Lateral Vibration of Shafts

Potential Energy = Kinetic Energy

$$\frac{g}{2}\left(m_1 d_1 + m_2 d_2 + m_3 d_3\right) = \frac{w_n}{2}\left(m_1 d_1^{\,2} + m_2 d^2{}_2 + m_3 d_3^{\,2}\right)$$

$$w_n = \sqrt{g\frac{\sum_i^n m_i d_i}{\sum_i^n m_i d^2{}_i}} = \sqrt{g\frac{\sum_i^n \frac{W_i}{g} d_i}{\sum_i^n \frac{W_i}{g} d^{\,2}{}_i}} = \sqrt{g\frac{\sum_i^n W_i d_i}{\sum_i^n W_i d^2{}_i}}$$

Includes only gravitational loads, not externally applied loads

Shaft whirl:

$$\frac{d}{e} = \frac{\left(w/w_n\right)^2}{1-\left(w/w_n\right)^2}$$

e = eccentricity --true mass center away from axis of shaft

What happens at $\omega = \omega n$?

Lateral vibration is forced, shaft whirl is self-excited.