Shear, Moments, and Deflection Curves

In designing machine components, stresses as well as deflections should be determined to predict and subsequently avoid failure.

There exist functional relationships between **the curvature** of a beam subjected to bending and the resulting deflection.

$$\frac{1}{r} = \frac{M}{EI}$$

where ρ is the **radius of curvature** of the beam.

From calculus, we determined that the **curvature** of a plane is:

$$\frac{1}{r} = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

y is the deflection of the beam. The **slope** of the beam at x is

$$q = \frac{dy}{dx}$$

<u>If the slope is very small</u> then the curvature is:

$$\frac{1}{r} = \frac{\frac{d^2y}{dx^2}}{[1+0]^{3/2}} = \frac{d^2y}{dx^2}$$

Now,

$$\frac{1}{r} = \frac{d^2y}{dx^2} = \frac{M}{EI}$$

The **shear** can be shown to be:

$$V = \frac{dM}{dx}$$

So,

$$V = \frac{d}{dx}\left(\frac{1}{EI}\frac{d^2y}{dx^2}\right) = \frac{1}{EI}\frac{d^3y}{dx^3}$$

The **load intensity, q**, any point is

$$q = \frac{dV}{dx} = \frac{d^2M}{dx^2} = \frac{1}{EI}\frac{d^4y}{dx^4}$$

So, given the loading on a beam, we can determine:

The loading intensity, q
The shear, V
The Moment, M
The slope, θ
The deflection, y

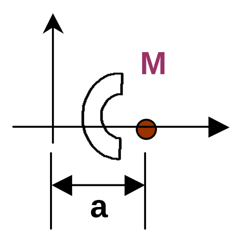
Singularity Functions

Instead of using free body diagrams for each section of the beam, singularity functions can be used.

Moment

$$\langle x-a\rangle^{-2}$$

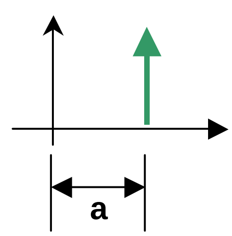
$$\begin{cases} x<a = 0 \\ x \ge a = \langle x-a\rangle^{-2} \end{cases}$$



Force

$$\langle x-a\rangle^{-1}$$

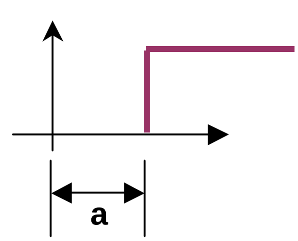
$$\begin{cases} x<a = 0 \\ x \ge a = \langle x-a\rangle^{-1} \end{cases}$$



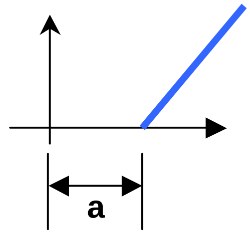
Unit Step

$$\langle x-a\rangle^{0}$$

$$\begin{cases} x<a = 0 \\ x \ge a = \langle x-a\rangle^{0} \end{cases}$$

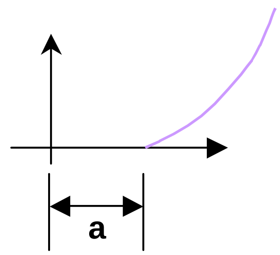


Ramp



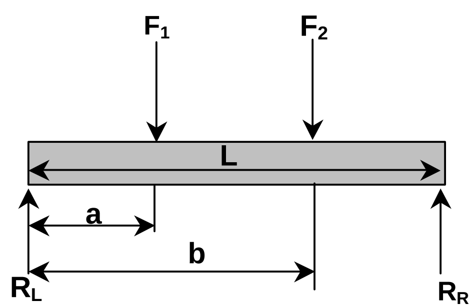
$$\langle x-a\rangle^{1}$$
$$\begin{cases} x<a = 0 \\ x\ge a = \langle x-a\rangle^{1} \end{cases}$$

Parabolic



$$\langle x-a\rangle^{2}$$
$$\begin{cases} x<a = 0 \\ x\ge a = \langle x-a\rangle^{2} \end{cases}$$

Example:



$$q(x) = R_L\langle x\rangle^{-1} - F_1\langle x-a\rangle^{-1} - F_2\langle x-b\rangle^{-1} + R_R\langle x-L\rangle^{-1}$$

$$V(x) = \int q(x) = \frac{1}{EI}\left(R_L\langle x\rangle^{0} - F_1\langle x-a\rangle^{0} - F_2\langle x-b\rangle^{0} + R_R\langle x-L\rangle^{0}\right)$$

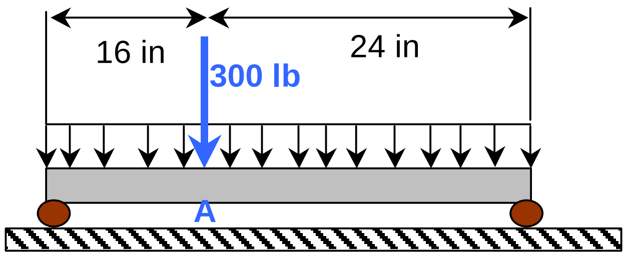
$$M(x) = \int V(x) = \frac{1}{EI}\left(R_L\langle x\rangle^{1} - F_1\langle x-a\rangle^{1} - F_2\langle x-b\rangle^{1} + R_R\langle x-L\rangle^{1}\right)$$

$$q(x) = \int M(x) = \frac{1}{EI}\left(\frac{R_L\langle x\rangle^{2}}{2} - \frac{F_1}{2}\langle x-a\rangle^{2} - \frac{F_2}{2}\langle x-b\rangle^{2} + \frac{R_R}{2}\langle x-L\rangle^{2} + C_1\right)$$

$$y(x) = \int q(x) = \frac{1}{EI}\left(\frac{R_L\langle x\rangle^{3}}{6} - \frac{F_1}{6}\langle x-a\rangle^{3} - \frac{F_2}{6}\langle x-b\rangle^{3} + \frac{R_R}{6}\langle x-L\rangle^{3} + C_1x + C_2\right)$$

In class assignment :

Find the deflection in the of the steel shaft at A.   Also, find the deflection at midspan. The shaft has a diameter of 1 ½ inches. The distributed load is 12 lb/in.



In – Class Assignment

Determine the maximum deflection of the beam shown below. The shaft has a diameter of 2 inches.