Ground Pivot Specification

$$\left(e^{i\alpha_2}\vec{R}_3 - e^{i\alpha_3}\vec{R}_2\right) - e^{i\beta_2}\left(\vec{R}_3 - e^{i\alpha_3}\vec{R}_1\right) + e^{i\beta_3}\left(\vec{R}_2 - e^{i\alpha_2}\vec{R}_1\right)$$
$$= \left(e^{i\alpha_2}\vec{R}_3 - e^{i\alpha_3}\vec{R}_2\right) + e^{i\beta_2}\left(e^{i\alpha_3}\vec{R}_1 - \vec{R}_3\right) + e^{i\beta_3}\left(\vec{R}_2 - e^{i\alpha_2}\vec{R}_1\right)$$
$$= Ae^{i\theta_A} + e^{i\beta_2}Be^{i\theta_B} + e^{i\beta_3}Ce^{i\theta_C}$$
$$= Ae^{i\theta_A} + Be^{i(\beta_2+\theta_B)} + Be^{i(\beta_3+\theta_C)}$$

Finding **A**

$$\left(e^{i\alpha_2}\vec{R}_3 - e^{i\alpha_3}\vec{R}_2\right)$$
$$\left(R_{3_x} + iR_{3_y}\right)\left(\cos\alpha_2 + i\sin\alpha_2\right) - \left(R_{2_x} + iR_{2_y}\right)\left(\cos\alpha_3 + i\sin\alpha_3\right)$$

Real:

$$R_{3_x}\cos\alpha_2 - R_{3_y}\sin\alpha_2 - R_{2_x}\cos\alpha_3 + R_{2_y}\sin\alpha_3 = A_x$$

Imaginary

$$R_{3_y}\cos\alpha_2 + R_{3_x}\sin\alpha_2 - R_{2_y}\cos\alpha_3 - R_{2_x}\sin\alpha_3 = A_y$$

$$\vec{A} = \sqrt{A_x^{\,2} + A_y^{\,2}}\angle\tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Finding **B**

$$\left(e^{i\alpha_3}\vec{R}_1 - \vec{R}_3\right)$$
$$\left(R_{1_x} + iR_{1_y}\right)\left(\cos\alpha_3 + i\sin\alpha_3\right) - \left(R_{3_x} + iR_{3_y}\right)$$

Real:

$$R_{1_x}\cos\alpha_3 - R_{1_y}\sin\alpha_3 - R_{3_x} = B_x$$

Imaginary

$$R_{1_y}\cos\alpha_3 + R_{1_x}\sin\alpha_3 - R_{3_y} = B_y$$

$$\vec{B} = \sqrt{B_x^{\,2} + B_y^{\,2}}\angle\tan^{-1}\left(\frac{B_y}{B_x}\right)$$

Finding **C**

$$\left(\vec{R}_2 - \vec{R}_1 e^{i\alpha_2}\right)$$

$$\left(R_{2_x} + iR_{2_y}\right) - \left(R_{1_x} + iR_{1_y}\right)\left(\cos\alpha_2 + i\sin\alpha_2\right)$$

$Real:$

$$R_{2_x} - R_{1_x}\cos\alpha_2 + R_{1_y}\sin\alpha_2 = C_x$$

$Imaginary$

$$R_{2y} - R_{1_y}\cos\alpha_2 - R_{1_x}\sin\alpha_2 = C_x$$

$$\vec{C} = \sqrt{C_x^{\ 2} + C_y^{\ 2}} \angle \tan^{-1}\left(\frac{C_y}{C_x}\right)$$

Solving the **non-linear** part of the problem-i.e., solving for β2 and β3 first.

$$Ae^{i\theta_A} + Be^{i\theta_B}e^{i\beta_2} + Ce^{i\theta_C}e^{i\beta_3} =$$

$$Ae^{i\theta_A} + Be^{i(\beta_2+\theta_B)} + Ce^{i(\beta_3+\theta_C)}$$

$Real$

$$A\cos(\theta_A) + B\cos((\theta_B + \beta_2) + C\cos(\theta_C + \beta_3) = 0$$

$Imaginary$

$$A\sin(\theta_A) + B\sin((\theta_B + \beta_2) + C\sin(\theta_C + \beta_3) = 0$$

Square Real and Imaginary and add

$$A^2\cos^2(\theta_A) + B^2\cos^2((\theta_B + \beta_2) + 2AB\cos(\theta_A)\cos(\theta_B + \beta_2) = C^2\cos^2(\theta_C + \beta_3)$$

$$+$$

$$A^2\sin^2(\theta_A) + B^2\sin^2((\theta_B + \beta_2) + 2AB\sin(\theta_A)\sin(\theta_B + \beta_2) = C^2\sin^2(\theta_C + \beta_3)$$

$$A^2\left(\cos^2(\theta_A) + \sin^2(\theta_A)\right) + B^2\left(\cos^2((\theta_B + \beta_2) + \sin^2((\theta_B + \beta_2)\right)$$

$$+2AB\left(\cos(\theta_A)\cos(\theta_B + \beta_2) + \sin(\theta_A)\sin(\theta_B + \beta_2)\right) =$$

$$C^2\left(\cos^2(\theta_C + \beta_3) + \sin^2(\theta_C + \beta_3)\right)$$

$$A^2 + B^2 + 2AB\cos\left(\mp(\theta_B + \beta_2) \pm \theta_A\right) = C^2$$

Solving for b2

$$\left(\frac{C^2 - A^2 - B^2}{2AB}\right) = \cos\left(\mp\left(q_B + \boldsymbol{b}_2\right) \pm q_A\right)$$

$$\cos^{-1}\left(\frac{C^2 - A^2 - B^2}{2AB}\right) = \mp\left(q_B + \boldsymbol{b}\right)_2 \pm q_A$$

$$\boldsymbol{b}_2 = \pm\cos^{-1}\left(\frac{C^2 - A^2 - B^2}{2AB}\right) - q_B + q_A$$