Polynomial Cams

It is possible to design a cam profile that will give us precisely the features we desire in kinematic behavior at the start and end of a cycle of cam rotation using polynomial cams (there are trade-offs too, however).

$$s = a_o + a_1\left(\frac{q}{b}\right) + a_2\left(\frac{q}{b}\right)^2 + a_3\left(\frac{q}{b}\right)^3 + a_4\left(\frac{q}{b}\right)^4 + a_5\left(\frac{q}{b}\right)^5$$

This particular polynomial requires the engineer to specify 6 different boundary conditions.

1. At $\theta = 0$, s = 0
2. At $\theta = 0$, s' = 0
3. At $\theta = 0$, s'' = 0
4. At $\left(\frac{q}{b}\right)=1$, s = h
5. At $\left(\frac{q}{b}\right)=1$, s'=0
6. At $\left(\frac{q}{b}\right)=1$, s''=0

How would you go about finding the polynomial coefficients? We need 6 equations. The 6 boundary conditions required, will yield the 6 equations needed.

$$s = a_o + a_1\left(\frac{q}{b}\right) + a_2\left(\frac{q}{b}\right)^2 + a_3\left(\frac{q}{b}\right)^3 + a_4\left(\frac{q}{b}\right)^4 + a_5\left(\frac{q}{b}\right)^5$$

The position equation at $\theta = 0°$ and $\left(\frac{q}{b}\right)=1$, s=h, gives us two equations.

$$s' = a_1\left(\frac{1}{b}\right) + 2a_2\frac{q}{b^2} + 3a_3\frac{q^2}{b^3} + 4a_4\frac{q^3}{b^4} + 5a_5\frac{q^4}{b^5}$$

$$s'' = 2a_2\frac{1}{b^2} + 6a_3\frac{q}{b^3} + 12a_4\frac{q^2}{b^4} + 20a_5\frac{q^3}{b^5}$$

The velocity equation at $\theta = 0°$ and $\left(\frac{q}{b}\right)=1$, s'=0, gives us two more equations.

The acceleration equation at $\theta = 0°$ and $\left(\frac{q}{b}\right)=1$, s'''=0, gives us the remaining two equations.

Putting in the first set of boundary conditions gives a0, a1, and a2 = 0. The second set of conditions, s = h at θ/β = 1; s'=0 at θ/β = 1 and s'' = 0 at θ/β = 1 gives the following 3 equations to solve for a3, a4, and a5.

$$s = h = a_3(1)^3 + a_4(1)^4 + a_5(1)^5$$

$$s' = 0 = 3a_3\frac{1^2}{b} + 4a_4\frac{1^3}{b} + 5a_5\frac{1^4}{b}$$

$$s'' = 0 = 6a_3\frac{1}{b^2} + 12a_4\frac{1^2}{b^2} + 20a_5\frac{1^3}{b^2}$$

Writing these three equations in matrix form, gives:

$$\begin{bmatrix} 1 & 1 & 1 \\ \frac{3}{b} & \frac{4}{b} & \frac{5}{b} \\ \frac{6}{b^2} & \frac{12}{b^2} & \frac{20}{b} \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} h \\ 0 \\ 0 \end{Bmatrix}$$

Solving for ao, a1, and a2 gives:

a0 = 10h
a1 = -15h
a3 = 6h

The final polynomial equation for follower displacement is:

$$s = 10h\left(\frac{q}{b}\right)^3 - 15h\left(\frac{q}{b}\right)^4 + 6h\left(\frac{q}{b}\right)^5$$