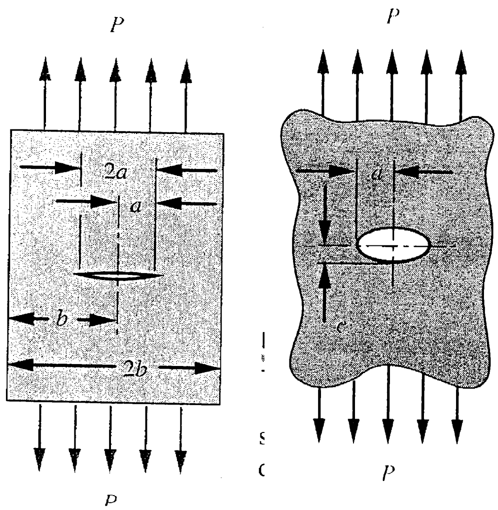
Fracture Mechanics

When a statically loaded part fails under stresses below those of its material strength, then it can be assumed that the failure resulted due to a pre-existing crack in the part. The science of fracture mechanics assumes that all parts contain cracks and that machine components can be designed to account for this.

An important assumption of Linear Elastic Fracture Mechanics is that the **plastic region around the crack itself is very small relative to the remainder of the part**. In other words, most of the material, with the exception of the zone around the crack, will behave according to Hooke's Law. If this assumption cannot be made, then the following discussion is not valid.

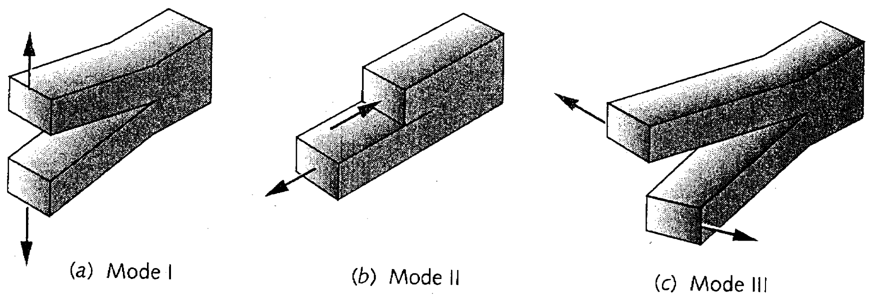
In the accompanying picture, a crack of elliptical shape is shown in a plate. The major radius, a, and minor radius, c, are indicated. The theoretical stress concentration factor, Kt, is given by:



$$\mathbf{K_t = 1 + 2(a/c).}$$

It is clear from this formula, that as c becomes smaller and smaller, that K_t becomes larger and larger. You recall from an earlier exercise that the nominal stress of a part, multiplied by Kt, yields the maximum stress. So, as c becomes very small the stress in the region of the crack becomes increasingly large. The crack becomes more and more elongated, i.e., less elliptical, and the crack tip becomes sharper and sharper,

The crack can be displaced either by opening it further in tension, shearing it in plane or shearing it out of plane (Mode I, Mode II, Mode III, respectively).



If the width of the part, b, is >>, a, (a/b≤ 0.4) the width of the crack then the stress intensity factor, K, can be determined for **a plate with a center crack**, from

$$K = s\sqrt{pa}$$

If crack width, a, is not small compared to the width of the part, and/or if the crack is not in the center of the plate, then an additional factor must be considered.

$$K = s\, b\sqrt{pa}$$

β depends on loading, geometry, and the ratio a/b. β for the **centered cracked plate** is:

$$b = \sqrt{\sec\left(\frac{pa}{2b}\right)}$$

If the **crack is on the edge** of the plate and a<<b, then β = 1.12.

To determine if a cracked part is safe, we compare its fracture toughness, Kc, to the stress intensity factor, K

N = Kc/K

Fracture toughness is determined for standard engineering materials and can be acquired from handbooks or vendors.

Example:

A 60 KN axial load is applied to a steel strap that has an edge crack in it. Determine the factor of safety for the strap before and after the edge crack. How big can the crack be before brittle failure? Will heat treating the cracked part help? Why or why not?

S_y = 540 M Pa
K_c = 66 M Pa $m^{(1/2)}$

l = 6 m
b = 80 mm
t = 3 mm
a = 10 mm (crack width)