Rules of Thumb for Shaft Design

- Keep shafts as short as possible
- Bearings should be close to applied loads
- Place any stress risers away from highly stressed regions of the shaft (local strengthening at risers may be helpful)
- Consider hollow shafts when weight is critical (weight to stiffness ratio should be low)
- For deflection concerns, choose a low carbon steel (why?)
- The first natural frequency of the shaft should be three times as high as the highest forcing frequency expected in service.

Design of Shafts based on loading modes

Fully reversed bending and constant Torsion

Shear stress due to torsion is steady—there will be a **mean component of shear**, **but not an alternating component of shear**

$$T_{\max} = T_{\min} = T$$

$$t_m = \frac{T_{\max}\, {}^{d}\!/_{2}(32) + T_{\min}\, {}^{d}\!/_{2}(32)}{pd^4}$$

$$= \frac{T(16) + T(16)}{pd^3} = \frac{32T}{pd^3}$$

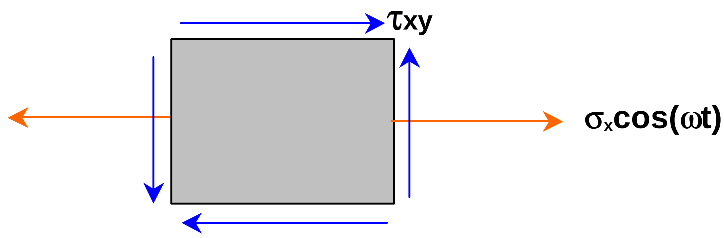
$$t_a = \frac{T_{\max}\, {}^{d}\!/_{2}(32) - T_{\min}\, {}^{d}\!/_{2}(32)}{pd^4}$$

$$= 0$$

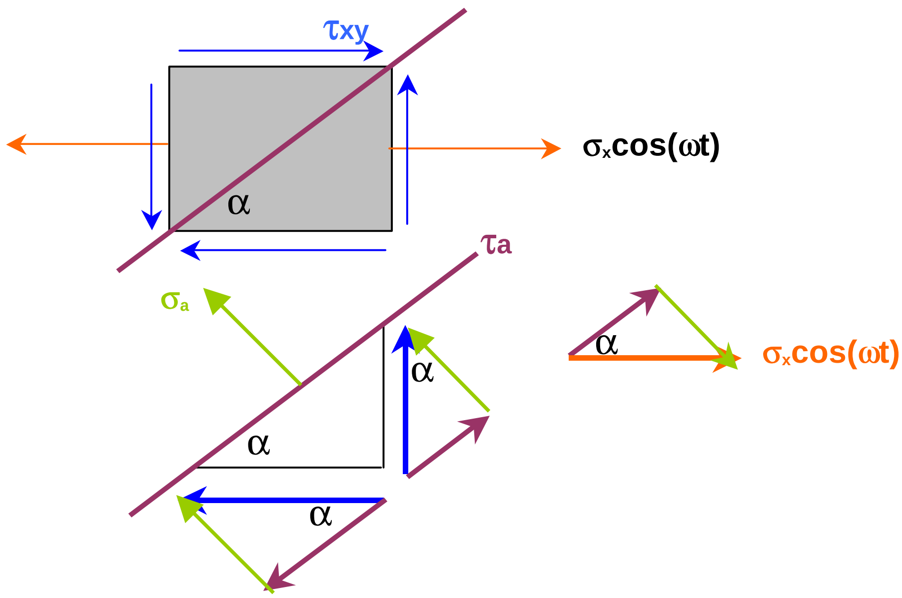
Stresses due to bending will yield an **alternating component**, **but not a mean moment**—see if you can convince yourself of this.

What happens if begin rotating the shaft at constant angular velocity of ω rads/s?

If the shaft is subjected to bending, an element will see tension and compression in a single revolution.



If we want to determine along which planes this element will fail, we can cut a slice through the element at angle α.



Now, add up all the forces in the τ_a direction :

$$t_a + t_{xy}\sin(a)\sin(a) - t_{xy}\cos(a)\cos(a) + s_x\sin(a)\cos(a)$$

$$t_a = -t_{xy}\sin(a)\sin(a) + t_{xy}\cos(a)\cos(a) - s_x\sin(a)\cos(a)$$

$$\cos^2(a) - \sin^2(a) = \cos(2a)$$

$$\sin(a)\cos(a) = \frac{\sin(2a)}{2}$$

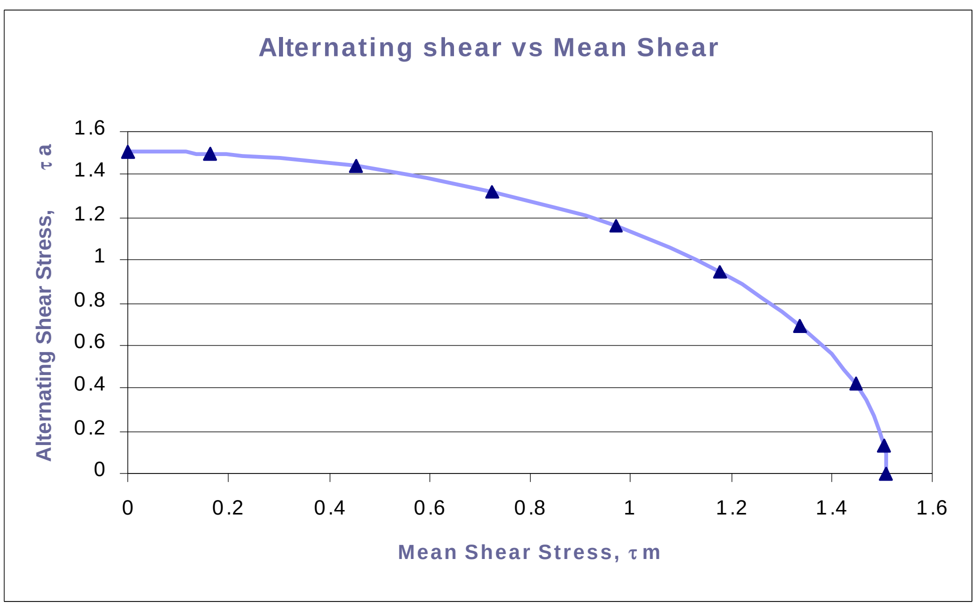
$$t_a = t_{xy}\cos(2a) - s_x\frac{\sin(2a)}{2}$$

$$t_a = \frac{T_m\, d\!/\!_2(32)}{pd^4}\cos(2a) - \frac{M_a\, d\!/\!_2(64)}{pd^4}\frac{\sin(2a)}{2}\cos(wt)$$

$$t_a = \frac{T_m(16)}{pd^3}\cos(2a) - \frac{M_a(16)}{pd^4}\sin(2a)\cos(wt)$$

The mean component of shear is 16 $T_m \cos(2\alpha)/(\pi\ d^3)$
The alternating component of shear is 16 Ma $\sin(2\alpha)/(\pi\ d^3)$

What happens when we plot the alternating component of shear vs. the mean component of shear, while varying α?



How do we know where we are safe against failure?

We will use a line parallel to the Goodman line and tangent to the ellipse created by varying α.

It can be shown that the angle a is:

$$a = \frac{1}{2}\tan^{-1}\left(\frac{M_a S_{ut}}{T_m S_f}\right)$$

Using the Maximum shearing stress theory, we can express the maximum shears in terms of σa and σm

$$s_a = 2t_a$$
$$= \frac{32M_a}{pd^3}\sin(2a)$$
$$s_m = 2t_m$$
$$= \frac{32T_m}{pd^3}\cos(2a)$$

The **load line** is then:

$$load\ \ line\ \ ratio = \frac{\frac{32M_a}{pd^3}\sin(2a)}{\frac{32T_m}{pd^3}\cos(2a)}$$
$$\frac{M_a\sin(2a)}{T_m\cos(2a)}$$
$$= \frac{M^2{}_a S_{ut}}{T^2{}_m S_e}$$

The diameter of the shaft can be shown to be:

$$d = \left\{ \frac{32n}{p} \left[\left(\frac{M_a}{S_f} \right)^2 + \left(\frac{T_n}{S_{ut}} \right)^2 \right]^{1/2} \right\}^{1/3}$$

including fatigue factors

$$d = \left\{ \frac{32n}{p} \left[\left(\frac{K_f \, M_a}{S_f} \right)^2 + \left(\frac{T_n}{S_{ut}} \right)^2 \right]^{1/2} \right\}^{1/3}$$

Example:

Given the following information, compute a safe diameter, d, the load line ratio, and the angle α corresponding to the location of the tangent to an ellipse indicating angles along which shear failures will occur.

The bending moment is fully reversed and its value is **1265 in-lb**. The torque between bearings at the critical section is **3300 in-lb** and it is constant. The fatigue strength , Sf, is 24 ksi, and the ultimate tensile strength, Sut, is 80 ksi. The factor of safety, n, is 1.80. The fatigue stress concentration factor, Kf, for the shoulder of the shaft at the critical section is 1.90