Review of calculating deflections in beams.

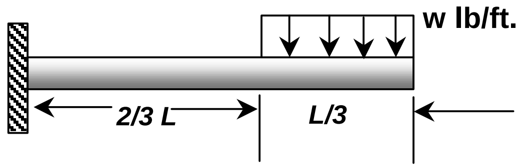
Method of Integration
Method of Superposition

Method of Integration

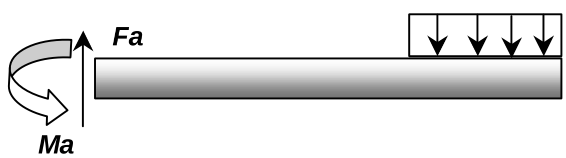
1. Select interval(s) to be used; place co-ordinate system at end of interval
2. Indicate range of interval (i.e., $0 \leq x \leq L$)
3. List all available boundary conditions in each interval
4. Solve $M(x) = EI\dfrac{d^2y}{dx^2}$ for slope and deflection
5. Use boundary conditions to determine constants of integration

Example:

Determine the deflection of the right-hand end of a beam loaded and supported as shown:



Free Body diagram (entire beam)



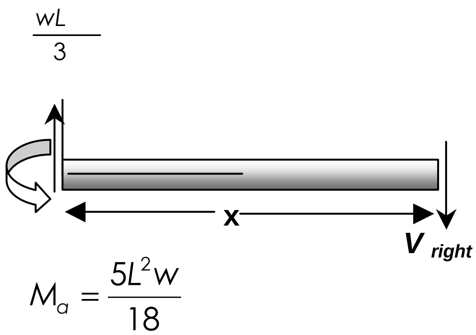
$$\sum F_y = F_a - w(\frac{L}{3})$$

$$\sum M = w\left(\frac{L}{3}\right)\left(\frac{2}{3} + \frac{1}{6}\right)L - M_a$$

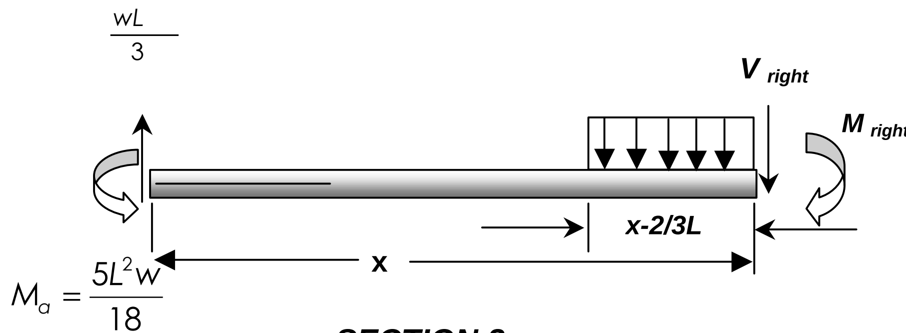
$$F_a = w\frac{L}{3}$$

$$M_a = w\frac{5L^2}{18}$$

Interval 1 ($0 \leq x < 2/3L$)



SECTION 1



SECTION 2

What are the **boundary conditions**?

At the left end, the slope, θ, dy/dx, and the deflection, y, are both 0.0

Where the sections meet, conditions must match. For example where section 1 and section 2 interface, the slope at the right end of section one, must match the slope at the left end of section 2.

Section 1

$$EI\frac{d^2y}{dx^2} = -\frac{5wL^2}{18} + \frac{wL}{3}x$$

$$EI\frac{dy}{dx} = -\frac{5wL^2}{18}x + \frac{wL}{6}x^2 + C_1$$

$$EIy = -\frac{5wL^2x^2}{36} + \frac{wL}{18}x^3 + C_1x + C_2$$

$$Y(0) = 0;\ C_2 = 0$$

$$\frac{dy}{dx}(0) = 0;\ C_1 = 0$$

Section 2

$$EI\frac{d^2y}{dx^2} = -\frac{5wL^2}{18} + \frac{wL}{3}x - w\left(x - \frac{2}{3}L\right)\left(\frac{x - \frac{2}{3}L}{2}\right)$$

$$EI\frac{dy}{dx} = -\frac{5wL^2}{18}x + \frac{wL}{6}x^2 - \frac{w}{6}\left(x - \frac{2L}{3}\right)^3 + C_3$$

$$EI\,y = -\frac{5wL^2x^2}{36} + \frac{wL}{18}x^3 - \frac{w}{24}\left(x - \frac{2L}{3}\right)^3 + C_3x + C_4$$

$$\frac{dy}{dx}\left(\frac{2}{3}L\right) = -\frac{5wL^2}{18}x + \frac{wL}{6}x^2 - \frac{w}{6}\left(x - \frac{2L}{3}\right)^3 + C_3 = -\frac{5wL^2}{18}x + \frac{wL}{6}x^2$$

$$C_3 = 0;$$

$$y\left(\frac{2}{3}L\right) = -\frac{5wL^2x^2}{36} + \frac{wL}{18}x^3 - \frac{w}{24}\left(x - \frac{2L}{3}\right)^4 + C_4 = -\frac{5wL^2x^2}{36} + \frac{wL}{18}x^3$$

$$C_4 = 0;$$

The deflection at the end of the beam is:

$$y(L) = -\frac{5wL^2(L)^2}{36} + \frac{wL}{18}(L)^3 - \frac{w}{24}\left(\frac{L}{3}\right)^4$$