EE 458, Homework 2: Due Tuesday, September 17, 2019

1. Assume that the fuel inputs R_1 and R_2, in MBTU/hour for units 1 and 2, respectively, are a function of unit MW output powers P_1 and P_2, respectively, and are given as

$$R_1 = 8P_1 + 0.024P_1^2 + 80$$

$$R_2 = 6P_2 + 0.044P_2^2 + 120$$

The minimum and maximum loadings for both units are 20 MW and 100 MW, respectively. All of the following plots should be given as a function of unit output power in MW.

   a. Plot the input-output curve for each unit.
   b. Plot the heat rate curves for each unit.
   c. Plot efficiency for each unit.
   d. Plot the incremental heat rate curves for each unit.
   e. Assume the cost of fuel is $2/MBTU. Plot the cost-rate curves for each unit.
   f. Plot the incremental cost-rate curves for each unit.

2. In class, we computed the emissions per unit electric energy produced by a power plant in lbs/MWhr as:

$$lbs/MWhr = EC \times \frac{3.41}{\eta}$$

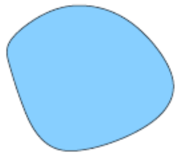
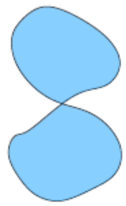
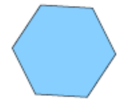
where EC is the CO_2 emissions content in pounds per short ton of the coal and η is the average full load efficiency. For example, for subbituminous coal, we computed:

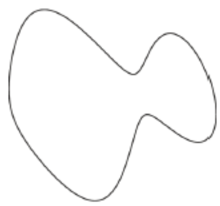
$$212.7\frac{lbs}{MBTU_{IN}} \times \frac{1MBTU_{IN}}{.39MBTU_{OUT}} \times \frac{3.41MBTU}{MWhr} = 1859.8 lbs/MWhr$$

   a. Evaluate the average full load heat rate of the power plant used in the above calculation; make sure to give units.
   b. Repeat the calculation for lbs/MWhr if the average full load heat rate is 10.0 MMBTU/MWhr.
   c. Repeat the calculation for lbs/MWhr using the original heat rate, i.e., the one computed in part (a), but assume that bituminous coal is used.
   d. The Regional Greenhouse Gas Initiative (www.rggi.org) conducts an auction to sell GHG allowances. The clearing price at the June 2011 auction was $1.89/allowance, where 1 allowance is a short ton of CO_2 emitted. Assume
      - the price of subbituminous (Powder River Basin) coal is $15/ton and its energy content is 17.45MMBTU/ton;
      - the price of bituminous coal is $60/ton and its energy content is 24MMBTU/ton.

      For each type, compute the percentage increase in cost of producing a MWhr. Use the original heat rate, i.e., the one computed in part (a).
   e. Go to the RGGI website and identify the clearing price for the 45^{th} RGGI auction on September 4, 2019/allowance.

3. Determine which of the below are convex sets.

a.  b.  c.  d. 

e.  f. 

4. Apply KKT conditions to the following optimization problems. Identify the solution(s) and the value of the objective function at the solution(s). Indicate whether your answer(s) is (are) a global optimum or not and how you know.
    a. Min $f(x_1,x_2)=x_1^2+x_2^2$ subject to $h(x_1,x_2)=x_1+2x_2=5$
    b. Min $f(x_1,x_2)=x_1^2+x_2^2$ subject to $h(x_1,x_2)=3x_1x_2^2=5$
5. A three-unit system is given by the following data. The total system demand is 1100MW. Generator constraints are $0 < P_{g1} < 550$, $0 < P_{g2} < 300$, $0 < P_{g3} < 300$

$$C_1(P_{g1}) = 0.010 \cdot (P_{g1})^2 + 0.3 \cdot (P_{g1}) + 1$$
$$C_2(P_{g2}) = 0.030 \cdot (P_{g2})^2 + 0.2 \cdot (P_{g2}) + 3$$
$$C_3(P_{g3}) = 0.020 \cdot (P_{g3})^2 + 0.9 \cdot (P_{g3}) + 5$$

(a) Identify the objective function for this optimization problem.
(b) Identify the Lagrangian function assuming no constraints are binding.
(c) Identify the KKT conditions assuming no constraints are binding.
(d) Find the solution to the problem assuming no constraints are binding.
(e) Find the solution to the problem accounting for any binding constraints.
(f) Find the total cost of supplying the 1100MW using the solution found in part (e)
(g) Approximate the total cost of supplying the 1100MW change if the upper limit on generator 1 was increased from 550MW to 560MW.