Stresses due to Combined Loading

$$s = \frac{My}{I}$$

normal stress

$$t = \frac{VQ}{It}$$

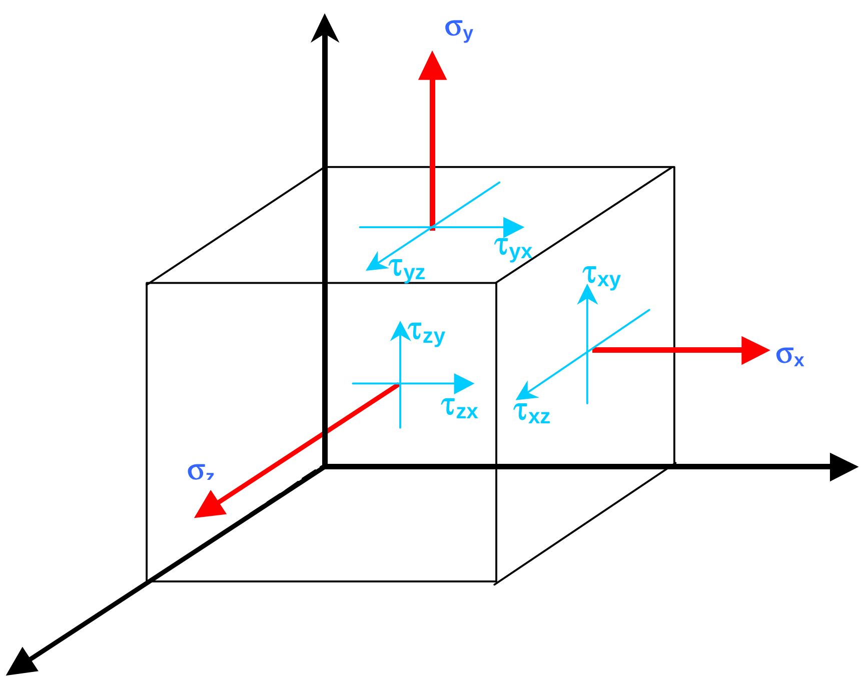
shear due to bending

$$t = \frac{M_t r}{J}$$

shear due to applied torque

Q is the first moment of area $Q = \int_A y dA$;

Stress at a point



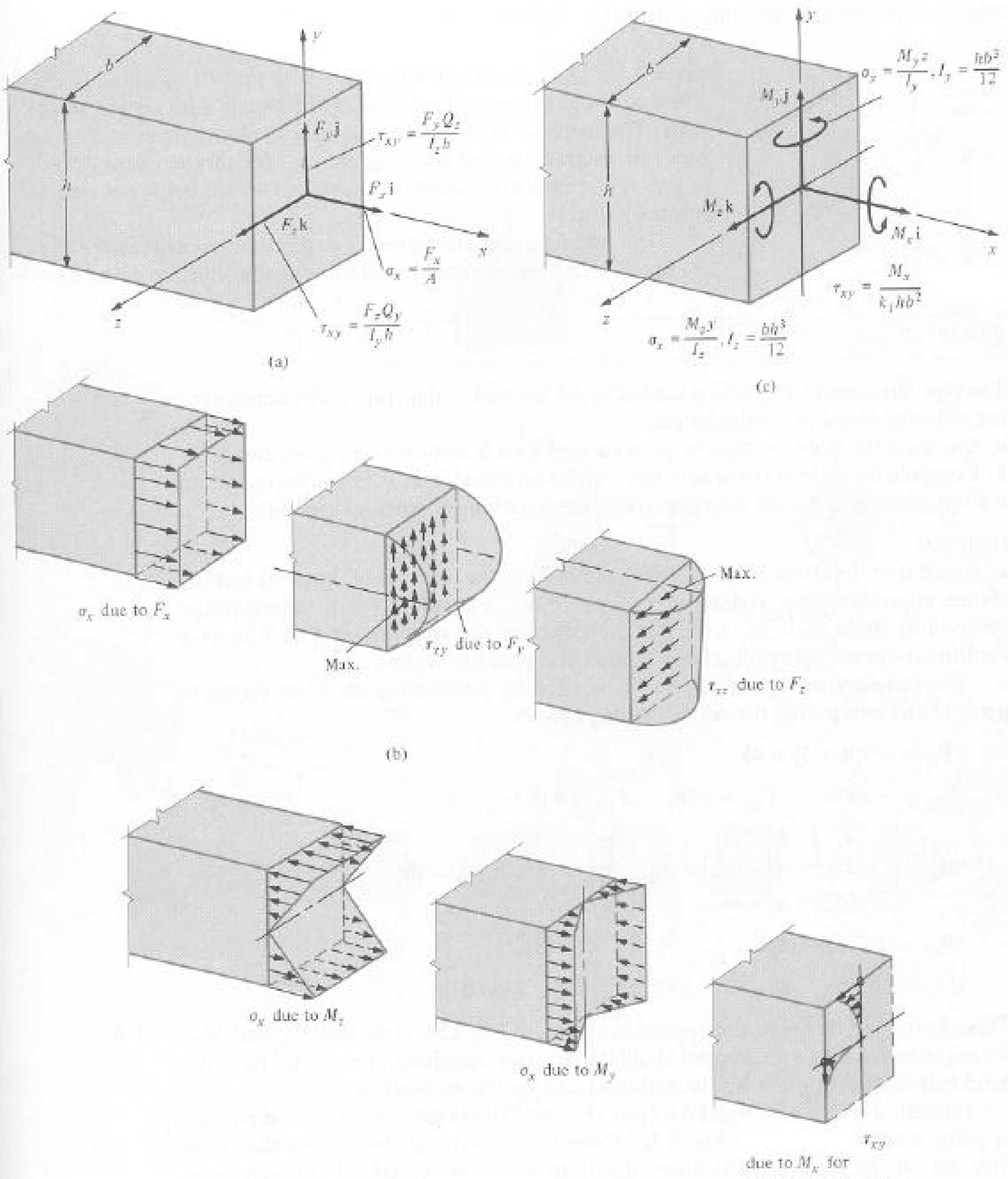
Mathematical representation: **Stress Tensor**

$$s = \begin{pmatrix} s_{xx} & t_{xy} & t_{xz} \\ t_{yxy} & s_{yy} & t_{yz} \\ t_{zx} & t_{zy} & s_{zz} \end{pmatrix}$$

To solve for stress at a point

1. Draw a free body of the cut section and identify all forces and moments acting at that section
2. compute stresses

The principle of superposition holds for stresses, i.e., stresses are additive.



y
b
h
$F_y\mathbf{j}$
$\tau_{xy} = \frac{F_y Q_z}{I_z b}$
$F_x\mathbf{i}$
$F_z\mathbf{k}$
x
$\sigma_x = \frac{F_x}{A}$
z
$\tau_{xy} = \frac{F_z Q_y}{I_y h}$
(a)
$\sigma_y = \frac{M_y z}{I_y}, I_y = \frac{hb^3}{12}$
$M_y\mathbf{j}$
$M_z\mathbf{k}$
$M_x\mathbf{i}$
$\tau_{xy} = \frac{M_x}{k_1 hb^2}$
$\sigma_x = \frac{M_z y}{I_z}, I_z = \frac{bh^3}{12}$
(c)
σ_x due to F_x
Max.
τ_{xy} due to F_y
Max.
τ_{xz} due to F_z
(b)
σ_x due to M_z
σ_x due to M_y
τ_{xy}
due to M_x for
noncircular members

Principal stresses

Normal and shear stresses vary with direction of the coordinate system. There are planes on which the shear stress components are zero. The normal stresses on these planes are called **principal stresses** and the planes themselves are called **principal planes**. The **principal shear stresses** are at 45° angles to the planes of the principal normal stresses.

The objective of the machine designer is to find (or predict) the largest stresses at critical sections of machine components.

We can use the following system of equations to determine principal stresses:

$$\begin{bmatrix} s_x - s & t_{xy} & t_{xz} \\ t_{yx} & s_y - s & t_{yz} \\ t_{zx} & t_{zy} & s_z - s \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Find where the determinant of the 3 x 3 matrix is 0.0

n_x, n_y, and n_z are **direction cosines**.

There will be three roots that will make the determinant of the above matrix 0.0; those three roots are σ_1, σ_2, and σ_3—the **principal stresses**. $\sigma_1 > \sigma_2 > \sigma_3$

The **principal shear stresses** are determined as follows:

$$t_{13} = \frac{|s_1 - s_3|}{2}$$

$$t_{21} = \frac{|s_2 - s_1|}{2}$$

$$t_{32} = \frac{|s_3 - s_2|}{2}$$

τ_{13} is the **maximum shear stress**.

For the 2-D case, or where $\sigma_z = 0.0$, the principal stresses are:

$$s_1, s_{21} = \frac{s_x + s_y}{2} \pm \sqrt{\left(\frac{s_x - s_y}{2}\right)^2 + t^2{}_{xy}}$$

Example