Slope and deflection of a shaft with varying cross-section

1. **Identify all stations**
   a. Every place there is a force, there is a station
   b. Every place there is a step, there is a station
   c. Every place the engineer wants the slope or deflection is a station

2. **Divide each station into regions**
   a. Measuring from the reference end of the shaft divide each station into halves and include two of the same x coordinate values for each station
   b. Number all of the coordinates beginning with 0 and end at N

3. **Determine the moment at every x coordinate**

4. **Determine the 2nd moment of area, I, for each x coordinate**
   a. At steps, I will change

5. **Calculate M/EI for each x coordinate**

6. Now use the following numerical integration scheme (trapezoidal rule) to **determine**

$$\int \frac{M}{EI(x)}dx = f$$

   a. For i = 1, N-2, 2

$$f_{i+2} = f_i + \frac{1}{2}\left(\frac{M_{i+1}}{EI_{i+1}} + \frac{M_i}{EI_i}\right)(x_{i+2} - x_i)$$

   slope

7. **Now use Simpson's Rule to determine** $\int\int \frac{M}{EI(x)}dx = y$

   Deflection

   a. For i = 1,N-4, 4

$$y_{i+4} = y_i + \frac{1}{6}(f_i + 4f_{i+2} + f_{i+4})(x_{i+4} - x_i)$$

8. **Determine constants of integration**
   a. Find two places on the shaft where the deflection is 0.0, x = x_a and x = x_b
   b. $C_1 = \frac{y_b - y_a}{x_a - x_b}$
   c. $C_2 = \frac{x_b y_a - x_a y_b}{x_a - x_b}$
   d. $y = y + C_1 x + C_2$
      $q = f + C_1$