Analytical Mechanism Synthesis

Ground Pivot Specification

Analysis procedures for motion generation using ground pivot specification are more involved than that for motion generation without ground pivot specification.   You will recall that this was the case for graphical synthesis as well (inversion was required for ground pivot specification).

Again we will design one dyad at a time. The vector equations we will use are as follows:

$$\vec{W} + \vec{Z} = \vec{R}_1$$
$$\vec{W}e^{i\beta_2} + \vec{Z}e^{i\alpha_2} = \vec{R}_2$$
$$\vec{W}e^{i\beta_3} + \vec{Z}e^{i\alpha_3} = \vec{R}_3$$

R$_1$, **R**$_2$, and **R**$_3$ are measured from the specified ground pivot. **W** and **Z** vectors are the two vectors making up the left dyad.

The set of equations given above represents a three-position problem. When we synthesized a motion generation mechanism before, without picking ground pivot points, we ended up with linear equations. You will recall that we had two free choices and we elected to pick β2 and β3, angular displacements of the **W** vector. However, with ground pivot specification, we have already made our two free choices-- R_{1x}, and R_{1y}. This means of course, we have non-linear equations to deal with. As you have learned (hopefully!) throughout the semester, it is sometimes possible to get closed form solutions for non-linear equations, and sometimes not possible. In this case, closed form solutions for the unknowns (β2, β3, **W**, and **Z**) are possible--but not easy nor straight forward to get!

The procedure for closed form solutions follows--watch carefully, as there are number of caveats that if missed, make the solution difficult to see.

First, notice that this problem is linear in **W** and **Z** and non-linear in β2 and β3. Realizing this, the system of equations can be written as follows:

$$\begin{bmatrix} 1 & 1 & \vec{R}_1 \\ e^{i\beta_2} & e^{i\alpha_2} & \vec{R}_2 \\ e^{i\beta_3} & e^{i\alpha_3} & \vec{R}_3 \end{bmatrix} \begin{Bmatrix} \vec{W} \\ \vec{Z} \\ -1 \end{Bmatrix} = \{\vec{0}\}$$ Isn't this true?

The only way for this statement to be true is if the matrix is singular, or phrased another way, if the determinant of the matrix is 0.0.

The determinant of the matrix is (expanding about the first column)

$$\left(e^{ia_2}\vec{R}_3 - e^{ia_3}\vec{R}_2\right) - e^{ib_2}\left(\vec{R}_3 - e^{ia_3}\vec{R}_1\right) + e^{ib_3}\left(\vec{R}_2 - e^{ia_2}\vec{R}_1\right)$$
$$= \left(e^{ia_2}\vec{R}_3 - e^{ia_3}\vec{R}_2\right) + e^{ib_2}\left(e^{ia_3}\vec{R}_1 - \vec{R}_3\right) + e^{ib_3}\left(\vec{R}_2 - e^{ia_2}\vec{R}_1\right)$$
$$= Ae^{iq_A} + e^{ib_2}Be^{iq_B} + e^{ib_3}Ce^{iq_c}$$
$$= Ae^{iq_A} + Be^{i(b_2+q_B)} + Be^{i(b_3+q_C)}$$

β2 and β3 are unknown. How would you solve for these two unknowns? Have you seen a similar vector equation before? Think back to the closed form solutions for the 4-bar and see is you can devise a way to solve for β2 and β3.

Now that β2 and β3 are known, we can solve for **W** and **Z** using any two of the three vector equations given earlier. Finding **W** and **Z** will much easier as the equations are now linear!