condition of no congestion, then

$$k \in gen: \quad \lambda = s_k \tag{35}$$

What does this mean?...

To understand what this means, it is important to understand that P_{gk} for which we differentiate to obtain (35) must be "regulating," i.e., it cannot be at its limit. We could have exposed this idea more clearly by including constraints on P_{gk} in the optimization problem formulation, in which case we would have obtained corresponding terms in the objective that would have vanished for regulating units and would have contributed for non-regulating units.

Now consider how an electricity market works. Each generation owner offers in their s_k with a corresponding range. The algorithm selects the lowest offer, and takes the full range of that offer, and then selects the next lowest offer, and then the next, and so on until the demand is met. Figure 1 illustrates.

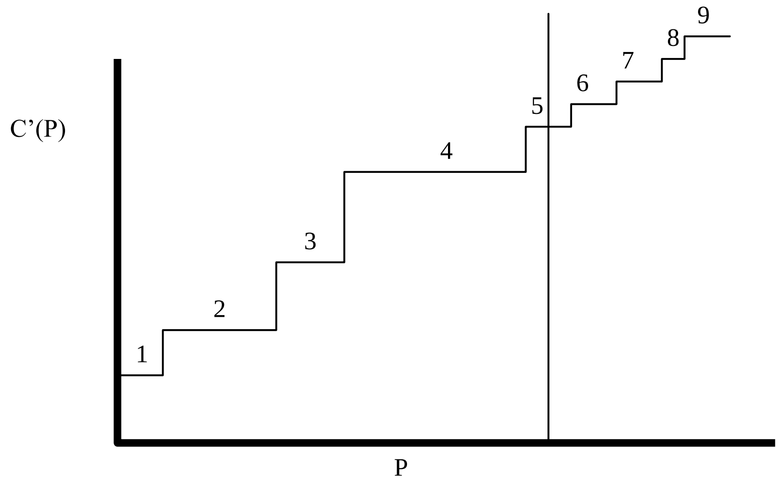

Fig. 1

The only unit that is selected, and is regulating, is unit 5. This is the unit for which $\lambda = s_k$. It is the unit that will pick up the extra demand when the demand is increased by 1 unit. We say that unit 5 is "on the margin."

8.0 Loss component

Consider the expression for LMP again, from (25)

$$k \in load: \qquad LMP_k = \lambda + \lambda \frac{\partial P_{loss}}{\partial P_{dk}} + \sum_{j=1}^{M} \mu_j t_{jk} \qquad (25)$$

Assuming no congestion, we have

$$k \in load: \qquad LMP_k = \lambda + \lambda \frac{\partial P_{loss}}{\partial P_{dk}} \qquad (36)$$

When we increase the demand at bus *k* by one unit, the losses will increase due to more current flowing through the network. Therefore the term $\frac{\partial P_{loss}}{\partial P_{dk}}$ will be positive. This results in each bus seeing a higher LMP than that set by the energy component λ.

For a particular bus *k*, the increase in LMP_k beyond λ will depend on how an increase in that buses demand P_{dk} would be compensated. The way it would *really* be compensated is that the marginal unit would increase its generation. This would require $\frac{\partial P_{loss}}{\partial P_{dk}}$ to be recomputed each time the marginal unit changes which is very frequent. What is really done is that $\frac{\partial P_{loss}}{\partial P_{dk}}$ is computed for each bus under an assumed compensation strategy. For example, reference [1][1] shows how to compute $\frac{\partial P_{loss}}{\partial P_{dk}}$ relative to a distributed slack bus reference. We will not cover this but will simply assume the availability of $\frac{\partial P_{loss}}{\partial P_{dk}}$.

9.0 Congestion component

Finally, we reconsider the expression for LMP once again, from (25)

$$k \in load: \qquad LMP_k = \lambda + \lambda \frac{\partial P_{loss}}{\partial P_{dk}} + \sum_{j=1}^{M} \mu_j t_{jk} \tag{25}$$

At this point, our interest is the last term. Let's ignore the losses, resulting in

$$k \in load: \qquad LMP_k = \lambda + \sum_{j=1}^{M} \mu_j t_{jk} \tag{37}$$

The summation in (37) will contain zero terms for all circuits *j* for which flow is not at the rating, i.e., the only non-zero terms in the summation will be for circuits that are at their rating, i.e., that are *congested*. Let's consider that there is only one such circuit in the network, circuit 5. Then

$$k \in load: \qquad LMP_k = \lambda + \mu_5 t_{5k} \tag{38}$$

[1] I have placed this reference on the web page. It is an excellent paper on LMP calculation.

The Lagrange multiplier (dual variable) μ_5 is on the flow constraint for circuit 5, and it will always be nonnegative. On the other hand, t_{5k}, the generation shift factor, representing the change in flow on circuit 5 for an increase in injection at bus k, may be positive or negative. Thus we see that congestion, although usually increasing LMPs for most buses, can also decrease LMPs under certain conditions.

We will study the effects of congestion on LMPs in some depth in the next set of notes.

---

[1] Eugene Litvinov, Tongxin Zheng, Gary Rosenwald, and Payman Shamsollahi, "Marginal Loss Modeling in LMP Calculation," IEEE Transactions On Power Systems, Vol. 19, No. 2, May 2004.